Understanding Strategic Interaction

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With the help of Martin Strobel

Understanding Strategic Interaction

Essays in Honor of Reinhard Selten

With 86 Figures

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Preface

On the 5th of October, 1995, Reinhard Selten celebrated his 65th birthday. To honor his scientific contributions, his influence and encouragement of other researchers the editors have asked prominent scholars for contributions. We are rather proud of the result, but the reader may judge this for her- or himself. Also on behalf of my co-editors I thank all authors who helped us to honor Reinhard Selten by contributing to this volume.

The main burden (most of the correspondence and all technical work) of editing the volume has rested on the shoulders of Martin Strobel who has been responsible for all technical aspects as well as for part of the correspondence involved. I gratefully acknowledge his kind help and fruitful cooperation, also as an inspiring co-author and as a co-interviewer.

Among my co-editors Eric van Damme has been most helpful by refereeing the papers like the others, but also by approaching authors and especially by interviewing Bob Aumann on the occasion of the Jerusalem Conference in 1995. Without the help and assistance of my co-editors the book would not be as interesting as it is - in my view - now.

Given the prominence of the laureate there was a considerable risk of overburdening the volume. To avoid this it was decided not to ask his present and former colleagues at the universities of Bonn, Bielefeld, Berlin and Frankfurt/M., nor his present and former assistants for contributions. So what the volume tries to accomplish is mainly a recollection of the scientific disputes, in which Reinhard Selten was actively and partly dominantly involved, as well as the methodologal approaches of a core group of researchers who continuously enjoyed the close cooperation with Reinhard Selten.

Initially our attempt was to have separate sections for game theory, applications of game theory, experimental economics and evolutionary game theory. Instead the contributions now are ordered more loosely in this way. The introductory essay offers a finer partitioning and provides some overview and guidance for the more selective readers.

Berlin, 1996

Werner Güth

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Interview with Elisabeth and Reinhard Selten*

Q: You, Mrs Elisabeth Selten, and your husband married in 1959. Do you have any advice for those who also would like to marry a future Nobel-laureate?

E: No! - Do not overestimate it. Often he is physically with you, but not mentally.

Q: You and your husband are both active members of the Esperanto-movement. Actually I think you met for the first time at an Esperanto-meeting.

E: This is true, yes. I was in Munich at that time and my sister was already in Frankfurt. I visited her and we went to the Esperanto-Club. There I met my husband who was studying in Frankfurt.

Q: Are you disappointed about the success of the Esperanto-idea, e.g. since it is not used as the official language of the United Nations or the European Community?

E: No, I wasn't disappointed because I don't expect it to become accepted that soon. You should not forget that people have to learn the language. It's easy to learn, but still you have to learn.

Q: Your husband contributed to many sciences, most notably to economics, politics, philosophy, psychology and biology. Is there a special field which you personally found the most interesting one?

E: I can only say which was the least interesting one. That was philosophy.

Q: Do you discuss his theoretical ideas with him?

E: To a degree not anymore. Some in biology.

Q: You have translated the collection of papers "Game Theory and Related Approaches to Social Behavior" (1964) edited by Martin Shubik[1] into German. Didn't you also help your husband to perform experiments?

E: Yes, but some time ago. He didn't have any assistance, so I helped him both as an assistant and as a secretary.

Q: You once said that the honorarium per page of translating this book edited by Martin Shubik was close to nothing.

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* The interview took place at the home of Elisabeth and Reinhard Selten on November 27th in 1995. After being typed it was slightly extended and revised by Reinhard Selten. The interviewers were Werner Güth and Martin Strobel.

1 Shubik, Martin (ed.) (1965): Spieltheorie und Sozialwissenschaften, Hamburg, S. Fischer Verlag

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E: It was better than what others got, it was the highest tariff I think. But if you would have to live from it ...! Well, actually it was really work.

Q: Many colleagues often enjoyed your hospitality and your skillful cooking. This must have been quite a burden for you. Didn't you sometimes complain? It also must have been more fun when you personally liked your visitors. Didn't you, since and then, ask to have meetings in the office rather than at your home?

E: No, I never did that. *(To Reinhard Selten)* Or do you remember anything? Sorry, you must ask Reinhard! The people whom I did not like never stayed here for longer. So, this answers the question.

Q: Thankyou, Mrs Selten!

Q: We expect the readers of this volume to be pretty well informed about Reinhard Selten's major contributions. What we would like to illuminate are the driving forces behind your work and how your research interests have developed over time. Let us start with high school. You visited the Gymnasium in Melsungen. Whoever knows you does not expect you to have been a crack in sports. You undoubtedly were more of the reading type. Which kind of literature were you reading mainly?

R: I was reading much world literature, belletristic, but also books about special subjects, nonfiction. And when I was in my second last year of high school I also had a full time job as a librarian in the America-house at Melsungen. At this time I got used to read English. In the library they had many books on different subjects, psychology, economics. - I cannot remember which topic was the most exciting one, I read a lot of books, among others a history of economic thought by Heimann, my first exposure to economic theory.

Q: After high school you have studied mathematics at the university of Frankfurt/M. You rarely showed up for the early morning lectures. Since the later Reinhard Selten hardly ever drops a session of his courses, e.g. by attending conferences, one wonders whether you have changed or whether you think teaching is more fun than being taught?

R: No, actually I don't think teaching is fun. The reason why I don't drop lectures, although I must do it occasionly, is that I think it is my duty to teach. It is not a question whether it's fun or not. This question never occured to me. It is my duty. I'm obliged to do it. This is what I'm paid for. As a student it was not really a duty to go to lectures, students are free to go there or not to go. But as a teacher you can't stay away.

Q: You were not only attending mathematics courses but also those of other disciplines. What were your main interests in addition to mathematics and what do you consider to be the main influences of this time on your later work?

R: I was interested in many subjects, especially psychology. I also took part in experiments both as a subject and as student helper.

Q: Did you sometimes think to change subjects and stop with mathematics?

R: There was a point where I thought about leaving the university. This was before the Vordiplom. I had the impression that I would not pass it. Therefore I went to the SAS. It was my idea to become a navigator. But they were not interested in me as a navigator, they were interested in me as a steward or something. *(Laughter)* No, no, I mean they didn't say what. Maybe also in their administration, I don't know. Maybe because I spoke a little bit Danish.

Q: Is it correct that Ewald Burger, who wrote a remarkable introduction to game theory[2] and who was your thesis advisor, has also been your main teacher in mathematics? Were your conceptual ideas already shaped by Ewald Burger or was he conceptually more neutral, e.g. by viewing cooperative and noncooperative game theory as related, but different fields of applied mathematics?

R: I was very much influenced by Ewald Burger, because he was a very good mathematican with a vast knowledge of the literature and he spent a lot of time talking to me. A that time I was still very young and not yet well trained in reading mathematical texts. It was very difficult for me to understand Shapley's paper on his value[3] and we went over this together. Burger was an excellent teacher, a very, very knowledgeable man. But he did not like conceptual discussions. He did not like to talk about interpretations and did this only where it was absolutely necessary. He did not feel secure about anything not mathematically precise. Later he concentrated on mathematical logic where he also had the same aversion against interpretations. Though I always had the impression that he is interested in game theory, he did not like to talk about the non-mathematical aspects of the field. Non-cooperative and cooperative game theory were not yet distinguished. It was all game theory. Nash introduced the notion of a non-cooperative game but in von Neumann's and Morgenstern's book[4] there is no distinction between cooperative games and non-cooperative games. There is a distinction between 2-person zero-sum games and all other games. It was the idea that 2-person zero-sum games have to be handled distinctly. In all other games there is scope for cooperation. What people largely believed in was not always expressed. It was some kind of "Coase conjecture": If there is scope for cooperation, cooperation will occur. This was the general feeling at this time, at least this is my impression.

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[2] Burger, Ewald (1966): Einführung in die Theorie der Spiele, 2. Aufl., Berlin

[3] Shapley, Lloyd S. (1953): A Value for n-Person Games, in H. W. Kuhn and A. W. Tucker (eds.), Contributions to the Theory of Games, Vol. II, Ann. Math. Stud. 28, Princeton, N.J., Princeton University Press, pp. 307 - 317

[4] von Neumann, John and Oscar Morgenstern (1944): Theory of Games and Economic Behaviour, Princeton University Press

Q: Although your doctoral degree (Doktor phil. nat.) is still in mathematics, you then changed to economics by joining the team of Heinz Sauermann at the economics department in Frankfurt/M. where you initiated the experimental tradition, known as the German School of Experimental Economics. Did you always plan to supplement normative game theory by a behavioral theory of game playing, e.g. due to your experiences with the experimental tradition in (social) psychology?

R: No, when I began to do experiments I was in the process of turning away from "naive rationalism", but I had not yet left this position completely behind me. The "naive rationalist" thinks that people must act rationally and it is only necessary to find out what is the right rational solution concept. At least in cooperative game theory as well as in oligopoly theory there was no agreement about the right rational theory. There were many cooperative theories. The problem of deciding between competing cooperative solution concepts may have induced Kalisch, Milnor, Nash, and Nering[5]to do their experiments on n-person coalition games. I was much impressed by their paper. It was already clear from their results that a purely rational theory is not a good theory in view of what happened in their experiments. Shortly after I had began to run oligopoly experiments I read Herbert Simon and I was immediately convinced of the necessity to build a theory of bounded rationality. At first I still thought it might be possible to do this in an axiomatic way, just by pure thinking, as you build up a theory of full rationality. But over the years I more and more came to the conclusion that the axiomatic approach is premature. Behavior cannot be invented in the armchair, it must be observed in experiments. You first have to know what really happens before forming an axiomatic theory.

Q: Two milestones of your academic career seem to be your contribution to "Advances in Game Theory"[6], and the famous 1965-article[7] where you introduce the concept of subgame perfect equilibria. Do these two papers also reveal a change in your conceptual views, e.g. from cooperative to noncooperative ways of modelling and solving strategic conflicts?

R: Yes and No. When I wrote the paper in 1965, I was really much motivated by my experimental work. This was a paper about a dynamic oligopoly game with demand inertia. We had played a somewhat more complicated game in the laboratory. I was interested in finding a normative solution, at least

[5] Kalisch, G. K., J. W. Milnor, J. Nash, E. D. Nering (1954): Some experimental n-person games. in: Thrall, R. M., Coombs, C. H. and Davis, R. L. (eds.), Decision Processes, New York, Wiley

[6] Selten, Reinhard (1964): Valuation of n-person games. In: Advances in Game Theory, Annals of Mathematics Studies no. 52, Princeton, N.J., 565 - 578

[7] Selten, Reinhard (1965): Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit, Teil I: Bestimmung des dynamischen Preisgleichgewichts, Teil II: Eigenschaften des dynamischen Preisgleichgewichts, JITE (Zeitschrift für die gesamte Staatswissenschaft) 121, 301 - 324, 667 - 689

for a simplified version. The reason why I took a non-cooperative approach goes back to the early oligopoly experiments by Sauermann and myself[8]. In the fifties many oligopoly theorists believed in tacit cooperation. The idea was that oligopolists would send signals to each other by their actions - they would speak a language, someone called "oligopolese" - and thereby arrive at a Pareto-optimal constellation. But this did not happen in our experiments. What we observed had much more similarity to Cournot theory. This was confirmed by my second paper on oligopoly experiments. However, in the dynamic game with demand inertia, it was not clear, what is the counterpart of the Cournot-solution. It took me a long time to answer this question. Finally I found an equilibrium solution by backward induction. However while I already was in the process of writing the paper, I became aware of the fact that the game has many other equilibrium points. In order to make precise what is the distinguishing feature of the natural solution I had derived, I introduced subgame perfectness as a general game theoretic concept. This was only a small part of the paper but, as it turned out later, its most important one.

Q: You have met John C. Harsanyi for the first time in 1961. Although there exist only two publications by the two of you[9,10], there has been a continuous exchange of ideas and mutual inspiration. Would you have developed differently, e.g. by concentrating more on your experimental research, without the influence of John C. Harsanyi?

R: It is very hard to say what would have happened. Probably I would have formed a relationship to somebody else and this could have driven me in another direction. What we first did, John Harsanyi and I, was dealing with 2-person bargaining under incomplete information. Then, after we had finished the paper for the fixed threat case[11], it originally was our intention to go on to the variable threat case. However, I proposed that we should first develop a general theory of equilibrium selection in games. This theory should then be applied to the variable threat case. He agreed to proceed in this way and the problem of equilibrium selection kept us busy for decades. Of course, I am much influenced by John Harsanyi. He is older, ten years older than I am. He was more mature when we met and he was a very systematic thinker. He has a clear view of what he wants to achieve. I felt motivated to subordinate my own inclinations to his aims as a system builder.Without his influence I

[8] Sauermann, Heinz and Reinhard Selten (1959): Ein Oligopolexperiment, Zeitschrift für die Gesamte Staatswissenschaft, 115, pp. 417 - 471

[9] Harsanyi, John C. and Reinhard Selten (1972): A generalized Nash-solution for 2-person bargaining games with incomplete information, Management Science 18, 80 - 160

[10] Harsanyi, John C. and Reinhard Selten (1988): A general theory of equilibrium selection in games, M.I.T. Press, Cambridge Mass

[11] Harsanyi, John C. and Reinhard Selten (1972): A generalized Nash-solution for 2-person bargaining games with incomplete information, Management Science 18, 80 - 160

might have worked less on full rationality and more on bounded rationality. As you probably know I am a methodological dualist and I think that both descriptive and normative theory are important. I am intringued by problems in both areas.

Q: You have impressed many scholars and shaped their way of studying social interaction, regardless whether they rely on the game theoretic or the behavioral approach or even on evolutionary stability. Who has impressed the Nobel-laureate Reinhard Selten?

R: Of course, I was impressed by my teacher Ewald Burger. And then as game theorists von Neumann and Morgenstern were very important. I knew Morgenstern personally. He visited Frankfurt sometimes. We had some discussions because he always wanted to talk to me when he came to Frankfurt. I didn't know von Neumann. Among the game theorists I was always very much impressed by Shapley and also by Aumann. Shapley was a leading, as far as I remember, he was the leading game theorist after von Neumann and Morgenstern. Nash also, but Nash was not available at this time. Nash was not visible. Of course, he was very important for me, both for my work on cooperative and my work on noncooperative theory, but personally it was Shapley and Aumann. There were also many other people, I already mentioned Herbert Simon. But I don't really know him personally, only superficially. You can also say that I was impressed by my students.

Q: You are approaching official retirement although you remain to be the director of the computer laboratory at the university of Bonn. If time allows, what do you still want to accomplish and what do you encourage younger researchers to study most of all?

R: I hope that during my lifetime I will have the opportunity to see the emergence of a strong body of behavioral economic theory which begins to replace neoclassical thinking. In my view experimental and empirically based theoretical work on bounded rationality is an important and fruitful research direction I can recommend to younger colleagues. Of course I myself will also try to achieve further progress in this area.

Q: Let us conclude by asking you what inspired your interests in evolutionary game theory to which you contributed some pioneering and pathbreaking studies.

R: I was lucky enough to come into contact with biological game theory very early in its development. I was fascinated by the subject matter and it became clear to me that there are important open theoretical problems of evolutionary stability in extensive games. I think that I succeeded to contribute something worthwhile in this area. More recently the task of writing

a survey paper[12] induced Peter Hammerstein and myself to look at the population genetic foundations of evolutionary stability. But I also would like to mention that my interests in biological game theory are not entirely concentrated on abstract problems. I enjoy my cooperation with Avi Shmida, a botanist at the Hebrew University of Jerusalem with whom I like to discuss the interaction of flowers and bees.

Q: Thank you for the interview!

[12] Hammerstein, Peter and Reinhard Selten (1994): Game Theory and Evolutionary Biology, in R. I. Aumann and S. Hart (eds.) Handbook of Game Theory, Amsterdam, Elsevier, pp. 929 - 993

On the State of the Art in Game Theory: An Interview with Robert Aumann*

Q: We could take your talk yesterday on "Relationships" as a starting point. That talk is a follow-up to your paper "What is Game Theory Trying to Accomplish"[1], which you presented in Finland some years ago. Selten acted as a discussant for that paper. He agreed with some of the points you made, but disagreed with others. It would be nice to make an inventory of what has happened since that time. Have your opinions gotten closer together or have they drifted further apart? Do you agree that this is a good starting point for the interview?

A: Fine.

Q: Let us recall your position. Yesterday you said that science is not a quest for truth, but a quest for understanding. The way you said this made it clear that you have to convince people of this point; not everybody agrees with your point of view, maybe even the majority does not agree. Is that true?

A: You are entirely right. The usual, naive, view of science is that it is a quest for truth, that there is some objective truth out there and that we are looking for it. We haven't necessarily found it, but it is there, it is independent of human beings. If there were no human beings, there would still be some kind of truth. Now I agree that without human beings there would still be a universe, but not the way we think of it in science. What we do in science is that we organize things, we relate them to each other. The title of my talk yesterday was "Relationships". These relationships exist in the minds of human beings, in the mind of the observer, in the mind of the scientist. The world without the observer is chaos, it is just a bunch of particles flying around, it is "Tohu Vavohu" — the biblical description of the situation before the creation[2]. It is impossible to say that it is "true" that there is a gravitational force: the

* This interview took place one day after the close of the "Games '95" Conference, which was held in Jerusalem between June 25 and 29, 1995. There are several references in the interview to events occurring at the conference, in particular to Paul Milgrom's lecture on the spectrum auction on June 29, to Aumann's lecture on "Relationships" on June 29, and to Aumann's discussion of the Nobel prize winners' contributions on Sunday evening, June 26. The interviewer was Eric van Damme, who combined his own questions with written questions submitted by Werner Güth. Most of the editing of the interview was done at the State University of New York at Stony Brook, with which Aumann has a part-time association. Editing was completed on June 19, 1996.

1 In *Frontiers of Economics*, K. Arrow and S. Honkapohja, eds., Basil Blackwell, Oxford, 1985, 28-76.

2 Genesis 1,2. "Tohu" is sometimes translated as "formless." It seems to me that the idea of "form" is in the mind of the observer; there can be no form without an observer.

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gravitational force is only an abstraction, it is not something that is really out there. One cannot even say that energy is really out there, that is also an abstraction. Even the idea of "particle" is in some sense an abstraction. There certainly are individual particles, but the idea that there is something that can be described as a particle — that there is a class of objects that are all particles — that is already an abstraction, that is already in the mind of the human being. When we say that the earth is round, roundness is in our minds, it does not exist out there, so to speak. In biology, the idea of "species" is certainly an abstraction, and so is that of an individual. So all of science really is in our minds, it is in the observer's mind. Science makes *sense* of what we see, but it is not what is "really" there.

Let me quote from my talk yesterday: Science is a sort of giant jigsaw puzzle. To get the picture you have to fit the pieces together. One might almost say that the picture *is* the fitting together, it is not the pieces. The fitting together is when the picture emerges.

Certainly, that is not the usual view. Most people do not see the world that way. Maybe yesterday's talk gave people something to think about. The paper, "What is Game Theory Trying to Accomplish?" had a focus that is related, but a little different.

Q: What I recall from that paper is that you also discuss "applied" science. You argue that from understanding follows prediction and that understanding might imply control, but I don't recall whether you use this latter word. Perhaps you use the word engineering. Your talk yesterday was confined to understanding: what about prediction and engineering?

A: We heard a wonderful example of that yesterday: Paul Milgrom talking about the work of himself and Bob Wilson and many other game theorists in the United States — like John Riley, Peter Cramton, Bob Weber, and John Macmillan — who were asked by the FCC (Federal Communications Commission) or one of the communications corporations to consult on the big spectrum auction that took place in the US last year, and that netted close to 9 *billion* dollars[3]. This is something big and it illustrates very well how one goes from understanding to prediction and from prediction to engineering. Let me elaborate on that. These people have been studying auctions both theoretically and empirically for many years. (Bob Wilson wrote a very nice survey of auctions for the Handbook of Game Theory[4].) They are theoreticians and they have also consulted on highly practical auction matters. For example, Wilson has consulted for oil companies bidding on off-shore oil tracts worth upwards of 100 million dollars each. These people have been studying auctions very seriously, and they are also excellent theoreticians; they use concepts from game theory to predict how these auctions might come out,

[3] See Milgrom, P. (1996): *Auction Theory for Privatization*, Oxford: Oxford University Press (forthcoming).

[4] Wilson, R. (1992): "Strategic Analysis of Auctions," in *Handbook of Game Theory, Vol. I,* R. Aumann and S. Hart, eds., Amsterdam: Elsevier, 227-279.

and they have gotten a good feel for the relation between theory and what happens in a real auction. Milgrom and Weber[5] once did a theoretical study, looking at what Nash equilibrium would predict on a qualitative basis for how oil lease auctions would work out, and then Hendricks and Porter[6] did an empirical study and they found that a very impressive portion of Milgrom's predictions (something like 7 out of 8) were actually true; moreover these were not trivial, obvious predictions. So these people make predictions, they have a good feel for theory and they have a good feel for how theory works out in the real world. And then they do the engineering, like in the design of the spectrum auction. So one goes from theory, to prediction, to engineering. And as we heard yesterday, this was a tremendously successful venture.

Another example of how understanding leads to prediction and to engineering comes from cooperative game theory; it is the work of Roth and associates on expert labour markets. This is something that started theoretically with the Gale-Shapley[7] algorithm, and then Roth found that this algorithm had actually been implemented by the American medical profession in assigning interns to hospitals[8]. That had happened *before* Gale and Shapley published their paper. It had happened as a result of 50 years of development. So there was an empirical development, something that happened out there in the real world, and it took 50 years for these things to converge; but in the end, it did converge to the game theoretic solution, in this case the core. Now this is amazing and beautiful, it is empirical game theory at its best. We're looking at something that it took smart, highly motivated people 50 years to evolve. That is something very different from taking a few students and putting them in a room and saying: OK, you have a few minutes to think about what to decide. These things are sometimes quite complex and take a long time to evolve; people have to learn from their mistakes, they have to try things out.

This was the beginning of a very intensive study by Roth and collaborators, Sotomayor and others[9], into the actual working of expert labor markets.

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[5] "The Value of Information in a Sealed Bid Auction," *Journal of Mathematical Economics*, 10 (1982), 105-114; see also Engelbrecht-Wiggins, R., P. Milgrom, and R. Weber: "Competitive Bidding and Proprietary Information," *Journal of Mathematical Economics* 11 (1983), 161-169.

[6] "An Empirical Study of an Auction with Asymmetric Information," *American Economic Review* 78 (1988), 865-883.

[7] Gale, D. and L. Shapley (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly* 69, 9-15.

[8] Roth, A. (1984): "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory," *Journal of Political Economy* 92, 991-1016.

[9] See Roth, A. and M. Sotomayor (1990): *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, Econometric Society Monograph Series, Cambridge: Cambridge University Press; Roth, A. (1991): "A Natural Experiment in the Organization of Entry-Level Labor Markets: Regional Markets for New Physicians and Surgeons in the United Kingdom," *American Economic Review*

And now they are beginning to consult with people and say: Listen, you guys could do this and that better. So again we have a progression from theory to empirical observation to prediction and then to engineering.

Q: From these examples, can one draw some lessons about the type of situations in which one can expect game theory to work in applications?

A: What one needs for game theory to work, in the sense of making verifiable (and falsifiable!) predictions, is that the situation be structured. Both the auctions and the markets that Roth studies are highly structured. Sometimes when people interview me for the newspapers in Israel, they ask questions like, can game theory predict whether the Oslo agreement will work or whether Saddam Hussein will stay in power. I always say: Those situations are not sufficiently structured for me to give a useful answer. They are too amorphous. The issues are too unclear, there are too many variables. To say something useful we need a structured situation. Besides the above examples, another example is the formation of governments. For years I have been predicting the government that will form in Israel once the composition of the Israeli parliament is known after an election. That is a structured situation, with set rules. The parliament has 120 seats. Each party gets a number of seats in proportion to the votes it got in the election. To form a government, a coalition of 61 members of parliament is required. The president starts by choosing someone to initiate the coalition formation process. (Usually, but not necessarily, this "leader" is the representative of the largest party in parliament.) The important point is that the process of government formation is a structured situation, to which you can apply a theory. That is true also of auctions and of labor markets like those discussed above. They work with fixed rules, and this is ideal for application. Now, if you don't have a structured situation, that doesn't necessarily mean that you cannot say anything, but usually you can only say something qualitative. For example, one prediction of game theory is the formation of prices; that comes from the equivalence theorem[10]. That kind of general prediction doesn't require a clear structure. On the other hand, the Roth example and the spectrum auction are examples of structured situations, and they are beautiful examples of applications.

Q: Could you explain what exactly you mean by a structured situation? Is it that there are players with well-defined objectives and well-defined strategy

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81, 415-440; and Roth, A. and X. Xing (1994): "Jumping the Gun: Imperfections and Institutions Related to the Timing of Market Interactions," *American Economic Review* 84, 992-1004.

[10] See, e.g., Debreu, G. and H. Scarf (1963): "A Limit Theorem on the Core of a Market," *International Economic Review* 4, 235-246; Aumann, R. (1964): "Markets with a Continuum of Traders," *Econometrica* 32, 39-50; Aumann, R. and L. Shapley (1974): *Values of Non-Atomic Games*, Princeton: Princeton University Press; and Champsaur, P. (1975): "Cooperation versus Competition," *Journal of Economic Theory* 11, 394-417.

sets, that there is perhaps even a timing of the moves? In short, is a structured situation one in which the extensive or strategic form of the game is given?

A: No, that is not what is meant by "structured". It means something more general. A structured situation is one that is formally characterized by a limited number of parameters in some definite, clear, totally defined way. That implies that you can have a theory that makes predictions on the basis of this formal structure, and you can check how often that theory works out, and you can design a system based on those parameters. That holds for auctions, and for Roth's labor markets, and for the formation of a governmental majority in a parliamentary democracy. In each of those cases, important aspects of the situation are described by a formal structure, a structure with clear, distinct regularities. Of course, there are also aspects that are not described by this formal structure, but at least *some* of the important aspects are described by this formal structure.

That is not so for the Oslo agreement and it is not so for Saddam Hussein. Those situations are in no sense repeatable, there is no regularity to them, there is nothing on which to base a prediction, and there is no way to check the prediction if you make it. Of course the prediction could be right or wrong, but there is no way to say *how often* it's right, because each prediction is a one-time affair, there's no way to generalize.

For instance, in the governmental majority matter, one can set up a parliament as a simple game in the sense of Von Neumann and Morgenstern's cooperative game theory, where we model the party as a player; we get a coalitional worth function that attributes to a coalition the worth 1 when it has a majority and the worth 0 otherwise. And then one can work out what the Shapley values are; the structure is there, it is clear, and one can make predictions. Now there are all kinds of things that are ignored by this kind of procedure, but one can go out and make predictions. Then, if the predictions turn out correct, you know that you were right to ignore what you ignored.

No matter what you do you are going to be ignoring things. This is true not only in game theory, it is true in the physical sciences also; there are all kinds of things that you are ignoring all the time. Whenever you do something out there in the real world, or observe something in the real world, you are ignoring, you are sweeping away all kinds of things and you are trying to say: Well, what is really important over here is this and that and that; let's see whether that's significant, let's see whether that comes out.

Q: In this example of coalition formation, you make predictions using an algorithm that involves the Shapley value. Suppose you show me the data and your prediction comes out correct. I might respond by saying that I don't understand what is going on. Why does it work? Why is the Shapley value related to coalition formation? Is it by accident, or is it your intuition, or is it suggested by theory?

A: There are two answers to that. First, certainly, this is an intuition that arises from understanding the theory. The idea that the Shapley value does represent power comes from the theory. Second, for good science it is not important that you understand it right off the bat. What is important, in the first instance, is that it is correct, that it *works*. If it works, then that in itself tells us that the Shapley value is relevant.

Let me explain this a little more precisely. The theory that I am testing is very simple, almost naive. It is that the leader — the one with the initiative — tries to maximize the influence of his party within the government. So, one takes each possible government that he can form and one looks at the Shapley value of his party within that government; the intuition is that this is a measure of the power of his party within the government. This maximization is a nontrivial exercise. If you make the government too small, let's say you put together a coalition government that is just a bare majority with 61 members of parliament — a minimal winning coalition — then it is clear that any party in the coalition can bring down the government by leaving. Therefore, all the parties in the government have the same Shapley value. So the hypothesis is that a wise leader won't do that. That is also quite intuitive, that such a government is unstable, and it finds its expression in a low Shapley value for the leader. On the other hand, too large a coalition is also not good, since then the leader doesn't have sufficient punch in the government; that also finds its expression in his Shapley value. Consequently, the hypothesis that the leader aims to maximize his Shapley value seems a reasonable hypothesis to test, and it works not badly. It works not badly, but by no means a hundred percent. For example, the current (June 95) government of Israel is very far off from that, it is basically a minimal winning coalition. In fact, they don't even have a majority in parliament, but there are some parties outside the government that support it; though it is really very unstable, somehow it has managed to maintain itself over the past 3 years. But I have been looking at these things since 1977, and on the whole, the predictions based on the Shapley value have done quite well. I think there is something there that is significant. It is not something that works 100% of the time, but you should know that nothing in science works 100% of the time. In physics also not. In physics they are glad if things work more than half the time.

Q: Would you like to see more extensive empirical testing of this theory?

A: Absolutely. We have tried it in one election in the Netherlands, where it seems to work not badly; but we haven't examined that situation too closely. The idea of using the Shapley value is not just an accident, the Shapley value has an intuitive content and this hypothesis matches that intuitive content.

Q: Are game theoretical applications to unstructured situations doomed to fail?

A: No, we already discussed the example of competitive equilibrium, the idea of price formation. It is successful although it is not structured. Even to diplomatic negotiations, which we discussed above, one can make contributions, in that one can point out certain things to be aware of. Let's consider an example. We have these negotiations with Syria. Now, the president of Syria, Mr. Assad, has stated publicly again and again that he is not sure that he is at all interested in reaching any kind of accommodation with Israel. The reason, he says, is that the status quo is not too bad for Syria; it is OK. And he is right, because we are not talking about a population in the Golan Heights, there are almost no Syrian people there. The few Syrian villages that were there and that were displaced in the 1967 war were resettled in Syria; no problems there, it was a small population. Most of the current inhabitants of the Golan Heights are Israeli Jews, all except for two or three villages right near the Syrian border, and there are no problems. These people are essentially satisfied. There is nothing that is burning under Assad's pants over there, and really under anybody's pants. So, Assad quite openly says: The status quo is not bad for me. Now, from the game theoretic point of view we have to ask ourselves, is the status quo in the set of feasible agreements? It could be that the status quo is outside the convex set of all possible pairs of utilities to agreements. It might be beyond the North-East boundary of that set. In that case there is no sense in pursuing a possible accommodation. Assad is quite happy the way it is; and while we would like to have a peace treaty with Syria, obviously that should not be at any cost. So the question is: Is there a possible peace treaty that is worthwhile for Assad, and that simultaneously is acceptable to us? I don't think that anybody in our government has answered that question, or even asked it. Professional diplomats don't think in those terms. They think in terms that are totally different; they don't have to do with payoffs, with outcomes. A diplomat will think in terms of vaguely defined objectives, like building trust between leaders, or peace, or signing a document, or making a statement to the press. This kind of thing is a totally different world. In this case, while game theory cannot make a prediction, it can say let us think in certain terms, in certain directions.

By the way, here again the discussion centers on the cooperative theory. It is the idea of a set of possible accommodations, and a status quo point, and where is that status quo point in relation to the set of possible agreements. That is cooperative game theory.

That brings to mind another point. I don't remember who it was, perhaps Kissinger, somebody in the early fifties said that a major contribution of game theory is just the idea of a strategy (or payoff) matrix. Just have the matrix in front of you. Just say: "look, we can do something, and they can do something; let's write down everything that we can do, and everything that they can do, and then let's look it over from that point of view". Whereas before game theory, people had only been thinking about what *we* can do. The example of Syria and Israel is similar in that respect, because it says:

Can we reach an accommodation that we can buy and they can also buy? Or, among all the accommodations that will be acceptable to them, will there be one that is acceptable to us? So we are putting ourselves in both shoes at the same time. That is already a giant step forward. So in this kind of unstructured situation, game theory does have something to contribute. But it is conceptual: ways of thinking, approaches – not specific predictions.

Q: I think that one of the persons who wrote in the fifties that game theory's most important contribution was the introduction of the payoff matrix was Thomas Schelling, and perhaps, he provides a nice, natural way to move to the next topic. Specifically, I would like to move on to disagreements between you and Reinhard Selten, differences of viewpoint. Yesterday you stressed the importance of relationships, of establishing links between the complex and the simple. Now one might argue that perhaps, relationships are the more easy to establish the more abstract the setting in which one is working. If I interpret Selten's discussion of your earlier paper correctly, then he fears somewhat that by moving to a more abstract level one is going to lose the connection with the real world. The reason I thought this relates to Schelling is that Schelling has argued in the past that game theory should stand on two strong legs, one is a strong empirical leg and the other is a deductive, formal leg. You are stressing very much this second leg, of establishing formal relationships, whereas Schelling and Selten would argue that we need to develop more the empirical leg. Do you see a conflict there?

A: No, that is a misunderstanding. The term "relationship" does not at all imply that we are talking about theory only. Empirics are vital, and I agree entirely with the view that you just attributed to Schelling. Up to now in this interview we have discussed *only* empirics, so the importance that I attribute to that side should be clear.

Let me point out that at least three major areas of my talk yesterday touched on the importance of empirics. The very first item in my talk was an empirical item. The Naval electronics problem[11] was my entrance into decision theory and from there into game theory. The naval electronics problem is a highly empirical problem with both legs in the real world; we are talking about real pieces of Naval equipment worth hundreds of thousands of 1955 dollars each. We are talking altogether about billions of 1995 dollars, and we went in there and we used methodology from decision theory, and from linear programming, and some methodology that Joe Kruskal and I developed for this very purpose ourselves, which later turned out to be connected with utility theory. We went in there with these tools, which are fairly sophisticated, to solve this real empirical problem; we did an engineering job, and I think it was a useful one. Yesterday I stressed the relationships within this engi-

[11] Aumann, R. and J. Kruskal (1959): "Assigning Quantitative Values to Qualitative Factors in the Naval Electronics Problem," *Naval Research Logistics Quarterly* 6, 1-16.

neering job, to go from simple hypothetical assignment problems to complex real ones. So that is the first empirical item.

The second one was my analysis of the concept of Nash equilibrium. You will recall that I used three slides. The black slide on the bottom, which had just the payoff matrix and the word "equilibrium" written on it; and two red slides, which I superimposed on the black slide, one after the other. The first red slide explained the payoff matrix in terms of conscious decision making, and the other one explained it in terms of evolution. In one case the equilibrium was strategic, and in the other case it was a population equilibrium: two completely different concepts with the same mathematical structure. Now the evolutionary interpretation of Nash equilibrium is certainly closely related to empirics. Let me mention in this connection the astoundingly beautiful work of Selten's associate Peter Hammerstein[12], who did this study of spiders in the American Southwest: a game theoretic study of the behavior of spiders in which mixed strategies turned up in precisely (or almost precisely) the right proportion when fitnesses were measured in terms of egg-mass; a mindblowing study! This is real empirics and that is empirical game theory.

The third item from yesterday's talk that is really empirically rooted is the study of the Talmud[13], using the idea of consistency and using the idea of the nucleolus. There we take something that happened 2000 years ago and happened in the real world; law is part of the real world. We are talking about a document that was written by people who had no idea of the discipline of game theory. They had something in mind, for 2000 years nobody understood it, and now it is understood. That is empirics. They were talking about a real bankruptcy problem, they had a real decision problem. This is a relationship; but not a theoretical relationship, it is something that is really happening out there. So that is the third area.

We're doing pretty well, when you take into account also all the other examples that we have mentioned, like the work about auctions, and about matching people to jobs, and the emergence of prices and many items we have not mentioned, like signalling[14], and labor relations[15], and other matters. The connection with the real world is indeed very important, and we have it. We have it and we are developing it further and it is closely associated with the

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[12] Hammerstein, P. and S. Riechert (1988): "Payoffs and Strategies in Territorial Contests: ESS Analysis of Two Ecotypes of the Spider *Agelenopsis Aperta*," *Evolutionary Ecology* 2, 115-138.

[13] Aumann, R. and M. Maschler (1985): "Game Theoretic Analysis of a Bankruptcy Problem from the Talmud," *Journal of Economic Theory* 36, pp. 195-213.

[14] See, e.g., Kreps, D. and J. Sobel (1994): "Signalling," in *Handbook of Game Theory, Vol. II*, R. Aumann and S. Hart, eds., Amsterdam: Elsevier, 849-867.

[15] See, e.g., Kennan, J., and R. Wilson (1993): "Bargaining with Private Information," *Journal of Economic Literature* 31, 45-104, and Wilson, R. (1994): *Negotiation with Private Information: Litigation and Strikes*, Nancy L. Schwartz Lecture, J.L. Kellogg School of Management, Evanston: Northwestern University.

theory; far from rejecting that, I embrace it. Many of the relations I discussed yesterday were real world relations.

Having said that, I'll say something else also. At some level the theory has to be several stages in advance of empirics. So we could be doing very complicated theory, let us say equilibrium refinement a la Mertens[16] with cohomology and all that, and at the same time be doing empirical studies and empirical engineering like Milgrom was discussing (the spectrum auction). It is very important that we have those two levels. Because it is not possible to do the things that Milgrom was discussing without having the Mertens cohomology at the same time. This is a somewhat subtle idea, so let me explain it. There has to be a body of knowledge out there that has been internalized by a body of people. Perhaps Milgrom, as an individual, does not necessarily have to understand Mertens's cohomology-based refinement theory, but the body of science has to develop in such a way that there can exist a Milgrom. (As it happens, Milgrom and Wilson are both very strong theorists; with all his involvement in the spectrum auction, Wilson has just written a highly theoretical piece about Mertens refinement[17].). In some sense it wouldn't be possible for him to work out these very practical rules without knowing something like Mertens's cohomology refinement; perhaps not necessarily that, but something of similar depth. You have to swim in it. You are not going to be able to swim from the shore to an island that is a kilometer away if it's the first time you are in the water. It has to be a part of your background. As I said yesterday, these things are all hitched together; they are part of a milieu. You have to have a lot of experience, and this experience has to be both practical and theoretical. You can't keep your nose too close to the ground. There has to be some "spiel", some room, and there has to be some background, and the theory always has to be far in advance.

Q: The second point of difference of view might perhaps be the following. You seem to be saying that every relationship is good, the more relations – the tighter the web – the better. But some of these relationships might actually be misleading. For example, take the rationalistic and evolutionary interpretations of Nash equilibrium. Maybe one interpretation applies in one context, while the other applies in a completely different context. Insights relevant in one context need not necessarily be relevant in the other context. Nevertheless, the relation might be used to import insights from one context

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[16] Mertens, J-F. (1989): "Stable Equilibria - A Reformulation, Part I: Definition and Basic Properties," *Mathematics of Operations Research*, 14, 575-625, and Mertens, J-F. (1989): "Stable Equilibria - A Reformulation, Part II: Discussion of the Definition and Further Results," *Mathematics of Operations Research*, 16, 694-753.

[17] Govindan, S. and R. Wilson (1996): "A Sufficient Condition for the Invariance of Essential Components," *Duke Journal of Mathematics*, to appear; see also Wilson, R. (1992): "Computing Simply Stable Equilibria," *Econometrica*, 60, 1039-1070.

to the other. One shouldn't mix these things, one should keep them separate. Selten advocates a sharp distinction between descriptive game theory and rationalistic game theory, where the former is based on less strong rationality assumptions. Hence, this is related to the issue of bounded rationality in general. Could you comment on these points?

A: You say "one shouldn't mix these things... Selten advocates a sharp distinction". Well, I disagree. When there is a formal relationship between two interpretations, like the rationalistic (conscious maximization) and evolutionary interpretations of Nash equilibrium[18], then one can say, "Oh, this is an accident, these things really have nothing to do with each other". Or, one can look for a deeper meaning. In my view, it's a mistake to write off the relationship as an accident.

In fact, the evolutionary interpretation should be considered the fundamental one. Rationality – Homo Sapiens, if you will – is a *product* of evolution. We have become used to thinking of the "fundamental" interpretation as the cognitive rationalistic one, and of evolution as an "as if" story, but that's probably less insightful.

Of course, the evolutionary interpretation is historically subsequent to the rationalistic one. But that doesn't mean that the rationalistic interpretation is the more basic. On the contrary, in science we usually progress, in time, from the superficial to the fundamental. The historically subsequent theory is quite likely to *encompass* the earlier ones.

The bottom line is that relationships like this are often trying to tell us something, and that we ignore them at our peril.

Having said this, let me add that game theorists argue too much about interpretations. Sure, your starting point is some interpretation of Nash equilibrium; but when one is doing science, you have a model and what you must say is: What do we *observe*? Do we observe Nash equilibrium? Or don't we observe it? Now, *why* we observe it, that's a problem that is important for philosophers, but less so for scientists. It's like the old joke of the town where they wanted to build a bridge over the river. They started arguing in the town council. The merchants wanted the bridge because it would be good for commerce. People from each side wanted it because it would be easier to go to the other side. The religious people wanted the bridge because then they could reach the synagogue more easily. The lovers wanted it because they could make romance on a full moon night. They started arguing and fighting with each other over why to build the bridge, and in the end they couldn't agree and they did not build the bridge.

So, the story of *why* we have Nash equilibrium, it's an important philosophical problem, but as science it is not that important. You are quite right,

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[18] Aumann, R. (1987): "Game Theory," in *The New Palgrave, Vol. II*, J. Eatwell, M. Milgate, and P. Newman, eds., London: Macmillan, 460-482. See specifically subsection ii of the section entitled 1970-1986, on page 477, near the bottom of the left column.

when we had a discussion this week, I was a little surprised when Reinhard said, "Yes, we must distinguish...". I don't quite understand why.

As far as descriptive Game Theory is concerned, I certainly agree that we must describe what we actually observe. But to divorce this from the theory would be counterproductive. After all, the theory is meant to *account* for what we observe. Imagine a theoretical physics that is divorced from observational physics – that would be absurd! There may be – indeed there *are* – discrepancies between theory and observation, both in physics and in game theory; but fundamentally, the aim must be to bring theory and observation together, not to distinguish between them. And as we have already discussed, in game theory we are doing quite well in this regard.

By the way, the evolutionary interpretation does not support only subgame perfect equilibrium, or, for that matter, trembling hand perfection. It also supports non-perfect equilibria. One beautiful example of that is in the ultimatum game. One can understand what is observed in the ultimatum game[19] on the basis of non-perfect equilibria. This is not an empty prediction. As you know, of course, every possible offer is a part of an equilibrium in the ultimatum game, so one could say that this does not mean very much; but that is not correct, because when you say that you have an equilibrium, then you say that this is in a sense a norm of behavior. That *can* be checked, and it *was* checked in the work of Roth, Prasnikar, Okuno-Fujiwara and Zamir[20], and they found regularities of behavior in various different places in the world, which were *different* in the different places. That is a beautiful verification of the evolutionary interpretation of Nash equilibrium. Far from the ultimatum game being some kind of rejection of game theory, it is a beautiful verification of it, a corroboration!

Q: You said yesterday that you thought that this was the direction to go in strategic game theory, that is, to further develop the evolutionary approach. At present, the models that we have are based on agents who do not have any cognition at all. Do you view this as a first step towards other models in which there is limited cognition, towards richer models of bounded rationality?

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[19] Güth, W., R. Schmittberger, and B. Schwarze (1982), "An Experimental Analysis of Ultimatum Bargaining," *Journal of Economic Behavior and Organization* 3, 367-388. In this experiment, two players were asked to divide a considerable sum (varying as high as DM 100). The procedure was that P1 made an offer, which could be either accepted or rejected by P2; if it was rejected, nobody got anything. The players did not know each other and never saw each other; communication was a one-time affair via computer. The only subgame perfect equilibria of this game lead to a split of 99-1 or 100-0. But there are many Nash equilibria, of the form "P1 offers x, P2 rejects anything below x."

[20] "Bargaining and Market Behavior in Jerusalem, Ljubljana, Pittsburgh, and Tokyo: An Experimental Study," *American Economic Review* 81 (1991), 1068-1095. Roughly speaking, it was found in this experiment that in each of the four different venues, the accepted offers clustered around some specific x, which differed in the different venues.

A: No, actually not. That is muddying the waters. The evolutionary interpretation is best understood as standing on its own legs, not as a stepping stone leading to rational or cognitive interpretations. That would not be a good way to go. We have here two separate – but completely parallel, completely isomorphic – interpretations of the *same* mathematical system. A population equilibrium in the evolutionary interpretation corresponds *precisely* to a strategic equilibrium in the cognitive interpretation. In a population equilibrium, each genotype that constitutes a positive proportion of the population is *best possible* given the environment. Similarly, in a strategic equilibrium, each pure strategy with positive probability is *best possible* given the mixed strategies of the others. In both cases we have *full* maxima; there is no way that the equilibria in the evolutionary interpretation could somehow be improved if we made them "more rational."

In brief, one should not try to transform an evolutionary model first into a boundedly rational model and then into something like a full rationality model; that's not the way to go. One should have both models in mind. Both are important.

Actually, the formal similarity between the two models probably reflects an important conceptual relationship. Rational thinking may have evolved *because* it is evolutionarily efficient. Let me clarify that. Evolutionary biologists keep asking about everything: What is the evolutionary function of this, what is the evolutionary function of that? One question, raised by Dawkins in his book[21] is: What is the evolutionary function of consciousness? What I'd like to suggest is that consciousness evolved because it serves a very important evolutionary function, namely that it enables people to think rationally in the sense of utility maximization. You are not only a computing machine; you have goals to achieve, utilities to maximize. For that you must be conscious, you must be able to experience, you must be able to say this is pleasurable, this is painful.

Just an example of that: What is the function of taste in food? Why must it have a taste? Well, it must have a taste because if it did not taste good we would not be motivated to eat it and then we would starve to death. We have to be conscious for that, we can't taste something without being conscious. The consciousness enables us to experience pleasure and pain. It has long been recognized that pain has a very important evolutionary function. I'm suggesting that pleasure also does.

Another example is, of course, sex. Why is sex pleasurable? What is the evolutionary function of that? Not of sex itself, that's obvious, but of the *pleasure* in sex? It is to induce the organism to have sex. On a deeper level, then, it is quite possible that the combination of the logical thinking in the brain and the consciousness which enables us to have utility, to have desires, is an outcome of an evolutionary process. So rational thinking, purpose oriented, utility maximizing behavior is an *outcome* of the evolutionary process. Not

[21] *The Selfish Gene*, Oxford: Oxford University Press, 1976.

the other way around – like we usually say that evolution is "as if" organisms were thinking! No, the thinking comes *because* of the evolution. What I'm trying to say is that in the end the two interpretations of Nash equilibrium really could be two sides of the same coin.

In his book, *The Growth of Biological Thought*[22], Ernst Mayr distinguishes between two kinds of explanation in biology, corresponding to the questions "how" and "why". Sometimes people confuse these terms. For example, the question "why do we see" might be answered in two ways. One is, "we see because we have eyes with lenses and retinas and neural connections to the brain." The other is, "we see because it helps us enormously in getting along in the world." Mayr says that the first answer really responds to the question "*how* do we see;" the term "why" is more appropriate to the second answer. "How" refers to the mechanism, "why" to the function. Needless to say, both questions are legitimate, and both answers are correct.

What I'm suggesting is that to the extent that it really occurs, conscious, rational utility maximization is a *mechanism*, an answer to the "how" question. Consciousness, the desire to maximize, the ability to calculate – these are traits that have evolved, like eyes. Their *function* is to enable us human beings to adapt, in a flexible way, to the very complex environment in which we operate. Because of its complexity and variability, it would be much more difficult to operate in this environment with genetically predetermined, "hard-wired" modes of behavior.

Q: Let us now turn to the questions that Werner Güth prepared in writing. The first one is this: One can distinguish two major tasks of game theory, namely

– formally to represent situations of strategic interactions, i.e., to define adequate game forms, and
– to develop solution concepts specifying the individually rational behavior in such games.

A: Well, let me take issue with that right away. It is not only strategic interaction. Yesterday we were talking about cooperative and noncooperative game theory and I said that perhaps a better name for cooperative would be "outcome oriented" or "coalitional," and for non-cooperative "strategically oriented". The way Güth's question is set up already points to the strategic direction. That, of course, is a very important part of game theory, but it is just one side of it. Game Theory develops not only solution concepts that specify rational behavior of individuals, but also solution concepts that specify outcomes. It is not just a question of behavior, but also of outcomes. And this is very very important, because in order to do game theory you would be very restrictive if you confined yourself to situations that are strategically well defined. In applications, when you want to do something on the strategic

[22] Cambridge, Mass.: Belknap Press, 1982.

level, you must have very precise rules; for example, like in an auction. An auction is a beautiful example of this, but it is very special. It rarely happens that you have rules like that. What other situations do you have that are strategically well-structured? OK, sometimes you have a parliament where you have rules of order and rules of introducing bills; Ferejohn[23] has actually done some work on applications of noncooperative theory to political science in political situations that are strategically well-defined. Another example is strategic voting[24]. Those situations are basically strategic and there really the non-cooperative side is very important. But such situations, with very precisely defined rules, are relatively rare.

Cooperative theory allows the rules of the game to be much more amorphous. If you wanted to do the Roth labour market, you could not do that non-cooperatively, the rules are not sufficiently well specified. If you wanted to do a non-cooperative analysis of coalition formation, you can't do it, it is not sufficiently well-specified. Who talks first, who talks second? It matters enormously in noncooperative game theory.

To sum it up, it is not just behavior that matters; it is also outcomes, and at least as much outcomes as behavior. So I disagree with the phrasing of the question.

Let's go on and read the second sentence of Werner's question.

Q: "In the social sciences one presently relies nearly exclusively on non-cooperative game models."

A: But that's absolutely incorrect. I can't disagree more. There have been important analyses of noncooperative models, but as the discussion up to now has shown, many of the models that we have been talking about are cooperative and many of them are successful ones. Roth's work on labour markets[25] is mindblowing, this is game theory at its best. If anything, the cooperative theory has played a *more* important role in applications than the non-cooperative theory.

Q: Maybe social sciences should be narrowed down to economics and the question has been motivated by the fact that there are books about game theory that don't refer to the cooperative branch at all.

A: That is true, but it is unfortunate. The fact that there are books published doesn't mean that that is what the world is about. The people who write these books are missing some very important sides of game theory; they are representing only their own knowledge and their own interests. These books do not give a balanced summary of what game theory has accomplished.

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[23] See, e.g., Ferejohn, J. and J. Kuklinski (eds.) (1990): *Information and Democratic Processes*, Urbana: University of Illinois Press.

[24] Of course, as we mentioned, voting also has a cooperative side.

[25] op. cit. (Footnotes 8 and 9).

By the way, there is an excellent book, recently published, by Osborne and Rubinstein[26]. These authors made most of their theoretical contributions on the strategic side, and yet they devote a nice portion of their book to cooperative game theory. I recommend this book highly, it is beautifully done, and it recognizes the importance of the cooperative theory.

Incidentally, the Rubinstein alternating offers model[27] is an example of a "bridge": a noncooperative, behavioral model leading to the Nash bargaining solution, which, as we know, is a cooperative, outcome-oriented concept. Not only in TU-situations; it also leads to the Nash solution in an NTU-setup, as Binmore has shown[28].

Summing up, one should look at what is significant, not at what is fashionable.

Q: Perhaps we can now move to the third and last part of this question. It reads: Although the noncooperative focus brought about a very thorough analysis of many institutional aspects (e.g. of economic markets), it seems that nearly every phenomenon can be explained by designing an adequate model (e.g. by relying on an infinite time horizon as in the case of the so-called Folk Theorems). Do you see this more as a problem (we need more selective predictions!) or as an advantage (we have to find out the relevant institutional details!)? Is there some hope for a revival of cooperative game theory, which avoids the specification of many facets?

A: There are several things to say to that. First, a word about institutions. Institutions can be treated as exogenous or endogenous. Güth's question ("we have to find out the relevant institutional details") treats them as exogenous: Given this or that institution, what are the equilibria? Though this has some interest, it is more interesting to treat institutions *endogenously*. *Why* did a given institution come about? What are the underlying forces that led to its formation?

This dichotomy is discussed, for example, in the work by Jacques Dreze and me about rationing[29]. Before this work, rationing was treated exogenously: Given the institution of rationing, what parameter values would bring the economy into equilibrium? Dreze and I asked a different question: What led to the institution of rationing? If it is a matter of excess demand or supply, why specifically rationing, and what form of rationing? Could other institutions handle this?

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[26] *A Course in Game Theory*, Cambridge, Mass.: MIT Press, 1995.

[27] Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica* 50, 97-109.

[28] "Nash Bargaining Theory II," in *The Economics of Bargaining*, K.G. Binmore and P. Dasgupta, eds., Oxford: Basil Blackwell 1987, 61-76.

[29] "Values of Markets with Satiation or Fixed Prices," *Econometrica* 54 (1986), 1271-1318.

An even more basic example is the equivalence theorem[30] itself. Rather than taking the institutions of money and prices as given, the equivalence theorem predicts the emergence of these institutions.

Note that both these examples come from cooperative game theory, so we are led naturally to your question about a "revival" of the cooperative theory. Cooperative theory is actually doing quite well. I've already said in this interview that many of the most interesting applications of game theory come from the cooperative side. In his invited talk at this conference[31], Mike Maschler discussed over thirty significant contributions to the cooperative theory that have been produced over the last few years. Yes, I suppose one could call that a "revival." And I agree with you that an important advantage of the cooperative theory is that it takes a broader, more fundamental view, that it is not so obsessed with procedural details; its fundamental parameters are the ***capabilities*** of players and coalitions.

Adam Brandenburger, who teaches at the Harvard Business School, has told me that the students there consider the cooperative theory a lot more relevant to business than the non-cooperative theory. In my opinion, both are important, perhaps for different kinds of applications.

Let's come now to your question about "nearly every phenomenon can be explained by designing an appropriate model." First of all, as you say, that happens less with cooperative models. But beyond that, the importance of a model depends on the sum total of its applications, on how the specific kind of model of this kind has been applied in other places. One has to get a feel for the applications that are covered by a certain kind of model. Perhaps you could design a model for anything, but that does not mean that it is an interesting model. It is interesting if you design it, and then it has applications A, B, C, D; it applies to labour, it applies to auctions, it applies to search, it applies to signalling, it applies to discrimination. When you have a lot of applications, then you can start saying, well, this is an important model. This is what I tried to point out yesterday in my lecture on relationships, that when you get something which gives you important things in many different applications, like the Shapley value, or the nucleolus, then it gets some credibility.

Q: Your answer brings up a related issue. In the survey that you wrote for The New Palgrave[32] you describe the game theoretic approach as being very different from the classical approach in economics. The game theoretic approach is unifying: the same concept is applied in various contexts. In contrast, in the economic approach, a model and solution concept is tailored to the situation at hand. What we currently see in game theoretic applications to economics is something like a half-way house: There is a unifying solution concept –

[30] That is, the equivalence between competitive equilibrium in markets and game theoretic concepts like core and value.

[31] "Games '95," Jerusalem.

[32] op. cit. (Footnote 18).

Nash equilibrium – but there is a great deal of freedom in constructing the extensive or strategic form model. There is a lot of flexibility.

A: Well, that applies to the non-cooperative theory. On the cooperative side, there are three or four central solution concepts – value, core, nucleolus, stable sets – but much less flexibility in constructing the model. The model is much better defined.

Even on the non-cooperative side, it's not clear that Nash equilibrium can be called a "unifying" solution concept. What about all the refinements and other variations? We still have a way to go, the smoke still has not cleared on the refinement battle field, or maybe it is not a battle field, but a refinement factory. I don't know what the product there is. We have a lot of refinements; which is the right one? Eric, actually what do *you* think? Now it is 1995, we are more than a decade into the refinement business, almost two decades. Do you see one or two or three refinements emerging as the accepted ones? What do you think?

Q: I think that some, like backwards induction, subgame perfect equilibrium, are there to stay. Others, like proper equilibria probably not, since they don't satisfy properties like invariance. As far as the variety of stability-like concepts, that were introduced by Kohlberg and Mertens[33], is concerned, it is still unclear, the dust hasn't settled yet. For example, there is still discussion about the admissibility requirement, is it necessary for strategic stability or not? Should one really look at strategy perturbations as, for example, Nash did in his original work on bargaining[34]? The discussion is still open, I think. And then there is the concept of persistent equilibria, due to Kalai and Samet[35], which I think belongs to a different category; it might show up in a learning context, in what Nash calls the "mass-action" interpretation of equilibria. That is, a learning process might converge to a persistent equilibrium.

A: Your answer is very useful; it underscores the difference between your viewpoint and mine, as representatives of different schools of thought.

For example, in discussing refinements just now, you were concerned about assumptions: admissibility is too strong, too weak, and so on. That is not my concern. I have never been so interested in assumptions. I am interested in conclusions. Assumptions don't have to be correct; *conclusions* have to be correct. That is put very strongly, maybe more than I really feel, but I want to be provocative. When Newton introduced the idea of gravity, he was laughed at, because there was no rope with which the sun was pulling the earth: Gravity is a laughable idea, a crazy assumption, it still sounds crazy today. When I was a child and was told about it, I could not believe it. It does not make

[33] "On the Strategic Stability of Equilibrium," *Econometrica* 54 (1986), 1003-1034.
[34] "The Bargaining Problem," *Econometrica* 18 (1950), 155-162.
[35] "Persistent Equilibria in Strategic Games," *International Journal of Game Theory* 13 (1984), 129-144.

any sense; but it does happen to yield the right answer. In science one never looks at assumptions; one looks at conclusions. Where do these refinements *lead*? Take an application, what do Kohlberg/Mertens[36] have to say about this application? What do Kreps/Wilson[37] have to say about this application? What do Kalai/Samet[38] have to say about this application? What do Cho and Kreps[39] have to say about it? I don't care what the assumptions are. So, I get a feel for what the concept says not by its definition but by where it leads. This is a fundamental difference between what I am trying to promote and the school that you are representing (which, of course, also has its validity).

Beyond that I agree with you that what's here to stay is subgame perfect equilibrium. That says a lot and we know what that says, but we don't know it from the definition. We know it from the applications. What else is here to stay, I don't know. I agree entirely with what Güth says that, given the situation, there is a lot of room for constructing a game tree, and it matters very much how you construct the game tree and I think that that is one of the problems. And that limits the applicability of the strategic approach altogether. It does not eliminate it, it has very important applications, but it limits it.

Q: As far as applications are concerned, one can point to signalling games. For example, take Spence's original model of education as a signal[40]. The game model of that situation has many equilibria, but the stable equilibrium outcome corresponds with the unique one that was originally singled out by Spence. I believe we can empirically verify the predictions of this equilibrium, for example, the overinvestment in the signal. I think we could also verify these predictions in other signalling contexts, for example, in financial markets.

A: That's the kind of result I am looking for. When you ask what are the right refinements, you have to say: What are the refinements that give us the Spence signalling equilibrium? That's what we have to ask.

Q: Well, in order to get that answer you have to apply strong solution concepts, you need basically the full strength of the Kohlberg/Mertens[41] 1986 Econometrica stability concept.

A: Fine, we need more applications like that. We need to have applications out there, not just Beer-Quiche; that's very important, but it is an example.

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[36] op. cit. (Footnote 33).
[37] Kreps, D. and R. Wilson (1982): "Sequential Equilibrium," *Econometrica*, 50, 863-894.
[38] op. cit. (Footnote 35).
[39] "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics* 102 (1987), 179-221.
[40] Spence, A. (1974): *Market Signaling*. Cambridge, Mass: Harvard University Press.
[41] op. cit. (Footnote 33).

We need a class, a kind of model to which to apply these things. By the way, there is an excellent article of Kreps and Sobel[42] on signalling in the Handbook. There are a lot of important applications out there, there is no question about it. But, as I said, the smoke has not cleared yet. It hasn't cleared on the assumptional side and it hasn't cleared on the conclusional side.

Q: What is definitely a drawback of some of these solution concepts is that they are very difficult to work with. Just as it is difficult to work with Von Neumann-Morgenstern stable sets in the cooperative theory, it is also difficult to see what the implications of the Mertens[43] stability concept are.

A: That is a very important point and it is one that I endorse entirely. It is something that I also stressed in the article "What is Game Theory Trying to Accomplish?": It is important to have an applicable model. It sounds a little like the man who had lost his wallet and was looking under the lamppost for it. His friend asked him: Why do you look only under the lamppost? And he answered: That's because there is light there, otherwise I wouldn't be able to see anything. It sounds crazy, but when you look at it more closely it is really important. If you have a theory that somehow makes a lot of sense, but is not calculable, not possible to work with, then what's the good of it? As we were saying, there is no "truth" out there; you have to have a theory that you can work with in applications, be they theoretical or empirical.

Incidentally, there is a man whom I love and respect very much and with whom I have worked closely and extensively, but who disagrees with me on this issue, and that is Mike Maschler. He has the opposite opinion. He wants to know what the "truth" is, and if it is difficult to calculate, he doesn't care, he has to find the truth. I don't have this notion of truth. It is really odd to see him wanting to find the "right" bargaining set. He has this idea that some of the bargaining sets are wrong and some of them are right. I don't have this idea. I have the idea of usefulness, not right or wrong. So, he would disagree with me. He says, if it is hard to calculate, it is just too bad, but we must find out what it is. My own viewpoint is that inter alia, a solution concept must be calculable, otherwise you are not going to use it.

Q: Whereas Reinhard Selten has tried to refine the equilibrium concept, as originally proposed by Cournot and Nash, you have developed your well-known coarsening idea of correlated equilibria[44]. Do you think that solution concepts should allow for more behavioral possibilities rather than select more and more specifically?

A: Well, my response to that is that the word "should" is out of place. I can't answer the question because I disagree with its formulation. This is again a

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[42] op. cit. (Footnote 14).
[43] op. cit. (Footnote 16).
[44] "Subjectivity and Correlation in Randomized Strategies," *Journal of Mathematical Economics* 1 (1974), 67-96.

somewhat different viewpoint from that of many people. Game theory is not a religion. In religion one says: One should observe the Sabbath, one should give charity. But game theory is not a religion, so the word "should" is out of place. I love correlated equilibria and I also love subgame perfect equilibria; I have written papers, which I hope the world will enjoy, on both subjects. A few years ago I wrote about correlated equilibria, and recently I published a paper[45] called "Backward Induction and Common Knowledge of Rationality". Backward induction, of course, is a subgame perfect equilibrium. So I do both, and I think that one "should" not do this or that exclusively, but one should develop both concepts and see where they lead. There are important things to say about correlated equilibria and there are important things to say about subgame perfect equilibria. This idea that correlated equilibria represent the truth, or that subgame perfect equilibria represent the truth, is an idea that I reject.

Q: If game theory is not religion, is it a tool?

A: Yes, absolutely, it is science. It is a tool for us to understand our world.

Q: Let us now move on to the notion of consistency, which played an important role in your talk yesterday. It is a property that is satisfied by many solution concepts, among them Nash equilibrium: If we fix some of the players in the game at their equilibrium strategies, then the combination of equilibrium strategies for the remaining players constitutes an equilibrium of the reduced game. When writing your paper for The New Palgrave[46] you were apparently already aware that this property could be used to axiomatize the concept. I would like to discuss this property in connection with equilibrium selection. There is a paper[47] that shows that no solution concept that assigns a unique equilibrium to each finite strategic game can be consistent. I don't know how to deal with this.

A: Well, that is certainly a very interesting result, and it is also important. But I can't say that it is distressing. If there is one thing that we have learned from axiomatics, it is that if you write down all things that are obviously something that you would want, then you almost always find a contradiction. So let me start by saying that I won't lose sleep over that.

Having said that, let me respond substantively. Equilibrium selection in the style of Harsanyi and Selten[48] is a beautiful venture, it is very important. It is important for two reasons. One is, that from one point of view, equilibrium selection is an important facet of the background of equilibrium. Game theory is not a religion, and, equilibrium does not have to be "justified"; but

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[45] *Games and Economic Behavior* 7 (1995), 6-19.

[46] op. cit. (Footnote 18).

[47] Norde, H., J. Potters, H. Reynierse and D. Vermeulen (1996), "Equilibrium Selection and Consistency," *Games and Economic Behavior* 12, 219-225.

[48] Harsanyi, J. and R. Selten (1988): *A General Theory of Equilibrium Selection in Games*, Cambridge, Mass.: MIT Press.

for those people who do want to justify it, including Harsanyi and Selten, selection plays a fundamental role. (Let me say that though I hold certain methodological viewpoints, that does not mean that I reject the importance of other people's methodological viewpoints. I have said that assumptions are not important, but I can understand also people who argue that assumptions *are* important, and who want to justify equilibria, and who do think that game theory is a way of life, that's OK also.) So in that respect – to "justify" equilibria – selection theory *is* important. And it has other important sides to it, byproducts: risk dominance is a very important by-product of selection theory, and so is the tracing procedure.

On the other hand, selection theory has a sort of fantastic aura to it. Hundreds of pages, filled with extremely complex instructions as to how one picks one equilibrium in each game. And, you know, for 20 years before they published the book, they had a different version every year or two. And now that they *have* published the book, they still come out with new versions. It is a beautiful abstract structure, and it has important concrete implications, but it has also this fantastic Rube Goldberg aura to it.

In brief, that one cannot find a consistent selection is not terribly distressing or surprising. If anything, it is an additional criticism of selection theory, not of consistency. Though it is not devastating, it *is* something that you have to chalk up against selection theory.

Q: In your foreword to the book of Harsanyi and Selten, you write that one rationale of Nash equilibrium, that a normative theory that advises people how to play has to prescribe an equilibrium since otherwise it is self-defeating, essentially relies on the theory making a unique prediction, hence, one needs a selection to justify the equilibrium concept in this way.

A: Yes, that is precisely what I meant when I just said that selection plays a fundamental role in justifying equilibrium. Thank you for clarifying that. What was your question?

Q: It was a remark, that, if you need selection for this "justification" of the equilibrium concept and if there is no consistent selection, then this "justification" might be problematic.

A: Yes, it is indeed a little problematic. As I just said, the whole selection project is a little problematic, and this *is* a mark against it. But if you want to "justify" equilibrium from a cognitive point of view, then equilibrium selection is not the only way of doing it; you can do it in other ways also. Have you seen the recent paper[49] by Brandenburger and myself? It is a different cognitive approach to Nash equilibrium, not using selection. You get equilibrium when certain informational assumptions are satisfied; the equilibrium does not have to be unique.

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[49] "Epistemic Conditions for Nash Equilibrium," *Econometrica* 63 (1995), 1161-1180.

So, selection is just one way of "justifying" equilibrium. That it is inconsistent with consistency is unfortunate, or regrettable, but it is not the end of the world, certainly not the end of the world for equilibrium, not even the end of the world for selection, we often have inconsistencies, it is something one learns to live with, both as a physicist and as a game theorist.

There is one more point to be made in this connection. Harsanyi and Selten base their selections on the mathematical form (strategic or perhaps extensive) of the game. Thus in two games with the same mathematical form, the same equilibrium must be selected. Now *that* is surely *not* needed to "justify" the idea of equilibrium from the point of view of advising the player. When you are giving advice, you do know about the real-life context of the game, and you *can* base your advice on that. You could easily select different equilibria in different contexts. For example, in my controversy with Roth and Shafer about the NTU Shapley value, I discussed two games with precisely the same mathematical form – one in a trading context, the other in a political context – whose "natural" or "expected" equilibria are quite different[50]. For a simpler example, think of a two-person game where each player must choose "L" or "R"; each gets 1 if they choose the same, 0 otherwise. Suppose they are driving toward each other, and the "L" or "R" represents the side of the road on which they drive. If it's in England, I would select (L,L); if in the Netherlands, (R,R). Harsanyi and Selten suggest randomizing, which seems a little crazy[51].

In brief, there's no good reason to base the selection on the mathematical form only; the context, the history also matter.

Q: Werner also had a question about consistency. Could we turn to the last part of that question, which reads: Could you comment on the extremely opposite justifications of Nash equilibrium which partly require perfect rationality and partly deny any cognition?

A: I have already responded to that at some length in this interview. They are just different approaches. There is no problem with that; on the contrary, it sheds light on them. It helps to fortify, to corroborate the concept, to validate it. Güth thinks of them as one "versus" the other[52]; he thinks if one is "right," the other must be "wrong." I think they are both right, they both illuminate the concept, from different angles. And as I have said, I don't

[50] "On the Non-Transferable Value: A Comment on the Roth-Shafer Examples," *Econometrica* 53 (1985), 667-677; see specifically Section 8, 674-675.

[51] They might answer that I'm making too much of a degenerate, non-generic example. But that misses the point. Suppose we perturb the payoffs very slightly, so that the Harsanyi-Selten selection is pure. I would still select (L,L) in England and (R,R) in the Netherlands. They, of course, would select the same in both countries. That's almost as crazy.

[52] Güth, W., and H. Kliemt "On the Justification of Strategic Equilibrium — Rationality Requirements Versus Conceivable Adaptive Processes," DP 46, Economics Series, Humboldt University, Berlin, 1995.

believe in justifications; it's not religion and it's not law; we are not being accused of anything, so we don't have to justify ourselves.

Q: Let's move on to the next question on Güth's list: "In my view, the notion of a strategy with all its partly counterfactual considerations seems already a too demanding concept for a descriptive theory. Correlated equilibria are of an even more complex nature."

A: Well, both assertions sound strange. It's true that the notion of a strategy is an abstraction, but it's conceptually a simple object. Anyway, if you're going to challenge that, then you're challenging the whole conceptual basis of the noncooperative theory, including, of course, the notion of Nash equilibrium.

As for correlated equilibria, conceptually they are fairly simple objects. The set of correlated equilibria is always a convex, compact polyhedron with finitely many extreme points, it is a very simple object, very easy to work with, much easier than the set of Nash equilibria. Nash equilibria are algebraically very complex objects. Harsanyi once wrote an article describing the deep algebraic geometry of Nash equilibria[53]. By contrast, a schoolchild can work out the correlated equilibria. They are much simpler objects.

Q: By now I have understood that assumptions do not count.

A: It's not that assumptions don't count, but that they come *after* the conclusions; they are *justified by* the conclusions. The process goes this way: Suppose you have a set of assumptions, which logically imply certain conclusions. One way to go is to argue about the innate plausibility of the assumptions; then if you decide that the assumptions sound right, then logically you must conclude that the conclusions are right.

That's the way that I reject, that's bad science.

The other way is not to argue about the assumptions at all, but to look at the *conclusions only*. Do our observations jibe with the *conclusions*, do the conclusions sound right? If yes, then that's a good mark for the assumptions. And then we can go and derive other conclusions from the assumptions, and see whether *they're* right. And so on. The more conclusions we have that jibe with our observations, the more faith we can put in the assumptions.

That's the way that I embrace, that's good science. Logically, the conclusions follow from the assumptions. But empirically, scientifically, the assumptions follow from the conclusions!

Q: Let's move to the next question on Güth's list: Like in traditional evolutionary biology, in evolutionary game theory one often assumes a "genetically" determined behavior. In modern ethology this is rather debated (apes, for instance, are known to have well developed cognitive systems). Should

[53] Unpublished. See also Schanuel, S.H., L.K. Simon, and W.R. Zame (1991), "The Algebraic Geometry of Games and the Tracing Procedure," in Reinhard Selten (ed.), *Game Equilibrium Models II*, Berlin: Springer.

one not allow for a more continuous transition from no cognition at all (e.g. when studying primitive organs like plants) to more or less rational behavior (when studying the behavior of apes and humans) to resemble the rather gradual evolutionary process as, for instance, measured by DNA-differences?

A: We discussed that above. No, one should not allow for that. We don't have to talk about apes, we can talk about humans. Humans undoubtedly have well developed cognitive systems and, in spite of that, much human behavior fits into the evolutionary paradigm. Most human actions are not calculated. I know that I don't calculate. Maybe you think that I don't have a well developed cognitive system: be that as it may, I hardly ever calculate when making decisions. Much of behavior is not calculated; it is either genetically determined or it is determined by experience, by learning – which is very similar to genetic determination, in the sense of Dawkins's memes[54]. A meme and a gene behave mathematically almost the same. Moreover, as I said above, to the extent that we *do* calculate, it may be a result of evolution.

Q: You have been very influential in developing a rigorous definition of common knowledge of rationality (CKR) and elaborating its implications. Recently it has been argued that CKR is self contradictory where one, of course, relies on more centralised players like in the normal form and denies "trembles" in the sense of Selten's perfectness idea of equilibria. When do you think is the assumption of CKR justified?

A: Well, to answer this last question, it is almost never justified. It is a far reaching assumption. Please refer to my paper in the Hahn Festschrift[55], "Irrationality in Game Theory", which indicates how very very small departures from CKR can lead to behavior that is very different from that of CKR, for example in the centipede game. The departures from CKR are small both in the sense of being tiny probabilities and also in that the failure is at a high level of CKR. In other words, you get mutual knowledge of rationality to a high level, and after that level CKR fails only by a very small probability; and nevertheless, the results are very different from those under CKR. So, CKR is not "justified"; it does not happen. But that does not mean that CKR is unimportant. It is still very important to know what CKR says and what it implies, and to understand the connection between it and backward induction. Just like it is important to know how a perfect gas behaves, though there are no perfect gases; and it is important to know what perfect competition does, although perfect competition does not exist. It is important to know what happens in the ideal state, although ideal states don't exist.

No, CKR never happens.

Q: Is it self-contradictory?

[54] *The Selfish Gene*, op. cit. (Footnote 21).

[55] In *Economic Analysis of Markets and Games, Essays in Honor of Frank Hahn*, P. Dasgupta, D. Gale, O. Hart and E. Maskin, eds., Cambridge, Mass.: MIT Press, 1992, 214-227.

A: No, that is simply a mistake. The idea that CKR is self-contradictory was due to an inadequate model. If you build your model carefully and correctly, then CKR is not self-contradictory. Admittedly, I had to think for three years before coming up with the right model. It is a very very confusing business, it is very subtle, it takes a lot of thought. The reader is referred to my article on "Backward Induction and Common Knowledge of Rationality"[56]. That article should settle this issue, or go a long way to settle it.

No, CKR is not self-contradictory in games of perfect information.

Q: Well, I heard you lecture about this paper and I must admit I didn't understand the argument fully. I haven't read the paper yet, but I am going to do so.

A: Good. Read the paper; it is not difficult and there is a careful conceptual discussion at the end of the paper. The structure is not very elaborate. It has three building blocks: rationality, knowledge, and commonality of knowledge. You first examine each one separately, define it carefully; then you put the three together, and you get backward induction. So first you have to ask yourself: What is rationality? What does it mean to be rational? And that has a simple definition. Not: What does it mean to have common knowledge of rationality, just what does *rationality* mean. And then, what does it mean to have knowledge? What is the exact model of knowledge? And you ask that independently of what rationality is, and what common knowledge is. Just, what does *knowledge* mean? And you find a satisfactory answer to that. And then you have to ask yourself: What is *common* knowledge? And there you use the accepted definition, you just iterate the knowledge operator. The key is to think about knowledge separately, and about rationality separately, and only then, in the end, to go to common knowledge of rationality; and then you get the result. It is to keep the ideas separate, not to confuse them. That is what mathematics is about, that's where mathematics helps.

Q: Shall we conclude the interview here or is there something that you would like to add?

A: I would like to express my tremendous admiration for Selten, as a human being, and as a pioneer in a number of fields. First of all, in the equilibrium refinement business. The first refinements, and among those that are sure to survive, are the subgame perfect equilibrium and the ordinary perfect equilibrium, the trembling hand perfect. This will survive, this is a monumental idea. Another very important contribution is the idea of doing experiments. I don't put quite the faith in experiments that Selten does, but one very important function of experiments, which he realizes himself, is that to run an experiment, you really have to see what the rules of the game are. You really have to think about the game very very carefully. Just like when you write a computer program for anything, you have to ask yourself: what am I

[56] op. cit. (Footnote 45).

doing here? Now, when you run an experiment it forces you to say: What is this game? What the subjects actually do is perhaps not all that significant (although it can be suggestive), but when you are designing an experiment it is very important to design it right. So, that is an input that Selten recognizes and emphasizes, and it bears more emphasis.

And, as I said last Sunday evening when we were talking about the Nobel Prize winners, a very important contribution of Selten is his work on the evolutionary approach. He has made some important scientific[57] contributions on this, but the main importance is to bring this into the world of game theory. To bring in the work of Maynard Smith[58] and his associates. Another thing to be mentioned is that he had an important influence also on John Harsanyi in value theory. His thesis[59] was basically about the Harsanyi value[60], and that is a very important concept.

On a different level, another important idea is Selten's umbrella. I'm not sure that all the people at the dinner on Sunday understood what this is about. Selten lives in a world where rain is unpredictable; it can rain any day. Rather than continually investing mental energy in the question "should I take an umbrella or shouldn't I?", he has made it a rule of life always to take an umbrella. Now, if he starts thinking that when he comes to Israel he really does not need an umbrella because it never rains in the summer, then he is contravening this rule of life which is *not* to put thought into it. So he has this meme for always carrying an umbrella. He himself is an embodiment of the evolutionary approach to game theory.

So, I have tremendous admiration for Reinhard and I am very happy that he got the Nobel Prize and I wish him continued success in the future and he seems to be strong and going well.

Q: Thank you very much for this interview!

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[57] "A Note on Evolutionarily Stable Strategies in Asymmetric Animal Conflicts," *Journal of Theoretical Biology* 84 (1980), 93-101; and "Evolutionary Stability in Extensive Two-Person Games," *Mathematical Social Sciences* (*MSS*) 5 (1983), 269-363 (see also "Correction and Further Development," *MSS* 16 (1988), 223-266).

[58] Maynard Smith, J. (1982): *Evolution and the Theory of Games*. Cambridge: Cambridge University Press.

[59] Selten, R. (1964): "Valuation of n-Person Games," in *Advances in Game Theory*, Annals of Mathematics Study 52, R.D. Luce, L.S. Shapley and A.W. Tucker, eds., Princeton: Princeton University Press, 577-627.

[60] Harsanyi, J. (1963): "A Simplified Bargaining Model for the n-Person Cooperative Game," *International Economic Review*, 4, 194-220.

Working with Reinhard Selten
Some Recollections on Our Joint Work 1965-88

John C. Harsanyi

University of California at Berkeley

Reinhard Selten and I first met in Princeton in 1961 at a game-theory conference sponsored by Oskar Morgenstern. The earliest thing I remember about him is that he presented an excellent paper at the conference about the difficulty of defining a value concept for games in extensive form.

In 1965 both of us joined a small group of young game theorists studying the game-theoretic aspects of arms control as part-time consultants for the U.S. Arms Control and Disarmament Agency (ACDA) but formally employed by the Princeton research organization Mathematica founded by Morgenstern. Other members of our group were Bob Aumann, Harold Kuhn, Mike Maschler, Jim Mayberry, Herb Scarf, Martin Shubik, and Dick Stearns. (Sometimes also some other people joined our group for short periods.) Between 1965 and 1969 the members of our group met fairly regularly about three times a year in Washington, D.C., or nearby to discuss the work each of us did for the ACDA project since our last meeting.

As far as I remember, close cooperation by the two of us really started at the First International Workshop in Game Theory organized by Aumann and Maschler, and held in Jerusalem in 1965. It lasted only for about three weeks and had only 16 or 17 participants (Aumann, 1987, p. 476). But its small size facilitated a very lively and fruitful exchange of ideas among the participants.

It was at this Workshop that I first outlined my probabilistic model for games with incomplete information (see Harsanyi, 1967-68). As a result, I received many helpful comments and suggestions from the other participants, including a suggestion by Reinhard on how to extend my model to cases where the different subjective probability distributions about alternative states of the world entertained by different player types are inconsistent (cf. Harsanyi, 1967-68, section 15).

Both of us were great admirers of Nash's (1950 and 1953) theory of two-person bargaining games. Thus, we decided already in Jerusalem to work together on the problem of extending Nash's theory to two-person bargaining games with incomplete information. In undertaking this work, one of our objectives was to gain better game-theoretic understanding of the arms-control negotiations between the United States and the Soviet Union conducted at the time, and studied by our ACDA group.

Our first paper on this work was our "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information" , published in 1968 as Chapter V of the volume The Indirect Measurement of Utility, Final

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Report of ACDA/ST-143.[1] It has been reprinted as Chapter 3 of Mayberry et al. (1992).

After writing this paper, we continued our work on this problem. We made particularly good progress during the academic year 1967-68, when Reinhard was a Visiting Professor in our Business School in Berkeley. We reported our results in a second paper[2] with the same title as that of our first paper of 1968. It was published in Management Science, vol. 18 (January 1972, Part II), pp. 80-106 (see Harsanyi and Selten, 1972).

Soon after writing this paper, we came to the conclusion that, instead of defining specific solution concepts for particular classes of games, we should try to define a general solution concept for all games, both those with complete and those with incomplete information. As we agreed with Nash's (1951, p. 295) suggestion that cooperative games should be modeled as noncooperative games of special kinds, our formal objective was to define a solution concept applying to all noncooperative games.

This in turn meant finding rational criteria for selecting one specific Nash equilibrium as the solution of any given noncooperative game. In other words, it meant to suggest a credible general theory of equilibrium selection for all noncooperative games, including cooperative games modeled as noncooperative games.

In the end it took us 16 years of hard work to come up with a theory we felt to be worth presenting to the game-theorist community (see Harsanyi and Selten, 1988). Now, seven years after the publication of our book reporting our conclusions, I for one certainly do not want to claim that we have said the last word about the problem of equilibrium selection. Yet, I do think that we have made some real contributions to this problem – as well as some worth-while incidental contributions to the resolution of some other game - theoretic problems.

Most of our work on the equilibrium selection problem was done in the 1970s. Reinhard visited Berkeley for two or three periods of several weeks each during the decade. I visited him in Bielefeld for two full academic years, in 1973-74 and in 1978-79. We did a small amount of work also by correspondence and by telephone calls.

Over time we first accepted and then rejected at least three different theories of equilibrium selection, before we adopted our final theory described in our 1988 book.

Our first theory was based on the concept of diagonal probabilities. Let $s = (s_1, s_2, ..., s_n)$ be a Nash equilibrium of an n-person game G. Suppose that at first all n players use their s-strategies. Then, all $(n - 1)$ players

[1] Chapter VI of the same ACDA volume is another paper co-authored by Reinhard on a closely related topic. It is: Reinhard Selten and John P. Mayberry, "Application of Bargaining I-Games to Cold-War Competition" . Reprinted as Chapter 4 of Mayberry et al., op. cit.

[2] Our second paper deals only with incomplete-information bargaining games with fixed threats (Nash 1950).

j other than i deviate from their s-strategies with the same probability p. The smallest deviation probability p that will destabilize player i from his s-strategy s_i will be called his diagonal probability d_i. Then, we can measure the stability of s by the product $\pi = \Pi_i d_i$ of these diagonal probabilities. This stability measure can be regarded as an analog of the Nash product (Nash, 1950).

Yet, we rejected this theory based on these probability products when we found that it was not invariant to splitting a given player into two identical players.

Another theory of equilibrium selection we considered was based on trying to measure the influence that any given player had on the strategies used by the other players. (This "influence" concept was meant to be a generalization of the probability that a given type t_i^k of player i will be present in a game with incomplete information.)

A third theory we considered was based on the concept of formations (see Harsanyi and Selten, 1988, p. 195). This theory represented an important step toward the theory described in our 1988 book, in which the concept of formations, and particularly that of primitive formations, plays an important role.

I am very sorry that I no longer remember many of the details of the theories we later discarded. I did not make any notes of our discussions. Wulf Albers was kind enough to send me all Working Papers of the Institute of Mathematical Economics at the University of Bielefeld published during the relevant period. But unfortunately they did not contain any information about our pre-1988 theories. Neither did our Working Papers published in Berkeley.

I am reasonably sure that Reinhard kept much better records than I did about different stages of our work, and will eventually write them up for publication or will make them available to interested scholars.

Let me finish this short paper with the following remarks. I feel very lucky indeed that I have had Reinhard as a close personal friend and as a close fellow worker on many scientific problems of great importance over so many years. There are many scientific problems we managed to solve together, which I could not have done alone without his help. We often disagreed, as is inevitable in any serious scientific work, but our disagreements have never weakened our friendship. I enjoyed discussing many intellectual and social problems with him, even problems quite unrelated to our work. I likewise enjoyed his willingness, which I fully shared, to take views contrary to majority opinion when he felt to have good reasons to do so.

I feel particular gratitude to him for the special interest he showed in the intellectual and emotional development of my son, Tom, and all the three of us are very grateful for the friendship that both Reinhard and his wife Elisabeth have shown for us over so many years.

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Introduction and Survey

Werner Güth

Understanding strategic interaction can be interpreted in more than one way: One is to explain interaction phenomena by modelling them as games and derive their solutions by relying on perfectly rational players. Let us refer to this as the rationalistic approach to understanding strategic interaction.

Another possibility is the behavioristic approach which requires only bounded rationality and which usually relies on empirical observations. Paralleling this methodological dualism, which is so characteristic of Reinhard Selten's work, the contributions in this volume can be partitioned accordingly, e.g. by distinguishing the applications of game theory from experimental studies.

A third approach is that of evolutionary stability, i.e. one views the ways in which the individual members of some population interact as the stable constellation for some evolutionary dynamics. Again Reinhard Selten was among the pioneers who clearly understood how the concept of evolutionary stability is related to the equilibrium concept and how it can be generalized, e.g. to games in extensive form.

The contributions in this volume can also be distinguished into conceptual discussions and applications of established concepts where one may, furthermore, subdivide each group into smaller categories.

Although the volume is not partitioned into sections of related contributions, this introduction offers a way how the various contributions could be arranged. Actually the papers were ordered accordingly. We admit, however, that other ways of subgrouping and ordering the papers are possible. Our attempt is simply to supply a more structured survey than just describing the contents of the various contributions.

Conceptual Aspects of Game Theory

The major task of game theory is first to formally describe situations of strategic interaction – one usually speaks of game forms – and second to develop solution concepts describing more or less specifically how rational players decide in such a formally described situation. The contributions in section I are partly addressing restrictions which one should impose on the game form (the papers by Binmore and by Harstad and Phlips) and partly discussing solution concepts and their properties.

One standard assumption of most game theoretic models is that of perfect recall, i.e. every player remembers everything he knew before, especially his own previous moves. Clearly, the assumption is unrealistic. It can be defended normatively by arguing that perfectly rational players – unlike human beings – possess this ability. Inspired by the "absent-minded driver's problem" **Ken Binmore** discusses various ideas how to incorporate imperfect recall in game

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theoretic models. The essential idea of the absent-minded driver is that of an information set like: If player i goes R(ight) at the left node, he immediately forgets this and therefore does not know after this choice whether he is in the right or left decision node. Binmore discusses how the difficulties to define rational behavior for such games can be overcome by relying on repeated decision making or on a team approach. A more recent study (Aumann, Hart, and Perry, 1996) claims, however, that this may not be needed.

According to the so-called Harsanyi-doctrine incomplete information should be consistent – individual beliefs about others' types are the marginal distributions of the type vector generating distribution – if all players are perfectly rational. From the very beginning Reinhard Selten has suggested that one might want to allow for inconsistent incomplete information when studying actual situations where beliefs will not be revised until they are consistent. The idea is to introduce fictitious chance moves with vector-valued instead of single-valued probability assignments where, as usual, the probability assignments are commonly known. **Ronald M. Harstad** and **Louis Phlips** go one step further by allowing for doubly inconsistent priors on a futures market: agents entertain inconsistent beliefs about the inconsistent beliefs of their trading partners who all can either be optimists believing in a high futures price or pessimists who believe that this price will be low. Although ex-post-inefficiences occur, they show that an ex-ante-efficient outcome can be reached by subgame perfect equilibria.

Inconsistent incomplete information is also considered by **Michael Maschler** who suggests representing inconsistent incomplete information in extensive form by one game tree for each player (instead of the vector valued type generating distribution suggested by Reinhard Selten, 1982). Of course, to compute an equilibrium such that no type of no player can profitably deviate, a player has to know also the trees of the other players.

Strict equilibria – every player loses by deviating unilaterally – satisfy the most crucial stability requirements like being immune against small trembles of strategies and/or payoffs. Unfortunately, they do not always exist. The refinement approach, inspired by Selten's concept of perfect equilibria, essentially tries to find out which crucial properties of strict equilibria a refinement, whose existence is guaranteed, should preserve and on which properties one should not insist. **Robert Wilson** focusses on admissibility – an equilibrium should not rely on dominated pure strategies – and demonstrates that requiring admissibility may be harmful when selecting among equilibrium components although it may be useful when selecting within a component. In spite of the deep substructures of games explored the arguments of Wilson's thorough and – in our view – informative discussion can be easily understood since he illustrates them by special games.

Unlike in their earlier versions of general theories of equilibrium selection Harsanyi and Selten (1988) have used two very different ideas to discriminate among equilibria, namely risk dominance and payoff dominance. Their

mutual inconsistency is, furthermore, resolved by granting priority to payoff dominance which they did not require at all in their earlier versions. The intuition for this is that players will naturally focus attention on equilibria which everybody prefers. **Eric van Damme** considers team games in which all players always receive the same payoff since, in his view, the unique payoff undominated equilibrium in such a game is clearly focal. What he shows is that even in this special class of games risk dominance and payoff dominance are inconsistent and that the payoff dominant equilibrium may be risk dominated.

Roger B. Myerson is interested in "sustainable" equilibria which, if once culturally established, cannot be easily destabilised as, for instance, certain mixed strategy equilibria. After demonstrating his intentions with the help of some basic games like the battle of sexes – or stag hunt games he argues that persistent equilibria might be the best suited refinement for selecting "sustainable" equilibria. The attempt is rather to describe an open problem than to offer yet a rigorous definition.

Evolutionary Game Theory

It has been a great inspiration both for evolutionary biology as well as for game theory that under certain conditions evolutionary stability can be described by evolutionarily stable strategies. **Peter Hammerstein** tries to demonstrate in a non-technical way the conflict aspect of some important stages in the evolution of life, e.g. in the evolution of cells, why chromosomes have evolved, or in the origin of sexes. Together these examples illustrate why conflict analysis, i.e. evolutionary game theory, may be helpful in revealing the evolutionary logic in the design of life.

One way to justify the assumption of perfectly rational players is to abstract from cognition at all, e.g. by relying on genetically predetermined behavior as it is – until now – mostly assumed in evolutionary biology, and to assume that the frequency of the possible ways of behavior is governed by evolutionary dynamics, i.e. a more successful type of behavior becomes more frequent. What has to be discussed then is, of course, what kind of "evolutionary" dynamics are needed to justify certain ideas of rational behavior. **Jonas Björnerstedt, Martin Dufwenberg, Peter Norman,** and **Jörgen W. Weibull** explore whether strictly dominated strategies can survive in the long run when certain assumptions for the evolutionary dynamics in discrete and continuous time are met. For an overlapping-generations version of the well-known replicator dynamics it can be shown that the survival of strictly dominated strategies depends on the "generational overlap". Furthermore, it is shown that certain evolutionary dynamics in continuous time do not guarantee elimination of strictly dominated strategies.

Based on Reinhard Selten's extension of evolutionarily stable strategies to asymmetric animal conflicts **Avi Shmida** and **Bezalel Peleg** analyse animal conflicts with payoff irrelevant asymmetries. Reinterpreting Aumann's

lottery device in the definition of correlated equilibria as nature's choice allowing phenotypes to condition their behavior the authors show that strict and symmetric correlated equilibria can be viewed as distributions of evolutionarily stable strategies, i.e. a chance device in nature allows phenotypes to behave differently in different states in spite of the same genotype.

Applications of Non-Cooperative Game Theory

The recent success of game theory, which is by now the dominating methodology in economic theory, and perhaps also in the social sciences, is due to the insights of the rationalistic approach. One now understands more clearly the importance of certain institutional aspects like the time order of decisions, the exact informational conditions etc. The predominant tradition of the rationalistic approach has, furthermore, relied on the so-called noncooperative approach of game theory as suggested by Nash (1951) and more thoroughly discussed later on by Harsanyi and Selten (1988). It essentially means to model a social conflict situation as a strategic game and to solve it then by determining its equilibria.

Matthew Jackson and **Ehud Kalai** define a recurring bully game which can be informally described as a recurring chain store paradox with incomplete information where all previous decisions can be observed. The bully – the analogue of the chain store – can be of two types, namely a rational one who would yield when being challenged and an irrational one who would fight in such an event. The bully's type is chosen randomly according to one of several type generating distributions where challengers only know the probabilities of the various distributions, but not the one which has been actually selected. They, however, may draw inferences about this distribution by updating their beliefs in view of earlier decisions. Jackson and Kalai are especially interested in exploring whether the crazy perturbation or reputation-"solution" of the chain store pardox can be maintained. They therefore assumed that the selected distribution guarantees rational bullies with probability 1. With trembles their clearcut result is that in the long run rational bullies never fight and therefore will always be challenged regardless of the initial beliefs in the sense of the probabilities for the various type generating distributions. Thus if trembles ensure learning, building up a reputation of craziness requires craziness, i.e. irrational bullies must result with positive probability.

In repeated games many types of behavior are possible, purely adaptive forms of behavior and more rational kinds of strategy selection. **Thomas Quint, Martin Shubik,** and **Dicky Yan** compare so-called dumb bugs – who stubbornly believe that others repeat their previous choices regardless of any contradictory evidence – with various sorts of more or less rational types of behavior (equilibrium and maxmin-players) as well as with malicious anti-players. The basic idea is to compare how monomorphic - all players rely on the same type of behavior – populations fare when they do not play

just one game but a whole universe of games. Since a "universe" is defined by a class of games and a probability distribution over this class, it could be interpreted as a complex stochastic game where players are allowed to react to the characteristics of the selected game. For a specific example it is demonstrated that a society of bugs may not fare worse than a society of equilibrium players.

Eric J. Friedman and **Thomas Marschak** discuss the problem whether an organization with autonomous members will communicate less than the organization as a team which maximizes the sum of individual profits. Based on a class of special models, justified by "classic information theory", the authors show that team members may communicate too little as well as too much when they become independent choice makers. An interesting further conclusion is that the customary role of mixed behavior in assuring existence of equilibria may be lost when communication effort is measured in a way that depends on probabilities.

As in some of his earlier studies **Eyal Winter** analyses how the agenda determination is related to the results of bargaining. One model he explores assumes that the agenda is negotiated by the committee members before dealing with the actual issues; according to the other model each proposer can propose an alternative on any unsettled issue at any time. Whereas for the first model efficiency of the bargaining outcome cannot be guaranteed, this is possible for the second model if no more than two issues have to be decided and if members can be restricted to vote for or against an issue (yes-no voting).

A central theme of human cooperation is how to prevent the inefficient results which would prevail if everybody behaves opportunistically. The approach of **Akira Okada** is to allow players to change the "rules of the game", e.g. by establishing a costly enforcement agency in order to limit free-riding. Based on an n-person prisoners' dilemma he presents a game model whose rules describe how players by their free will can found an organization – a coalition of players – whose enforcement agent tries to discourage free-riding by the members of the organization. The multistage game is closely related to Reinhard Selten's distinction between small (not more than 4 sellers) and large (not less than 6 sellers) markets (Selten, 1973). It is shown that an organization with a prohibitive punishment policy has to be large enough, but – due to free-riding in forming the organization – usually no general cooperation will be achieved. In the case of symmetric players the only symmetry invariant equilibrium implying cooperative results is therefore one in mixed strategies. Okada also analyses a game with heterogeneous players and discusses two applications, namely environmental pollution and the emergence of the state.

Classic philosophers have studied the difficulties and possibilities to substitute anarchy by social order when all individuals are rational. **Werner Güth** and **Hartmut Kliemt** try to relate some of these philosophical ideas

to the conceptual innovations by Reinhard Selten. The problems discussed and illustrated by simple games are Bodin's attempts to understand what the lack of commitment power of the sovereign implies, de Spinoza's analysis of opportunism in trust relations, Locke's ideas to justify legal institutions by consensus and Hobbes' related efforts. It is shown that some of the conceptual and philosophical insights by Reinhard Selten were already anticipated although the general principles were not available what, in our view, illustrates their intuitive appeal.

Both, Elisabeth Selten and Reinhard Selten, are engaged in the Esperanto-movement and they both like to rely on this non-ethnic language whenever others speak Esperanto too. Together with Jonathan Pool Reinhard Selten has, furthermore, analysed classes of linguistic games (Selten and Pool, 1995) where communication can also be ensured when interaction partners learn some planned language which nobody has learnt so far and whose learning costs are much lower than those of natural languages. **Werner Güth, Martin Strobel,** and **Bengt-Arne Wickström** try to resolve a typical problem in such linguistic games, namely the multiplicity of their strict equilibria, by applying the theory of equilibrium selection (Harsanyi and Selten, 1988). The major intention is, of course, to find out under which conditions international communication should rely on Esperanto rather than on learning complicated ethnic languages like English.

Relating Cooperative and Non-Cooperative Game Theory

Although the rationalistic approach has predominantly used the noncooperative way of modelling and solving strategic interaction, one certainly has experienced and will experience some frustrations when relying on the noncooperative approach because of its enormous flexibility. By now it seems commonly accepted that nearly any type of interaction phenomena can be justified by carefully designing an appropriate noncooperative game model and focussing on one of its equilibria. This is most dramatically illustrated by the so-called Folk Theorems which, under appropriate conditions, can justify any individually rational behavior or by the so-called crazy perturbation approach which introduces special forms of incomplete information.

In view of these likely frustrations it seems important to keep in mind what cooperative game theory has to say about the results of strategic interaction. The cooperative approach does not specify the strategic possibilities of the interacting parties and may therefore imply a more robust type of results, provided that they are specific enough to offer some guidance.

Benny Moldovanu illustrates how graph-theoretical methods can be used to study the relations of stable demand vectors and core allocations.The basic idea of stable demands is that every player forms an aspiration of what he requires when joining a coalition. Such demands are stable if no player can raise his demand without excluding himself from all coalitions that meet

the demands by all its members and if nobody depends on somebody else (an agent depends on another agent if any coalition, which can guarantee his demand, includes the other agent, but not vice versa). The author concentrates on NTU (non transferable utility)-games whose players are buyers and sellers. Here each stable demand vector induces a graph whose vertices are the agents on the market and where a buyer and a seller are connected if their demands are compatible, i.e. if they agree on the sales price.

Using non-atomic exchange economies **Guillermo Owen** discusses three properties of efficient allocations: the well-known competitive allocations, consistent allocations based on equal weighted contributions as well as homogeneous allocations (coalition members receive the same vectors of net trade if their coalition becomes larger or smaller without changing its composition). It is shown that homogenity and consistency of efficient allocations correspond to competitiveness.

Starting with the early and truly impressive approach of Edgeworth an important and influential justification of competitive allocations is that they are the only core-elements when the number of agents becomes very large. **Joachim Rosenmüller** establishes the equivalence of the core and the set of competitive allocations by an approach, based on "non-degenerate" characteristic functions. Unlike previous positive results relying on transferable utility his aim is to tackle also non-transferable utility economies. The study, which is seen as a first step in this direction, provides a partial answer: for certain classes of exchange economies with piecewise linear utilities a finite convergence-theorem can be proved.

Philip J. Reny and **Myrna Holtz Wooders** refer to units of an organization as states and to the organization itself as a commonwealth if certain stability requirements are met. They study the stability of the organization if its units can threaten to leave it. The model employed is a cooperative game with side payments. A threat (against a payoff vector) of a unit to secede is called credible if no group of players can achieve their payoffs by leaving and if this would harm at least one other player. A payoff vector and a partition of the organization into states is a commonwealth if the payoff vector is in the core of the game, if in each state all members need each other and if no states can credibly threaten to secede.

Principles of Behavioral Economics

Unlike the rationalistic research tradition the behavioristic approach is usually based on empirical observations rather than on philosophical ideas of what rationality requires. By now there is a lot of evidence that people do not always choose the alternative which serves their own interests best. Very influential concepts trying to account for non-optimizing behavior are the norm of reciprocity and the theory of satisficing (instead of optimizing) and of aspiration adjustment.

Colin Camerer tries to discuss the major differences between mainstream experimental economics and the considerably longer experimental tradition in (social) psychology. Although he concedes that this distinction is not sharp, he states some crucial differences like a stronger orientation toward testing formal models or theories in experimental economics and the more thorough ways of experimental psychologists in analysing not only how people decide, but also why they do so. In our view, Camerer rightly concludes that these differences would have been sharper without the experimental work of Reinhard Selten who, since the beginning of his experimental work, has always tried to combine the two approaches. Both research traditions are needed to promote the behavioristic alternative of the still dominating homo oeconomicus.

The behavioral norm of reciprocity, whose importance has been demonstrated by many experimental studies, is discussed by **Elisabeth Hoffmann**, **Kevin McCabe**, and **Vernon L. Smith**. Here reciprocity means to react in kind, i.e. one rewards kind and punishes mean partners by actions which do not serve one's own interests in the best way. The authors stress the crucial role of reciprocity in organizing human cooperation and division of labor and outline several approaches to explain the evolution of reciprocity. Instead of speculating wildly they develop their arguments by relying on previous experimental results. Their article is therefore both a selective survey of influential experimental studies pointing out how relevant reciprocity is and an outline of how one might try to explain this norm.

One of the most influential ideas of the theory of bounded rationality is the theory of aspiration adaptation (Sauermann and Selten, 1962). Unlike in normative theory decision makers are not assumed to search for the best, but to satisfy certain aspirations levels where these aspiration have to be updated in the light of own and others' experiences. **Reinhard Tietz** has observed aspiration adaptation experimentally by asking the participants to engage in decision preparation by stating certain critical points, e.g. first demand, planned bargaining goal, attainable agreement, planned threat to break off, planned conflict limit in bilateral bargaining. He mainly relies on the results of his well-known Kresko-experiments where collective wage bargaining is based on a complex macroeconomic model. His major result seems to be the principle of aspiration balancing stating that both bargaining parties reach equally high aspirations.

Most theories of characteristic function bargaining – a very prominent topic in cooperative and experimental game theory with pioneering contributions by Reinhard Selten – simply state conditions for proposals (a coalition structure together with a proposed allocation of payoffs) to qualify as stable. Sometimes – like in the sequence "proposal, objection, counterobjection" on which the bargaining set is based – the dynamics of bargaining enter the picture, but only in a rudimentary way and often without affirmative empirical evidence. Based on own and other experimental studies **Wulf Albers** tries

to outline a more comprehensive model of characteristic function bargaining by experienced players who, based on previous experiences, do not try out anymore deviations with worse terminal results, but can correctly anticipate future consequences. An attempt is made to incorporate the prominence structure of the number system, the stronger loyalty of players with similar roles as well as the attribution of aspiration levels (attributed demands) from previously rejected proposals.

Common-pool resources (CPRs) are defined by **Elinor Ostrom**, **Roy J. Gardner,** and **James Walker** as natural, e.g. fisheries, or man-made, e.g. irrigation systems, resources in which exclusion is difficult and yield is subtractable. Because of the latter property common-pool resources are endangered by congestion and even destruction. The continuing and impressive research program is based on formal models of various institutional arrangements including rules how to assign users' rights and monitor users as well as on field and experimental observations of how these arrangements affect the sustainability of the common-pool resources. Field studies have to be supplemented by laboratory experiments since the latter allow for a better control of the relevant variables. The ideal combination of game theoretic modeling and related field and laboratory studies sets a high standard for model-based empirical research.

Experimental Studies

The behavioristic approach to understand strategic interaction should be based on principles which boundedly rational decision makers want to use and can apply. It is therefore important to provide empirical evidence guiding our attempts to develop behavioral theories of strategic interaction. Since quite often empirical field studies cannot offer the data needed for specifying behavioral theories, one has to rely on experimental studies to test hypotheses or – in case of more explorative experimental studies – to suggest hypotheses for boundedly rational decision making.

Although game theory started out by solving two person zero sum games, the question if and in which ways boundedly rational players deviate from the obvious maxmin-strategies in such games is still open. The inspiring experiment by **Ariel Rubinstein**, **Amos Tversky,** and **Dana Heller** explores two person zero sum games as well as two person coordination and discoordination games. The experimental setup is always a hider and seeker-design employing pictorial and verbal items of which one is always distinctively focal. Whereas maxmin-behavior requires that all items will be selected with equal probability in strictly competitive games, the observed behavior is significantly biased, e.g. by avoiding the endpoints or by relying on the focal item. The results further suggest that seekers tend to avoid endpoints more than hiders who, in turn, avoid focal items more often. Concerning the nonzero sum games players relied on focal items to coordinate whereas focal

items were nearly as likely as other items in discoordination games where both players are interested in choosing different items.

Two standard paradigms, demonstrating doubts about the predictive power of game theoretic solution concepts like (subgame) perfect equilibria, are the finitely repeated prisoners' dilemma game and the chain store paradox as originally introduced by Reinhard Selten (1978). Whereas the role of players is more or less the same in repeated prisoners' dilemma games, there is a crucial asymmetry in the chain store paradox since the chain store wants to deter entry of its many potential competitors. **James A. Sundali** and **Amnon Rapoport** report on experiments testing whether the intuitive deterrence idea (the chain store fights in case of early entries to discourage further entry) or the game theoretic backward induction solution is more in line with the experimental observations. Although an increase in the number of entrants from 10 to 15 increases deterrence, even this higher level of deterrence could not prevent most entrants from entering. It seems that the intuition underlying deterrence requires a more substantial number of potential entrants. The authors conclude "... Selten's intuition in choosing m = 20 (as the number of potential entrants) was right ..."

Intergroup conflicts are special team games where at least two groups compete and where the group outcomes are public goods which are nonexcludable concerning the group members. **Gary Bornstein**, **Eyal Winter**, and **Harel Goren** consider a situation where each player can either contribute or not and where the game within a team and the game between teams are prisoners' dilemma games. In their experiment two groups of three players either played an intergroup prisoners' dilemma or each of the two groups played an independent prisoners' dilemma game resembling the possible intergroup games of the intergroup conflict. As would have been predicted by psychological concepts like group or corporate identy intergroup competition increases cooperation. This effect, however, becomes weaker in later rounds when the games are played repeatedly.

Consider an experimental situation with the player roles described by the rules of the game. If two different individuals have to decide in the same role of such a game, their behavior will usually not only depend on the rules, but also on their individual characteristics. **Axel Ostmann** and **Ulrike Leopold-Wildburger** focus attention on bargaining styles which experimental participants employ in face to face-bargaining and which can be detected by recordable data. The basic idea is that bargainers quickly develop partner images to which – of course, also to the rules of the game – they adopt their communicative style. Relying on the 3-dimensional SYMLOG-space (dominance, friendliness, task orientation) the authors try to assess bargaining types (insistent, leadership, cooperation, low profile) based on videotapes and protocols. It can then be assessed which bargaining styles are more successful.

Based on the empirical evidence from the "Iowa Electronic Markets" **Joyce Berg, Robert Forsythe,** and **Thomas Rietz** discuss what makes markets predict well. In addition to field data the observations of the experimental markets offer another and for theoretical purposes more adequate way to analyse how markets reveal information. More specifically, the large-scale experimental markets provide data about traders' individual characteristics which is not available in field studies. The authors rely on the data from the US political markets to identify factors determining when these markets accurately predict election results. While the experimental markets predict election results quite well, the prediction error depends on certain factors like the number of major candidates and the pre-election market volumes.

Roy J. Gardner and **Jürgen von Hagen** define the budget process, e.g. of national governments, as the system of rules which govern the process like, for instance, the sequence of budget decisions for which two extreme possibilities exist: Bottom up-budgeting starts by collecting requests which are individually decided by voting, top down-budgeting starts with voting on the total budget which then is split up. The hypothesis the authors want to test experimentally claims that top down-budgeting leads to greater fiscal discipline. The interesting experimental design distinguishes four treatments, i.e. four distributions of ideal points of the five voters, and compares the predictive success of more or less normative theories (subgame perfect equilibrium as a point and as an area prediction and efficiency).

Charles R. Plott and **Theodore L. Turocy** speak of intertemporal speculation if one buys commodities now to resell them later. In their experiment future demand was unknown to non-speculating agents and speculators who could place unlimited orders without knowing the number of competitors nor their orders. Participants played repeatedly a two-period-game which allowed speculators to carry over inventory from the first to the second period. Markets were organized as computerized multiple unit double auctions and contained always 4 potential buyers, 4 sellers and 4 speculators where the latter could only gain by speculation in one treatment. In the other treatment speculators could act also as normal agents, i.e. buyers or sellers. Although there were only four experimental sessions, the rational expectations hypothesis seems to be supported what, once again, demonstrates the surprising information efficiency of the double auction procedure. We agree with the authors that one needs to explore more thoroughly the mechanism of information transfer in double auction procedures.

One of the so-called anomalies is the endowment effect according to which the same commodities are higher evaluated if they are part of one's endowment as compared to the situation where one still wants to acquire them. The typical empirical evidence is that one would sell one's own commodity only at a higher price than the price one is willing to accept when acquiring it. Whereas most of the previous experimental studies have used riskless commodities **Graham Loomes** and **Martin Weber** rely on risky assets.

They observed the buying prices for state contingent claims by auctioning such claims by a value-revealing auction mechanism. The way to detect an endowment effect in such a setup is to auction off various claims and to analyse the vector of bids. If, for instance, two state contingent claims taken together constitute a sure payoff or dividend, an endowment effect could be detected when the two bids for the two claims add up to less than that amount. In addition to endowment effects Loomes and Weber also test for framing effects (the same endowment is once framed as a small amount of money plus a state dependent claim and once as a larger amount of money minus a complimentary state dependent claim).

Altogether these contributions provide a broad picture of how one can model and analyse strategic interaction. It hopefully provides guidance and inspiration for scholars who want to follow the traces of Reinhard Selten. Furthermore, it should be clear by now that such scholars could originate from many scientific disciplines.

Of course, it seems difficult to equal Reinhard Selten's capabilities: his philosophical insights, mathematical abilities, profound knowledge of the literature and last, but not least his enormous creativity in modelling and analysing strategic interaction. But he also offers us many ways to join him on his way of improving our understanding of strategic interaction: It can be done by developing new concepts, by applying such concepts rigorously, by empirical research, especially experimentation, and by formulating behavioral theories based on empirical observations.

Join the crew if you have not yet done so!

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A Note On Imperfect Recall*

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Abstract. The Paradox of the Absent-Minded Driver is used in the literature to draw attention to the inadequacy of Savage's theory of subjective probability when its underlying epistomological assumptions fail to be satisfied. This note suggests that the paradox is less telling when the uncertainties involved admit an objective interpretation as frequencies.

Ce que j'ai appris, je ne le sais plus. Le
peu que je sais encore, je l'ai deviné.

Chamfort, *Maximes et Pensées*, 1795

1. Introduction

Von Neumann and Morgenstern [17] proposed modeling bridge as a two-person, zero-sum game in which each partnership is one of the two players. Modeled in this way, bridge becomes a game of imperfect recall, because the players forget things they knew in the past. For example, when bidding as North, the player representing the North-South partnership must forget the hand he held when bidding as South. A bank choosing a decentralized lending policy for its hundred branches confronts a similar problem. The boss can imagine himself behind each manager's desk as he interviews clients, but then he must forget the earlier lending decisions he made while sitting behind other managers' desks. The decision problem he faces is therefore one of imperfect recall.

Since Kuhn [9] pointed out that mixed and behavioral strategies are interchangeable only in games of perfect recall, little attention has been paid to the difficulties that can arise when recall is imperfect.[1] The orthodox approach has been to regard a person with imperfect recall as a team of agents who have identical preferences but different information. In the style of Selten [15], each agent is then treated as a distinct player in the game used to model the problem. For example, the orthodox approach models bridge as a four-player game with two teams and the banking problem as a hundred-player game with one team.

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[1] Notable exceptions are Isbell [7] and Alpern [1].

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As Gilboa [6] confirms, the consensus in favor of the team approach is very strong. Nevertheless, a recent paper of Piccione and Rubinstein [12] has revived interest in the problem of decision-making with imperfect recall. Their emphasis in this paper is on the straightforward psychological fact that most people know that they will forget things like telephone numbers from time to time. The orthodox approach dismisses such folk as irrational and thereby escapes the need to offer them advice on how to cope with their predicament. But I agree with Piccione and Rubinstein that people who know themselves to be absent-minded can still aspire to behave rationally in spite of their affliction. However, my own interest in the imperfect recall problem is mostly fuelled by the difficulties with the team approach outlined in Binmore [2].

Players stay on the equilibrium path of a game because of their beliefs about what would happen if they were to deviate. But how do players know what would happen after a deviation? The orthodox approach treats a player as though he were a team of agents, one for each information set at which he might have to decide what action to take. Such a viewpoint de-emphasizes the inferences that a player's opponents are likely to make about his thinking processes after he deviates. One agent in a team may have made a mistake, but why should that lead us to think that other agents in the same team are liable to make similar mistakes? Traditionalists see no reason at all, and hence their allegience to backward induction and similar solution concepts. However, a theory that treats a player as a team of independently acting agents is unlikely to have any realistic application, because we all know that real people are liable to repeat their mistakes. If I deviate from the equilibrium path, it would therefore be stupid for my opponents not to make proper allowance for the possibility that I might deviate similarly in the future.[2] The team approach is therefore not without its difficulties even in games of perfect recall. It therefore seems an unlikely panacea for imperfect recall problems.

Piccione and Rubinstein [12] do not claim to provide a theory of rational decision-making under imperfect recall. They seek only to comment on some of the issues that would need to be addressed in formulating such a theory. I am even less ambitious in that I shall simply be commenting on some of their comments. The problem of time-inconsistency raised by the one-player game that they aptly describe as the Paradox of the Absent-Minded Driver is particularly interesting.

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[2] Binmore [2] argues that one needs an explicit algorithmic model of the reasoning processes of a player in order to take account of such considerations. Finite automata have been used for this purpose in a number of papers. However, as Rubinstein [13] notes, one cannot model players as finite automata without introducing imperfect recall problems.

2. Paradox of the Absent-Minded Driver

The general issue of imperfect recall in games is discussed in my *Fun and Games* (Binmore [3]). Such textbooks explain how to interpret representations of imperfect recall problems like that shown in Figure 2.1(a).[3] One may imagine an absent-minded driver who must take the second turning on the right if he is to get home safely for a payoff of 1. If he misses his way, he will find himself in an unsafe neighborhood and receive a payoff of 0.

The driver's difficulty lies in the fact that he is absent-minded. At each exit, he forgets altogether what has happened previously in his journey. Since both exits look entirely the same, he is therefore unable to distinguish between them.[4]

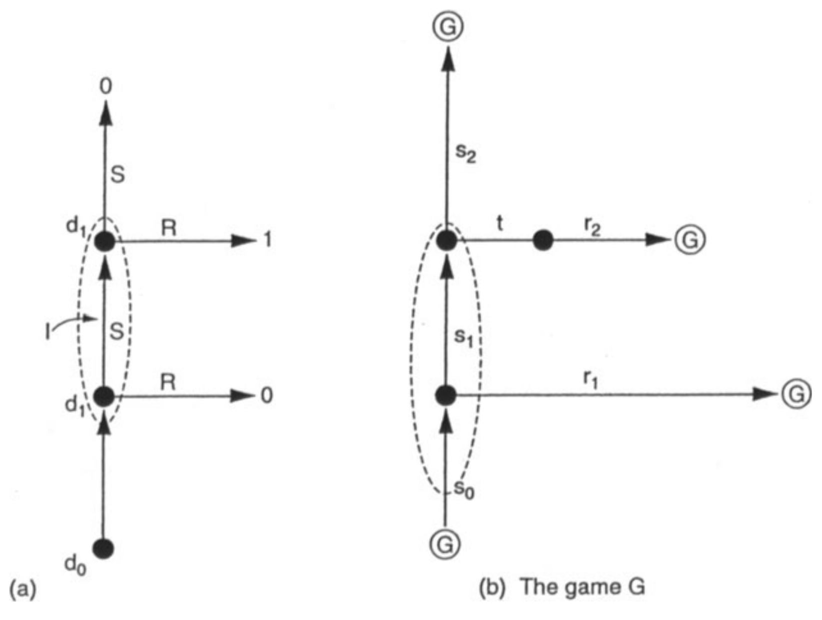

Fig. 2.1. The absent-minded driver's problem

[3] Von Neumann and Morgenstern [17] excluded cases like this by requiring that no play of a game should pass through an information set more than once. However, it has now become customary to accept such cases as particularly challenging examples of games of imperfect recall.

[4] One can, of course, invent ways in which he could supplement his memory. For example, he might turn on the radio on reaching an exit. He could then distinguish between the exits by noting whether his radio is on or off. But the introduction of such expedients is not allowed.

The driver has two pure strategies for this one-player game of imperfect recall, R and S. The use of either results in a payoff of 0. It follows that the same is true of any mixed strategy. However, in such games of imperfect recall, one can achieve more by using a behavioral strategy. A behavioral strategy requires the driver to mix between R and S each time he finds himself called upon to make a decision.[5] Let $b(p)$ be the behavioral strategy in which R is chosen with probability $1 - p$ and S is chosen with probability p. A driver who uses $b(p)$ obtains an expected payoff of $p(1 - p)$, which is maximized when $p = \frac{1}{2}$. According to this analysis, his optimal behavioral strategy is therefore $b(\frac{1}{2})$, which results in his receiving an expected payoff of $\frac{1}{4}$.

The paradox proposed by Piccione and Rubinstein [12] hinges on the time at which the driver chooses his strategy. The argument given above takes for granted that the driver chooses $b(\frac{1}{2})$ before reaching node d_1, and that he can commit himself not to revise this strategy at a later date. But such an attitude to commitment is not consistent with contemporary thinking in game theory. In particular, Selten's [15] notion of a perfect equilibrium, together with all its successors in the literature on refinements of Nash equilibrium, assumes that players will always be re-assessing their strategy throughout the game.[6] More precisely, the orthodox view is not that players *cannot* make commitments, but, if they can, their commitment opportunities should be modeled as formal moves in the game they are playing. However, once their commitment opportunities have been incorporated into the rules of the game, then the resulting game should be analyzed without attributing further commitment powers to the players.

So what happens in the absent-minded driver's paradox if the driver is not assumed to be committed to $b(\frac{1}{2})$? Following Piccione and Rubinstein [12], let us assume that he reaches the information set I and remembers that he previously made a plan to choose $b(\frac{1}{2})$ on reaching I. He then asks himself whether he wants to endorse this strategy now that he knows he has reached the information set I and hence may either be at d_1 or d_2. If he attaches probability $1 - q$ to the event of being at d_1 and probability q to the event of being at d_2, then choosing $b(p)$ at I results in a payoff

$$\pi = (1 - q)p(1 - p) + q(1 - p)\,. \tag{2.1}$$

This payoff is maximized when $p = (1-2q)/2(1-q)$. The driver will therefore only choose $p = \frac{1}{2}$ at I if he believes that $q = 0$. That is to say, in order that a time-inconsistency problem not arise, it is necessary that the driver deny the possibility of ever reaching the second exit. But to deny the possibility of reaching the second exit is to deny the possibility that he can ever get home!

[5] By contrast, a mixed strategy requires a player to randomize over his pure strategies once and for all before the game is played.

[6] McClennen [10] is one of a number of philosophers who insist that rationality includes the facility to commit oneself to perform actions in the future under certain contingencies that one's future self would regard as suboptimal.

3. Whence q?

To make progress with the Paradox of the Absent-Minded Driver, it is necessary to ask how the driver came to believe that the probability of being at the second exit d_2 is q. This question forces us in turn to face a philosophical question about the nature of probability. In the terminology of Binmore [5, p.265], is the driver's probability theory logistic, subjective or objective? A logistic theory treats a probability as the rational degree of belief in an event justified by the evidence. The subjective theory of Savage [14] is the basis for the familiar Bayesian orthodoxy of economics.[7] An objective theory regards the probability of an event as its long-run frequency.

The most satisfactory interpretation for q in the Paradox of the Absent-Minded Driver would be logistic, in the style attempted by Keynes [8]. However, I think it uncontroversial that no theory of this type has yet come near being adequate. My guess is that most economists would take for granted that q is to be interpreted as a subjective probability à la Savage [14]. But there are major difficulties in such an interpretation. In the first place, Bayesian epistomology—as described in Chapter 10 of Binmore [3]—fails in the absent-minded driver's problem.[8] Secondly, if the postulates of Savage's theory are to make sense, it is vital that the action space A, the space B of states of the world, and the space C of consequences have no relevant linkages other than those incorporated explicity in the function $f : A \times B \rightarrow C$ that determines how actions and states together determine consequences (Binmore [5, p.310]). But it is of the essence in the Paradox of the Absent-Minded Driver that states are not determined independently of actions. A rational driver's beliefs about whether he is at the first exit or the second must surely take account of his current thinking about the probability at which he would turn right if he were to reach an exit.

Personally, I think that the most important role for paradoxes like that of the absent-minded driver is to focus attention on the inadequacies of our current logistic and subjective theories of probability. However, I have nothing particularly original to propose on either front, and so follow Piccione and

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[7] Notice that I do not identify Bayesianism with Savage's theory. I distinguish those who subscribe to Savage's view from followers of Bayesianism by calling the former Bayesians and the latter Bayesianismists (Binmore [4, 5]). Bayesianismists argue that rationality somehow endows individuals with a prior probability distribution, to which new evidence is assimilated simply by updating the prior according to Bayes' Rule. Such an attitude reinterprets the subjective probabilities of Savage's theory as logistic. This may sometimes be reasonable in a small-world context, but Savage [14] condemns such a procedure as "ridiculous" or "preposterous" in a large-world context.

[8] The epistomology taken for granted by Bayesian decision theory is simply that a person's knowledge can be specified by an information partition that becomes more refined as new data becomes available. Binmore [3, p.457] discusses the absent-minded driver's problem explicitly in this connexion.

Rubinstein [12] in this paper by turning to the interpretation of q as an objective probability.

If q is to be interpreted objectively, one must imagine that the driver faces the same problem every night on his way home from work. After long enough, it is then reasonable to regard the ratio of the number of times he arrives at the second exit to the total number of times he arrives at either exit as a good approximation to the probability q. Of course, this frequency will be determined by how the driver behaves when he reaches an exit. If the driver always continues straight on with probability P at an exit, then the number of times he reaches d_2 will be a fraction P of the number of times he reaches d_1. It follows that

$$q = P/(1+P) \quad \text{and} \quad 1-q = 1/(1+P)\,. \tag{3.1}$$

One school of thought advocates writing the values for q and $1-q$ from (3.1) into (2.1) and then setting $p = P$ to obtain $\pi = 2p(1-p)/(1+p)$. The result is then maximized to yield the optimal value $p = \sqrt{2}-1$. However, such a derivation neglects the requirement that a decision-maker should maximize expected utility *given* his beliefs. In what follows, I therefore always treat a player's beliefs as fixed when optimizing, leaving only his actions to be determined. However, if (2.1) is maximized with $q = P/(1+P)$ held constant, then the maximizing p satisfies $p = (1-P)/(1+P)$. A time-inconsistency problem then arises unless $p = P$. Imposing this requirement leads to the equation $p^2 + 2p - 1 = 0$, whose positive solution is $p = \sqrt{2} - 1$ as before. In the next section, I plan to defend this result as the resolution of the Paradox of the Absent-Minded Driver in the case when it is possible to interpret q as a frequency.

4. Repeated Absent-Mindedness

One of the things that game theory has to teach is that difficulties in analyzing a problem can sometimes be overcome by incorporating *all* of the opportunities available to the decision-maker into the formal structure of his decision problem. The need to proceed in this manner has been recognized for a long time in the case of precommitment, and I think it uncontroversial to assert that the orthodox view among game theorists is now that each opportunity a player may have to make a precommitment should be built into the moves of a larger game, which is then analyzed without further commitment powers being attributed to the players.

If the absent-minded driver's decision problem is prefixed with such a move—at which the driver commits himself to choosing a probability p with which to continue straight ahead whenever he reaches an exit—then we have seen that the problem reduces to choosing the largest value of $p(1-p)$. However, if the problem is presented without a formal commitment move, as

in Figure 2.1(a), then the convention in game theory is to seek an analysis that does not attribute commitment powers to the driver.

It seems to me that the same should go for Piccione and Rubinstein's [12] assumption that the driver is able to remember a decision made in the past about which action he planned to take on encountering an exit. If there are pieces of information that are relevant to the decisions that might be made during the play of a game, then these should be formally modeled as part of the rules of the game. Otherwise, my understanding of the conventions of game theory is that an analysis of the game should proceed on the assumption that the unmodeled information is not available to the players. In particular, we should analyze the absent-minded driver's decision problem as formulated in Figure 2.1(a) without assuming that the driver remembers anything at all that is relevant to his decision on arriving at I. One could, of course, introduce a new opening move at which the driver makes a provisional choice of behavioral strategy for the problem that follows. However, personally I think that this issue is something of a red herring. As we all know when we dip our toes in the sea on a cold morning, the plans we made earlier when getting changed are not the plans that determine what we actually do. What we do now is determined by the decision we make now.

If I understand correctly, Piccione and Rubinstein [12] want the driver to remember his original provisional plan so that they have grounds for attributing beliefs to him about whether he is at d_1 or d_2. But why should his original plan determine his beliefs if he has no reason to believe that he will carry out his original plan once he reaches I? In my view, this and other issues in the case when q is to be interpreted as a long-run frequency are clarified by explicitly modeling the situation as a *repeated* decision problem, rather than leaving the repetitions to be implicitly understood. One is then led to present the problem as shown in Figure 2.1(b), where it is to be understood that time recedes into the infinite past as well as into the infinite future. In order to avoid the type of time-inconsistency problems pointed out by Strotz [16], it will be assumed that the driver discounts time according to a fixed discount factor δ $(0 < \delta < 1)$.

The label attached to an edge representing an action in Figure 2.1(b) will also denote the time the driver spends between the nodes joined by the edge. Nothing very much hinges on this point, but I think it helpful to imagine that no time at all is spent at a node, but that the driver does all his thinking while driving between nodes. The information set I in Figure 2.1(b) is therefore not quite the same as its cousin in Figure 2.1(a), since it includes not only the nodes d_1 and d_2, but also the open edge that joins d_0 and d_1 and the open edge that joins d_1 and d_2.

I envisage the driver moving through the tree reviewing his plan of campaign as he goes. As noted above, whether he actually remembers his previous plan or not seems irrelevant to the question since he will be reiterating all of the considerations each time he reviews his situation. When he reaches an

exit, he implements whatever plan he currently has in mind. To keep things simple, I assume that there is a probability of 1 that he will review his plan at least once on each edge of the tree. Notice that, as the problem is set up in Figure 2.1(b), the driver always remembers what happened when he faced the same problem in the past. Only within I does he forget something, and then the only relevant matter of which he is unaware is his location in I. (He is, of course, not allowed to carry a clock or to employ any other device that would help him to reduce his uncertainty while in I.)

Within this formulation, two simple points seem apparent to me. While suffering dreadfully with a hangover, I can recall swearing never to drink unwisely again. I can also recall making similar resolutions repeatedly in the past. But plans for the future made under such circumstances remain relevant only while the memory of the hangover is sharp. Once the memory has faded, the joys of convivial company again outweigh the anticipated suffering and one drinks too much again.[9] In brief, if we want to know what someone who is not a regular tippler will do when tempted to overindulge, we need to know how he will view the matter when he is tempted—not immediately after he has overindulged in the past. Similarly, in the problem of the absent-minded driver as modeled in Figure 2.1(b), it seems to me that only the plan the driver makes while in the set I is relevant.

The second point concerns his beliefs while in the set I. The driver always remembers all his past history up to his most recent entry into the set I. He can therefore compute the frequency P with which he continued straight ahead at exits in the past. This information does not tell him for certain what probabilities he should be using to estimate his location in I now, but it is the strongest possible evidence he could possibly have on this subject.[10]

Let v be the expected payoff to the driver, given that he is at node d_1 and will always play $b(p)$ at an exit now and in the future. Let w be the the expected payoff when he is to use $b(p)$ but is now at node d_2. Then

$$\begin{aligned} v &= (1-p)v\delta^{r_1+s_0} + pw\delta^{s_1} , \\ w &= (1-p)\delta^t + (1-p)v\delta^{t+r_2+s_0} + pv\delta^{s_2} , \end{aligned}$$

[9] Alcoholics Anonymous recommend permanent attendance at group sessions. Is this to keep memories sharp by renewing them vicariously through the experience of others?

[10] One might argue that we should model the driver as a computing program. Nothing would then prevent this program from using itself as a subprogram and hence simulating its own thinking processes. Could it not therefore dispense with external information and simply use an introspective analysis to determine what it must have done in the past? Binmore [2] points out that difficulties arise in such cases because of the Halting Problem for Turing Machines. In simple terms, a program that decides what to do on the basis of a prediction of what it is about to do will get into an infinite loop. However, the driver in the formulation given in the text has no need to face the problems that modeling such introspection creates, because he is provided painlessly with the data that the attempt at introspection is intended to generate.

from which it is easy to calculate v and w. One can experiment with various relative values of the time intervals of the model. In some fairly natural cases, it turns out that the driver's informational state is irrelevant, since he would make the same decision at d_1 and d_2 even if he knew his location. The issue is also trivialized by considering the case when the time intervals are fixed and $\delta \to 1$. However, I plan to consider the limiting case when δ is fixed but $r_1 = r_2 = s_2 = T$ and $T \to \infty$. This removes the influence of the future repetitions of the problem on the driver's current behavior while still allowing him access to his decisions in the same situations in the past. Under this simplifying hypothesis,

$$\begin{aligned} v &= p(1-p)v\delta^{s_1+t}, \\ w &= (1-p)\delta^t. \end{aligned}$$

If the driver knew he were between d_0 and d_1, he would want to maximize v and so would choose $p = \frac{1}{2}$. If he knew he were between d_1 and d_2, he would want to maximize w and so would choose $p = 0$. But once he is within the set I he does not know his location. But he can easily compute the conditional probability q that the latter case applies to be $q = Ps_1/(s_0 + Ps_1)$. I take $s_0 = s_1 = s$, so that this formula reduces to $q = P/(P+1)$ as in the analysis of Piccione and Rubinstein [12] discussed in the preceding section. The optimizing calculation for p then proceeds precisely as in their analysis, so that the maximizing p is $(1-P)/(1+P)$.

I see no more paradox in the fact that this value of p differs from that the driver would choose outside I than the fact that it differs from the p he would choose if he knew he were approaching d_1 or d_2. In both cases, he knows less at the time of decision than in the circumstances with which his actual decision is to be compared. In particular, within the set I, he does not know whether he is involved in a decision problem D_1 that starts at d_1 or a decision problem D_2 that starts at d_2. On the other hand, if any plan he made outside I stood a chance of still being in place when d_1 is reached, then he would be choosing in the knowledge that the problem to be solved is D_1.

It remains to argue that $p = P$. In accordance with my doubts about backward induction (Binmore [2]), I do not believe that it is possible to tackle this question adequately without considering out-of-equilibrium behavior. However, in the absence of an algorithmic model of the driver's thinking processes, it is only possible to offer a sketch of how a full argument would go.

Begin by considering the possibility that $p \neq P$. The driver now has a problem because the behavior that his calculation recommends for the infinite future is not consistent with his summary of his behavior in the infinite past. He therefore needs some theory of "mistakes" to explain why he did the wrong thing in the past or why he may not actually do what he has calculated to be optimal in the future.[11] When $p \neq P$, the driver will presumably accept that

[11] I hope that it is uncontroversial to suggest that Selten's theory of the "trembling hand" is too simple a story to be appropriate in the current context.

either P is flawed as a prediction of p or $P/(1+P)$ is flawed as a prediction of q. He will then need to employ more complicated functions of his history to generate the estimates he needs to make a sensible decision. [12] If these functions move the driver's estimates towards consistent values over time, then we have an equilibrium story to tell. In particular, on the equilibrium path we will find that $p = P$, so that $p = \sqrt{2} - 1$.

5. A Team Analysis

My doubts about modeling players as teams of agents with identical preferences have already been mentioned. But it cannot be denied that the methodology has its advantages in games of perfect recall in which no player ever moves more than once. In the presence of the latter proviso, it is irrelevant what would be inferred about a player's future play from a counterfactual deviation on his part from the equilibrium path. At the same time, the "single improvement property" for games of perfect recall ensures that there is no loss of efficiency in allowing agents to make decisions one-by-one (Piccione and Rubinstein [12]).

Gilboa [6] offers the orthodox case for modeling the the absent-minded driver as a team problem. The driver is treated as though he had a multiple personality, with two personalities or agents to be called Alice and Bob. They have the same preferences but different information. When one of the two agents is called upon to make a decision, the agent does not know whether he is at the first exit or the second, but he does know that the other agent will be making an independent decision at the other exit (should it be reached). Figure 5.1(a) shows a representation of the game of perfect recall that must then be played between Alice and Bob. Its opening move is a chance move that assigns control of the driver at the first exit to Alice or Bob with equal probability.[13]

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[12] For example, on his nth entry into the information set I he might perhaps arbitrarily estimate the probability that he has yet to reach node d_1 as the discounted sum

$$Q_n = (1 - \Delta)^{-1}(q_{n-1} + \Delta q_{n-2} + \Delta^2 q_{n-3} + \cdots),$$

where, for each $k < n$, $q_k = p_k/(1+p_k)$ and p_k is the probability with which he actually went straight ahead at intersections during his kth entry into I. On the assumption that Q_n is correct, he can then choose p_n optimally to be $(1 - 2Q_n)/(1 + 2Q_n)$. Then q_n is defined in terms of its predecessors by $q_n = p_n/(1+p_n)$. The question is then whether the sequence $\langle q_n \rangle$ converges.

[13] Although nothing in the specification of the Paradox of the Absent-Minded Driver would seem to provide a strong justification for setting the probability r with which the opening chance move assigns control to Alice equal to 1/2. But if $r \neq 1/2$, we shall not be led to the satisfying conclusion that Alice and Bob should each choose R or S with probability 1/2.

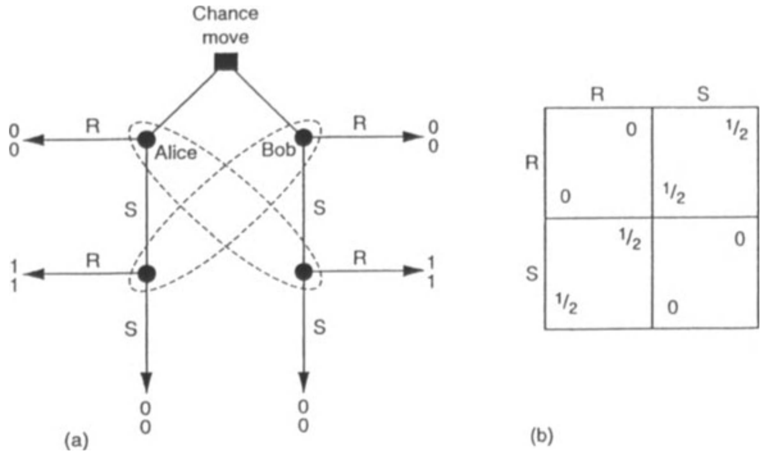

Fig. 5.1. Splitting the driver's personality

Figure 5.1(b) shows the strategic form of the game. Its unique equilibrium is easily calculated. Alice and Bob should each independently choose R or S with probability $\frac{1}{2}$. The outcome is then the same as when the driver commited himself to the behavioral strategy $b(\frac{1}{2})$ before reaching I in the analysis of Piccione and Rubinstein [12].[14] In particular, both Alice and Bob receive an expected payoff of $\frac{1}{4}$.

In spite of the welcome conclusion to which this analysis leads, I believe it to be wrong, because it neglects the fact that what a person does in certain circumstances must surely be evidence about what he will do if placed in exactly same circumstances later on. It is certainly true that arguments that take account of such reasoning can easily go astray. Somehow a line must be drawn between valid uses of the principle and invalid uses that lead authors like Nozick [11] to twinning arguments which supposedly demonstrate that cooperation is rational in the Prisoners' Dilemma (Binmore [5, p.205]). As in much else, my own view is that the right way or ways to proceed will remain mysterious until we have satisfactory *algorithmic* models of the players we study (Binmore [2]).

6. Conclusion

This paper has done no more than comment on the comments made by Piccione and Rubinstein [12] on their Paradox of the Absent-Minded Driver. It claims that the paradox is illusory when the driver has frequency data

[14] This is not an accident. Whatever payoffs are assigned to the possible outcomes in the game, the two analysis yield the same result.

to support his beliefs. But, as Gilboa [6] aptly observes, what counts as a paradox depends on the viewpoint of the observer. As for the paradox in the general case, I have nothing useful to say beyond the observation that it highlights the need for an adequate theory of decision-making under uncertainty to supplement the current Bayesian orthodoxy.

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Futures Market Contracting When You Don't Know Who the Optimists Are*

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Abstract. We have built models of speculative futures trading based upon inconsistent priors, analyzing games of inconsistent incomplete information. These models have assumed that the inconsistent priors are themselves common knowledge. In this paper, we explore the game-theoretic implications of treating doubly inconsistent incomplete information, in that inconsistent priors are private information, and traders attach inconsistent assessments to the probability that a trader will be an optimist.

The result is not arbitrary: the logic of a separating equilibrium can be specified via backwards induction. It is unlikely that subgame-perfect equilibria will exhibit pooling. The volume of speculative trading is reduced by informational constraints, but a sense is specified in which no ex ante agreed-upon Pareto improvements over separating equilibrium behavior can satisfy the information constraints.

1. Introduction

Reinhard Selten's most widely known publications ([1965], [1975], [1978], and the [1988] book with Harsanyi) naturally did not give a complete picture of his research interests. Like nearly all game-theoretic models with some dynamic structure, our analysis uses the concept of subgame-perfect equilibrium ([1965]). However, we rely more heavily on a less standard concept, games of inconsistent incomplete information, first formalized in Selten [1982]. Also, our paper points to a less-widely known feature of Selten's scholarly career: his continuing emphasis on the relevance of game-theoretic techniques to the modeling of applied problems (indeed, the original 1965 paper focuses on oligopoly models with repeat purchase loyalty). Our application concerns futures markets for basic commodities.

The sort of futures markets we seek to model exist for many commodities, including metals and crude oil. The market in mind when selecting model

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simplifications is the London International Petroleum Exchange for Brent Blend (a standardized mixture of North Sea crude oil, cf. Phlips [1992] for a description). It bears emphasis that this market (like many futures markets) does not offer opportunities to purchase today a quantity for delivery at some future date. It is a purely financial or "paper" market: a buyer of a futures contract pays a seller today the agreed-upon price of the contract; in return, the seller makes a purely financial payment to the buyer at the specified maturity date, with the amount of that payment determined by the spot market price prevailing on the maturity date. (There is no option to demand delivery of the commodity at maturity, nor any constraint that the total outstanding contracts must be less than the deliverable amount of the commodity.) A trader's "position" or, more precisely, net short position, is his total sales less his total purchases of futures contracts. In most trades, the buyer is simply gambling that the spot price will constitute a good return on the money invested in buying the futures contract, while the seller is gambling that it will not.

The standard literature on futures markets assumes all agents active in these markets have common priors. This assumption is needed to construct rational expectations models in which the futures price is an unbiased predictor of the spot price at maturity. Instead, we use the more realistic assumption that agents have different priors: their beliefs are *inconsistent*, reflecting differences in opinion, not in information. For concreteness, each agent is one of two *types*: either an "optimist" or a "pessimist." The optimist expects the spot price at maturity to be high; the pessimist expects it to be low. As these expectations are due to differences in opinion, in our model, an optimist would not adjust his estimate of the spot price upon learning that another agent is a pessimist. His estimate has already taken into account the possibility that other agents might have expectations he views as incorrect.

In addition, we suppose that agents know only their own type and "don't know who the optimists are." They assign subjective probabilities to other agents' types, and these probabilities are also inconsistent. There is thus a double inconsistency, in that agents have inconsistent beliefs about inconsistent beliefs of their trading partners. Our purpose is to show that a game-theoretic analysis of such a situation, which is described by Selten [1982] as one of *inconsistent incomplete information*, is possible and provides interesting insights into the working of futures markets.

We have studied futures markets for natural resources in circumstances featuring extractors with market power in the cash market (Phlips and Harstad [1991], Harstad and Phlips [1990], Phlips and Harstad [1992], listed in the order they were written). Those papers assumed inconsistent priors, which enabled explanations of some stylized facts about futures markets that are inconsistent with mainstream futures models, principally a nonzero volume of rational speculative trade. Here we address a key limitation of those papers, exploring the considerable complications that arise under the more reasonable assumption that the fact of inconsistent priors is common knowledge, but the priors themselves are

private information. When simplifying assumptions sidestep other issues, we will refer to one of the other papers addressing them.

We basically use the three-player, two-stage noncooperative game studied in Phlips and Harstad [1991]. Two risk-averse producers of an exhaustible resource simultaneously set market supplies in each of two successive spot markets. Prior to the first spot market, they and a risk-neutral speculator can trade futures contracts for the resource, contracts which mature as of the second spot market.

We show that the double inconsistency of beliefs has no impact on the producers' spot market activity, except through the futures contracts signed. Nonetheless, the speculator can (and must) evaluate the impact of the producers' spot market behavior on the expected profitability of his own futures position.

Subgame-perfect equilibria were found to reach Pareto-efficient outcomes when the players' types were common knowledge. Under the assumptions made here, the desired equilibrium outcomes are also located on contract curves. However, these curves are based on subjective probabilities. Consequently, the contracted futures positions may be larger or smaller than the "objectively" efficient ones. In a world characterized by uncertainty, however, an allocation mechanism is appropriately judged by its efficiency relative to the information available at the time the mechanism is put in place. In that sense, the game analyzed here reaches an ex ante efficient outcome in subgame-perfect equilibrium. Nonetheless, ex post, objective inefficiencies occur.

We emphasize that the behavior we analyze below is fully rational, and indeed, assumes players correctly handle extremely complex calculations. It is simply behavior that is not conditioned on adjusting one's own beliefs for the beliefs of other traders.

2. The Order of Moves

The game is complex despite our efforts to avoid tangential complications. To sort out detail, the order of moves is presented here, with the nature of incomplete information specified in the next section, and spot market cost and demand conditions following in section 4. Hence, specification of the producers' payoffs—certainty equivalent levels of sums of profits from futures market and spot market transactions—and the speculator's payoff—expected profit from his futures dealings—are delayed until equations $\langle 3 \rangle$ and $\langle 5 \rangle$ in section 4.

The set of players is $\{\mathcal{A}, \mathcal{B}, \mathcal{S}\}$. To begin, a futures market for the natural resource is opened at the beginning of period 1. The game unfolds as follows:

Step 1: Nature determines whether a player is optimistic or pessimistic, and privately informs each player of his type. Nature also determines a "commitment order," that is, a permutation of the order $(\mathcal{A},\ \mathcal{B},\ \mathcal{S})$, which is common knowledge.

Step 2: Each player simultaneously announces one futures contract offer, either a sale or a purchase offer. Each offer consists of a (*price, quantity*) pair, and is an all-or-nothing offer (neither price or quantity is negotiable). Once announced, the offers are irrevocable.

Step 3: The player designated *first* in the commitment order irrevocably accepts or rejects each contract offered by another player.[1]

Step 4: The player designated *second* in the commitment order irrevocably accepts or rejects each contract offered by another player and not yet accepted. (A contract accepted by one player is not available for later acceptance by another player.)

Step 5: The player designated *third* in the commitment order accepts or rejects each contract offered by another player and not yet accepted. The futures market then *closes*.

Step 6: The extraction subgame occurs, that is, producers $\mathcal{A}$ and $\mathcal{B}$ simultaneously determine what fraction of their stock of the resource to supply before the maturity date of the futures contracts. Technically, this ends the game.

Step 7: The market demand level is revealed and determines the spot price at maturity, given the extraction decisions in step 6.

Our analysis throughout will be limited to consideration of pure strategies, presumably not a serious restriction when a continuum of actions is available at step 2. Several simplifications follow from a traditional assumption: that a player indifferent between accepting and rejecting a contract in steps 3–5 accepts.

3. Inconsistent Incomplete Information

There are two sources of uncertainty. The first is about the level of aggregate demand: each player has an expectation $\widehat{p}_i$ $(i = \mathcal{A}, \mathcal{B}, \mathcal{S})$. These expectations are inconsistent, since drawn from different distributions. For simplicity, these expectations are of two types: either $\widehat{p}_i = \widehat{p}^H$ or $\widehat{p}_i = \widehat{p}^L < \widehat{p}^H$—a player is either an optimist or a pessimist. All optimists have the same mean expectation $\widehat{p}^H$. All pessimists have the same mean expectation $\widehat{p}^L$. The values $\widehat{p}^H$ and $\widehat{p}^L$ are common knowledge. An economic interpretation of $\widehat{p}$ is delayed until analysis of the extraction subgame in Section 4.

Second, each player knows his own type but does not know the types of the other players. They know that Nature determines whether a player is an optimist or a pessimist with probabilities λ and $(1 - \lambda)$. However, each player has his own personal evaluation of λ, λ_i, $i = \mathcal{A}, \mathcal{B}, \mathcal{S}$.

If a "true" λ were commonly known, all conditional subjective probabilities that any other player is an optimist or a pessimist would be equal to λ or $1 - \lambda$, respectively. To illustrate, consider the probability matrix of type combinations given in Table 1. An optimistic $\mathcal{S}$ is represented as $\mathcal{S}^H$, and a pessimistic $\mathcal{S}$ as $\mathcal{S}^L$, with types of $\mathcal{A}$ and $\mathcal{B}$ analogously represented. The sum of its 8 elements is 1. This matrix is uncorrelated only if $\lambda = 0.5$. If correlated, however, any given player, say $\mathcal{S}$, with belief λ_s, still attaches a conditional probability λ_s that any other given player, say $\mathcal{A}$, is an optimist.

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[1] That is, he chooses to accept both contracts, accept either contract, or reject both contracts offered.

Table 1. Probability Matrix of Type Combinations

	$\mathcal{A}^H$ $\mathcal{B}^H$	$\mathcal{A}^H$ $\mathcal{B}^H$	$\mathcal{A}^L$ $\mathcal{B}^H$	$\mathcal{A}^L$ $\mathcal{B}^H$
$\mathcal{S}^H$	λ^3	$\lambda^2(1-\lambda)$	$\lambda^2(1-\lambda)$	$\lambda(1-\lambda)^2$
$\mathcal{S}^L$	$\lambda^2(1-\lambda)$	$\lambda(1-\lambda)^2$	$\lambda(1-\lambda)^2$	$(1-\lambda)^3$

Consistent incomplete information is defined by the condition that all players know the true λ and therefore have the same subjective probabilities. *Inconsistent* incomplete information allows each player to have a different theory about how Nature selects types and, therefore, a different λ. Then we may have $\lambda_s \neq \lambda_a \neq \lambda_b$. Notice, however, that the values $\lambda_s, \lambda_a, \lambda_b$ and the corresponding probability matrices are common knowledge in the inconsistent as well as in the consistent case.

4. The Extraction Subgame

This section concludes the game's description by specifying conditions of the extraction subgame, step 6, and then begins analysis with that step. The two producers have exogenous initial stocks s_{a0} and s_{b0}, which are common knowledge. In period 1, each extracts and sells q_{i1} units ($i = \mathcal{A}, \mathcal{B}$). In period 2, that is, at maturity, they extract and sell the remainder

$$q_{i2} = s_{i0} - q_{i1}. \qquad \langle 1 \rangle$$

In both periods, the instantaneous inverse demand function is

$$p_t(\, q_{at} + q_{bt}\,) = \alpha - (\, q_{at} + q_{bt}\,), \qquad t = 1, 2. \qquad \langle 2 \rangle$$

The intercept $\alpha > 0$ is unknown. The price p_t is a present price, net of extraction cost. Risk aversion implies the certainty equivalent payoffs

$$W_a = \mathrm{E}_a[\pi_a] - \tfrac{M_a}{2}\,(\, s_{a0} - f_{ab} - f_{as}\,)^2, \qquad \langle 3.1 \rangle$$

$$W_b = \mathrm{E}_b[\pi_b] - \tfrac{M_b}{2}\,(\, s_{b0} + f_{ab} - f_{bs}\,)^2, \qquad \langle 3.2 \rangle$$

where

$$\pi_a = p_2(\, q_{a2} + q_{b2}\,)\,(\, q_{a2} - f_{ab} - f_{as}\,) + p_1(\, q_{a1} + q_{b1}\,)\, q_{a1}$$
$$+\, p_{ab} f_{ab} + p_{as} f_{as}, \qquad \langle 4.1 \rangle$$

$$\pi_b = p_2(q_{a2} + q_{b2})(q_{b2} + f_{ab} - f_{bs}) + p_1(q_{a1} + q_{b1})q_{b1}$$
$$- p_{ab}f_{ab} + p_{bs}f_{bs}, \qquad \langle 4.2 \rangle$$

$f_{ab} > 0$ is a futures position such that $\mathcal{A}$ sells to $\mathcal{B}$ ($\mathcal{A}$ "goes short" and $\mathcal{B}$ "goes long") at the futures price p_{ab} agreed upon, and similarly for f_{as} and f_{bs}. Of course, $f_{ba} = -f_{ab}$, so the futures market revenue terms in ⟨4.1⟩ and ⟨4.2⟩ reverse the sign on p_{ab}. All futures positions are closed out at price p_2 in period 2. Thus, equations ⟨4.1⟩ and ⟨4.2⟩ simply sum spot-market revenues and futures-market net revenues. Equations ⟨0.1⟩ and ⟨0.2⟩ use the mean-variance model (with constant absolute risk aversion parameters M_a, M_b) and the fact that $p_2(q_{a2} + q_{b2})$ and $p_1(q_{a1} + q_{b1})$ have a common uncertain parameter α. The variance of each producer's belief about α is normalized to 1, to simplify. There is no covariance term. Notice that a producer's degree of risk aversion affects his payoff so long as his net futures position leaves some of his initial stock unhedged.

To complete the specification of the game, the risk-neutral speculator's payoff is simply his net revenue on the futures market:

$$\phi_s = E_s[(p_2 - p_{as})f_{as} + (p_2 - p_{bs})f_{bs}]. \qquad \langle 5 \rangle$$

We now analyze behavior in the extraction subgame. With given futures positions, simultaneous maximization of W_a in q_{a1} and W_b in q_{b1} (given ⟨1⟩) gives the unique Cournot equilibrium strategies

$$q_{a1} = \tfrac{s_{a0}}{2} - \tfrac{f_{ab}+f_{as}}{3} + \tfrac{f_{bs}-f_{ab}}{6}; \quad q_{a2} = \tfrac{s_{a0}}{2} + \tfrac{f_{ab}+f_{as}}{3} - \tfrac{f_{bs}-f_{ab}}{6}; \qquad \langle 6.1 \rangle$$

$$q_{b1} = \tfrac{s_{b0}}{2} - \tfrac{f_{bs}-f_{ab}}{3} + \tfrac{f_{ab}+f_{as}}{6}; \quad q_{b2} = \tfrac{s_{b0}}{2} + \tfrac{f_{bs}-f_{ab}}{3} - \tfrac{f_{ab}+f_{as}}{6}. \qquad \langle 6.2 \rangle$$

Producer $\mathcal{A}$'s net short futures position is $f_{ab} + f_{as}$. Producer $\mathcal{B}$'s net short position is $f_{bs} - f_{ab} = f_{bs} + f_{ba}$. Each producer is seen to adjust the amount extracted prior to maturity (q_{i1}) for the net futures position he has taken and to partially counteract the adjustment his rival makes. If $f_{ab} + f_{as} > 0$ and $f_{bs} - f_{ab} < 2(f_{ab} + f_{as})$, then $\mathcal{A}$ makes his net short position more profitable by shifting extraction to at-maturity supply, driving down the spot price at maturity, and with it the value of the futures contracts he has (net) sold.

The equilibrium spot prices are

$$p_1 = \hat{p} + \tfrac{1}{6}(f_{as} + f_{bs}); \qquad p_2 = \hat{p} - \tfrac{1}{6}(f_{as} + f_{bs}), \qquad \langle 7 \rangle$$

where $(f_{as} + f_{bs})$ is the producers' combined net short position (or the speculator's net long position) and

$$\widehat{p} = \alpha - \tfrac{s_{a0}+s_{b0}}{2}.$$

This parameter can now be given a straightforward economic interpretation: $\widehat{p}$ is the spot price that would prevail in both periods in Cournot equilibrium, if the producers were both inactive in the futures market. It is a natural benchmark. Since it is also a linear function of α, it is convenient to express beliefs in terms of $\widehat{p}$ rather than α. The players' beliefs are thus expressed as the mean expectations $\widehat{p}_a, \widehat{p}_b$ and $\widehat{p}_s$.

It should be noted that, given futures market positions, the equilibrium extraction rates do not depend upon the producers' beliefs about α. A trading partner of a producer on the futures market can therefore predict how the producer will shift extraction in reaction to contracts accepted without knowing the producer's expectations. It remains to be seen whether he can predict if the producer will accept the contract.

5. Subgame-Perfect Acceptable Contracts

In determine subgame-perfect actions in steps 5 to 2, we first incorporate the extraction subgame's equilibrium strategies ⟨6.1⟩-⟨6.2⟩ and the equilibrium prices ⟨7⟩ in the payoff functions W_a and W_b of the producers and ϕ_s of the risk-neutral speculator. The result is

$$W_a = s_{a0}\,\widehat{p}_a + V_a, \qquad \langle 8.1\rangle$$

$$W_b = s_{b0}\,\widehat{p}_b + V_b, \qquad \langle 8.2\rangle$$

$$V_s = (\widehat{p}_s - p_{as})\,f_{as} + (\widehat{p}_s - p_{bs})\,f_{bs} - \tfrac{1}{6}\,(f_{as} + f_{bs})^2, \qquad \langle 8.3\rangle$$

where

$$V_a = (p_{ab} - \widehat{p}_a)f_{ab} + (p_{as} - \widehat{p}_a)f_{as} + \frac{1}{18}(f_{as} + f_{bs})^2 - \frac{M_a}{2}(s_{a0} - f_{ab} - f_{as})^2, \qquad \langle 9.1\rangle$$

$$V_b = (\widehat{p}_a - p_{ab})f_{ab} + (p_{bs} - \widehat{p}_b)f_{bs} + \frac{1}{18}(f_{as} + f_{bs})^2 - \frac{M_b}{2}(s_{b0} + f_{ab} - f_{bs})^2. \qquad \langle 9.2\rangle$$

We focus on V_a, V_b and V_s, because the expected value of the stock, $s_{i0}\,\widehat{p}_i$, is unaffected by futures trading or by extraction rates.

5.1. Calculation of Barely Acceptable Contract Terms

When announcing a particular offer (a price and a quantity), a player aims at obtaining contract terms that are most favorable to himself and yet acceptable to the player for whom the announced contract is intended. Each player therefore

has to determine which terms are just acceptable to the other side of the market, namely, the highest price acceptable to the other party if an offer to sell is being announced, and the lowest price acceptable to the other party if an offer to purchase is being announced. Player $\mathcal{B}$, say, can do this by solving

$$V_a - V_a \mid_{f_{ab}=0} \geq 0.$$ [2]

If $\mathcal{B}$ wants to purchase from $\mathcal{A}$ ($f_{ab} > 0$), any price equal to or higher than the price that satisfies the equality is acceptable to $\mathcal{A}$, since

$$V_a - V_a \mid_{f_{ab}=0}$$

is the contribution of the futures position f_{ab} to $\mathcal{A}$'s expected profit and any nonnegative contribution will be preferable to not trading with $\mathcal{B}$. The solution is

$$p_{\boldsymbol{a}b} \gtrless [\widehat{p}_a - M_a(s_{a0} - f_{as})] + \tfrac{M_a}{2} f_{ab} \qquad \text{as} \quad f_{ab} \gtrless 0, \qquad \langle 10.1 \rangle$$

The boldfaced italic subscript indicates the player for whom the price is acceptable. By the same reasoning,

$$p_{a\boldsymbol{b}} \gtrless [\widehat{p}_b - M_b(s_{b0} - f_{bs})] - \tfrac{M_b}{2} f_{ab} \qquad \text{as} \quad f_{ab} \gtrless 0, \qquad \langle 10.2 \rangle$$

$$p_{\boldsymbol{a}s} \gtrless [\widehat{p}_a - M_a(s_{a0} - f_{ab}) - \tfrac{1}{4} f_{bs}] + (\tfrac{M_a}{2} - \tfrac{1}{18}) f_{as} \qquad \text{as} \quad f_{as} \gtrless 0, \qquad \langle 10.3 \rangle$$

$$p_{a\boldsymbol{s}} \gtrless [\widehat{p}_s - \tfrac{1}{3} f_{bs}] - \tfrac{1}{6} f_{as} \qquad \text{as} \quad f_{as} \gtrless 0, \qquad \langle 10.4 \rangle$$

$$p_{\boldsymbol{b}s} \gtrless [\widehat{p}_b - M_b(s_{b0} + f_{ab}) - \tfrac{1}{4} f_{as}] + (\tfrac{M_b}{2} - \tfrac{1}{18}) f_{bs} \qquad \text{as} \quad f_{bs} \gtrless 0, \qquad \langle 10.5 \rangle$$

$$p_{b\boldsymbol{s}} \gtrless [\widehat{p}_s - \tfrac{1}{3} f_{as}] - \tfrac{1}{6} f_{bs} \qquad \text{as} \quad f_{bs} \gtrless 0, \qquad \langle 10.6 \rangle$$

The difficulty for $\mathcal{B}$, in evaluating $p_{\boldsymbol{a}b}$, for example, is that $\mathcal{B}$ does not know whether $\mathcal{A}$ is an optimist ($\widehat{p}_a = \widehat{p}^H$, or $\mathcal{A}^H$ for short) or a pessimist ($\widehat{p}_a = \widehat{p}^L$, $\mathcal{A}^L$). The two possibilities define two boundaries for the price-quantity combinations just acceptable to $\mathcal{A}$. The same is true when $\mathcal{A}$ evaluates the terms acceptable to

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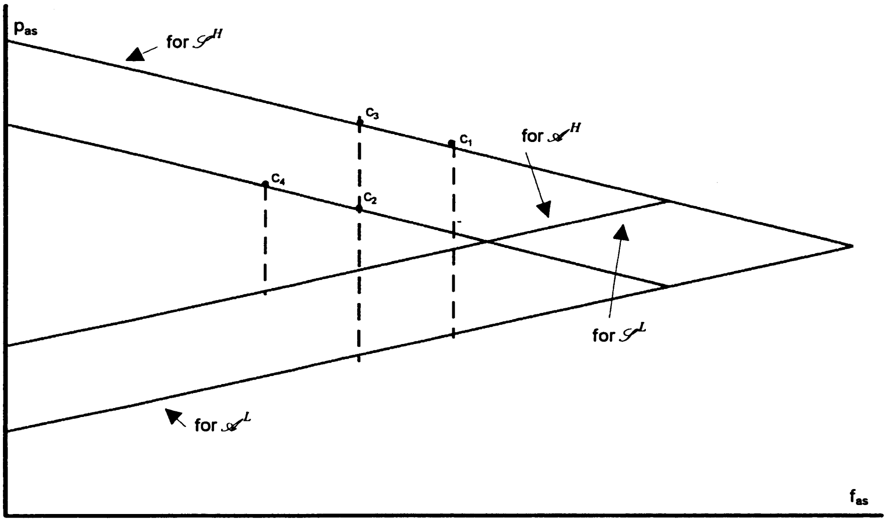
[2] This equation assumes that $\mathcal{B}$ has not accepted and will not accept any contract offered by $\mathcal{A}$. Violations are pathological, and analyzed similarly; the only simplification lost is the linearity of what would otherwise be parallel quadratic curves in Figures 1 and 2.

$\mathcal{B}$ according to $\langle 10.2 \rangle$. For each of the possible contracts, a figure similar to Figure 1 can be drawn.

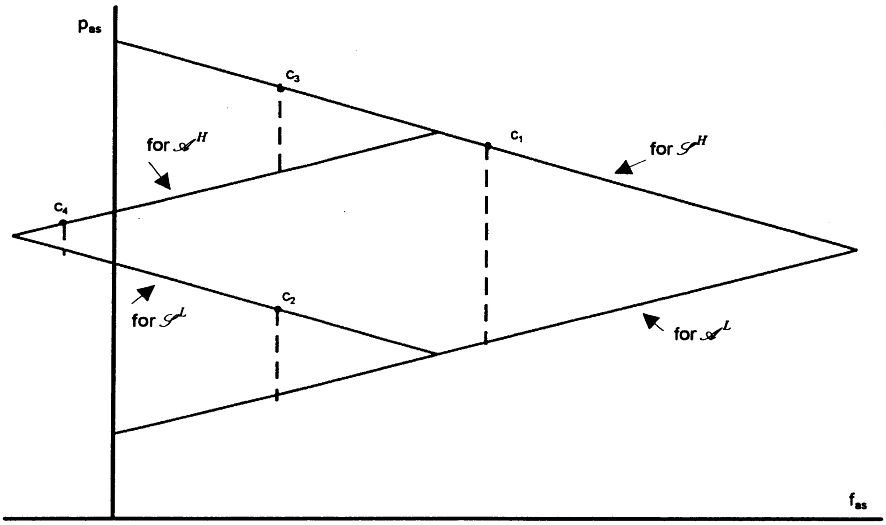
Figure 1



5.2. The Basic Geometry of Acceptable Contracts

Figure 1 illustrates the case of a sale by $\mathcal{A}$ to $\mathcal{S}$ ($f_{as} > 0$). The two upward-sloping lines represent the lowest selling prices type $\mathcal{A}^H$ can accept (i.e., for $\widehat{p}_a = \widehat{p}^H$) and (lower) $\mathcal{A}^L$ can accept. The two downward-sloping lines represent the highest prices at which types $\mathcal{S}^H$ and $\mathcal{S}^L$ are each willing to buy. Slopes of the lines simply reflect risk aversion. The vertical distance between pairs of parallel lines is $\widehat{p}^H - \widehat{p}^L$ (for both pairs). The relative positioning of the pairs of lines reflects sizes of stocks, extent of risk aversion, and, most importantly, positions on contracts with the third player ($\mathcal{B}$, in the case shown where a contract between $\mathcal{A}$ and $\mathcal{S}$ is being analyzed), as indicated in equations $\langle 10 \rangle$. When a sufficiently large share of the potential gains to trading futures between $\mathcal{A}$ and $\mathcal{S}$ have been usurped by contracts one or the other player reaches with $\mathcal{B}$, this positioning may become tight enough to switch from Figure 1 to Figure 2. If information limitations can be surmounted, trading is possible whenever an upward-sloping and a downward-sloping line cross, and for price-quantity combinations within the triangle formed by these lines and the vertical axis.

Figure 2



6. Contract Acceptances in the $\mathcal{ABS}$ Order

Description of subgame-perfect equilibrium, and the associated inferences about types, quickly gets complex. Nonetheless, analytic steps have geometric interpretations, given below. As the qualitative properties do not depend upon the commitment order, we will present the analysis only for the order $\mathcal{ABS}$. This substantially reduces repetition and adds some useful concreteness; the cost is a suggestion of higher payoff for $\mathcal{A}$ and lower payoff for $\mathcal{S}$, in general, than occurs across commitment orders. Where it aids concreteness, we will assume parameter configurations with the property that, were it common knowledge that all three players were of the same type (all optimists or all pessimists), then $\mathcal{A}$ would take a net short position, $\mathcal{S}$ a net long position, and $\mathcal{B}$ an intermediate position. This would be accomplished by giving $\mathcal{A}$ a larger stock than $\mathcal{B}$, and setting their risk postures close approximations to each other, relative to stocks. While these parameter configurations are not essential to the analysis, they simplify presentation, and fit with the intuitive value of imagining f_{ab}, f_{as}, f_{bs} all being positive. (In particular, they underlie presentation of figures with a positive horizontal axis.)

6.1. Notation for Pooling or Separating Announcements

The analysis of subgame-perfect equilibrium proceeds, as is usual, backwards from Step 5. Before beginning, however, it is essential to note forward inferences resulting from Step 2 behavior. It is common in games of incomplete information to distinguish between "pooling" and "separating" equilibria: in a pooling equilibrium, a player's type is not revealed by his behavior, as both types choose

the same strategy in equilibrium. In a separating equilibrium, each type selects a different strategy, and no uncertainty about a player's type remains after his action is observed.

Intermediate "semi-pooling" cases can arise here. Specifically, one or two players could choose announcements which reveal their type, with the rest of the players choosing pooling announcements in Step 2. During Steps 3, 4 and 5, all players will face no uncertainty in predicting the behavior of a player who made a separating announcement, but may be unsure of which contracts, if any, will be acceptable to a player who chose a pooling announcement. To handle all possibilities, the following notation will be employed. Player i's announcement made in step 2 will be denoted $c_i = (p_i, f_i)$, $i = \mathcal{A}, \mathcal{B}, \mathcal{S}$, where f_i is a short position: $f_i > 0$ implies i is offering to sell a quantity f_i of futures contracts at price p_i, while i is offering to buy a quantity $-f_i$ of futures contracts (go long) at price p_i if $f_i < 0$. The information available in step 3 about players' types as revealed in equilibrium announcements $c = (c_a, c_b, c_s)$ is summarized by $I = (I^{a\tau}, I^{b\tau}, I^{s\tau})$, where $I^{i\tau}$ takes the value I^{iH} if i made a separating announcement revealing type i^H (optimist), I^{iL} if i made a separating announcement revealing type i^L, and I^{i0} if both types of i made the same announcement.

6.2. Summary of Futures Contract Acceptance Behavior

This subsection summarizes subgame-perfect behavior in steps 5, then 4, then 3.[3] Whether $\mathcal{A}$ in step 3 or $\mathcal{B}$ in step 4 gains by accepting an available contract depends on the anticipated contract acceptance behavior to follow. Only if a player has identified his type via a separating announcement can his contract acceptance behavior be predicted with certainty.

PROPOSITION 1: Under doubly inconsistent incomplete information, any set of contract announcements yields a unique subgame-perfect equilibrium continuation.

Uniqueness depends upon the convention that a player indifferent between accepting and rejecting a contract accepts. The point, though, is that doubly inconsistent incomplete information does not create a game in which any behavior whatsoever can be rationalized. Contract acceptance behavior is as uniquely determined by the rationality postulates supporting subgame-perfect equilibrium as in a game of consistent incomplete information. Probabilistically, it is also just as determinable as if information were consistent.

PROPOSITION 2: In subgame-perfect equilibrium continuation following any set of contract announcements, no player alters his contract acceptance behavior to avoid revealing his type.

Separating equilibrium behavior in steps 3–5 stems from two sources: independence of extraction behavior from beliefs, and independence of payoffs from rivals' types, given the contracts that are accepted by rivals.

[3] Propositions stated here are available to the reader in Harstad and Phlips [1993], the fuller version of this paper.

PROPOSITION 3: Contract acceptance behavior of any player i when any rivals deciding after i have revealed their types is essentially a matter of anticipating which contracts will be accepted following any behavior by i, and then simply deciding whether i would prefer the pattern of acceptances that includes his acceptance of an available contract to the pattern that includes his rejection of that contract.

PROPOSITION 4: Contract acceptance behavior of any player i when some rival deciding after i has not revealed his type involves anticipating the subjectively expected pattern of acceptances following any behavior by i, and then simply deciding whether i would prefer the stochastic pattern of acceptances that includes his acceptance of an open contract to the stochastic pattern that includes his rejection of that contract. When the player committing first makes this determination, he anticipates the behavior of the player committing second by anticipating the behavior of the player committing third using the second player's prior beliefs (with which he disagrees), and then using his own prior beliefs to predict both rivals' behavior.

7. Step 2 Announcements in the $\mathcal{ABS}$ Order

It is almost a tautology that the details of subgame-perfect decisions about which contracts to offer cannot be less complex than the details of acceptance behavior. We again offer a summary:

PROPOSITION 5: In any subgame-perfect equilibrium, the player committing first makes a separating announcement in step 2, revealing his type.

Since the announcements are simultaneous, $\mathcal{A}$'s rivals in the $\mathcal{ABS}$ order cannot gain in step 2 from learning $\mathcal{A}$'s type. Proposition 0 has already shown that $\mathcal{A}$ will choose to separate, revealing his type, in step 3 before his rivals take their next actions. So in a pooling announcement, at least one type of player $\mathcal{A}$ would be sacrificing payoff.

Proposition 5 prevents the existence of a "completely pooling" equilibrium in which all three players make pooling announcements. The principal results of this section characterize "completely separating" equilibria in which all three players announce separating contract offers:

PROPOSITION 6: If the second (j) and third (k) players in the commitment order make separating announcements (as does the first player i), then i:

[a] examines each pair of rivals' types for which an offer is accepted by k with some given probability and determines the conditionally efficient contract for each pair;

[b] determines, among these efficient contracts with k, the one that has the highest expected profit;

[c] repeats steps [a] and [b] to determine an efficient offer to j;

[d] finally selects the offer (directed to j or k) that provides the highest expected profit.

Players j and k follow essentially the same logic.

PROPOSITION 7: Proposition 6 implies that targeting any announcement to the player with whom the greatest gains from trade are at stake is typically best response behavior. In general, the resulting equilibrium is not unique.[4]

7.1. Remarks on Existence of a Separating Equilibrium

To our knowledge, the literature on existence of a separating equilibrium has not offered a proof covering the case of an extensive form with three players simultaneously choosing from an unordered continuum of signals prior to a sequential-move subgame; this paper will not change that situation. By and large, adding realism to the deliberately demonstrative model considered here would surely not ease these complications.

It is also possible that the nonconvexities constitute the sort of "controlled discontinuities" for which Dasgupta and Maskin [1986] offer existence theorems. At least generic use of such an approach may be possible via the following sort of truncation argument. For arbitrary behavior in the futures market, there exists a unique subgame-perfect continuation in the extraction subgame, so the game can be truncated by replacing this subgame with its payoffs. For arbitrary contract offers in step 2, the convention that offers are accepted when the player is indifferent makes the subgame-perfect continuation uniquely determined. Thus, payoffs can in principle be specified so that the game can be truncated after step 2. The resulting game is a simultaneous-move game in which each player's strategies form a compact subset of $\mathbb{R}_2$,[5] and payoffs are quadratic functions of strategies except on sets of measure zero.

We found a one-parameter family of subgame-perfect equilibria in our similar model where inconsistent beliefs were common knowledge (Phlips and Harstad [1991]); this is also encouraging for the existence of a separating equilibrium here.

7.2. Remarks on Existence, Nature and Plausibility of a Semi-Pooling Equilibrium

Proposition 5 has shown the nonexistence of completely pooling equilibrium, as the player who commits first does not pool. Thus, the issue discussed here is the possibility of a semi-pooling equilibrium, in which both types would make the same contract offer announcement, and then separate in their contract acceptance behavior, for one or both of the players committing last. In this section, for this model, that is what the term "pooling equilibrium" will mean.

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[4] The reader interested in the detailed logic of equilibrium announcements is referred to Harstad and Phlips [1993].

[5] Compactness is no problem. Since the V_i functions all involve quadratic terms in futures positions with negative coefficients, there must exist a finite number which is a nonbinding absolute bound on the f_i coefficient of any best response. With the f_i's artificially but harmlessly bounded, it is then similarly possible to obtain a nonbinding, finite upper bound on the p_i component of best responses. Without loss of generality, the strategy sets can be altered to incorporate these bounds.

The canonical signaling games (see van Damme [1987], ch. 10, for an introduction) achieve pooling equilibria by having the receiving player's payoff depend directly on the signaling player's type, and by having the receiver respond to an out-of-equilibrium message based upon an assumption about sender's type that is unfortunate for one type of sender. As a result, that type of sender's predominant interest is in imitating the equilibrium message of the other type of sender.

It does not require the sort of subtle thinking debated in the refinements literature (van Damme [1987] and references cited) to restrict acceptance behavior following out-of-equilibrium contract offer announcements. A player's payoff depends upon a rivals' type only through the contracts that are accepted; no player changes his own contract acceptance behavior directly as a result of his belief about a rival's type. The only possible impact is that during step 3 or 4, a player may alter his contract acceptance behavior in response to a belief that a rival's type will lead to particular acceptance behavior in following steps; in what follows, we call this the "pooling impact." Acceptance behavior that serves directly to reduce the payoff of a rival whose type is uncertain, or is believed to be revealed by an out-of-equilibrium move is clearly disequilibrium behavior in this model.

Thus, there is no particular incentive for types of a player to pool in order to avoid revealing their types, other than the pooling impact. Its possible benefit is limited by the binary nature of the sequential decisions. It seems most likely, in a pooling equilibrium scenario, that a pooling impact beneficial to one type of a player is harmful to that player's other type. It follows that type t' will not benefit from an effort of type t to imitate type t', so t' will only alter his contract announcement from a separating equilibrium choice if and only if doing so separates. Hence, t's benefit from the pooling impact would have to increase his equilibrium payoff more than the decrease that resulted from announcing the contract that was a best response for t' rather than for t. We have no proof that such pooling equilibria exist only for unlikely parameter constellations, but this seems the most natural conclusion to conjecture.

Should both a separating and a pooling equilibrium exist, refinements that are only well-defined for finite games or single-signaler, single-receiver games will not directly offer useful distinctions. However, we can envision an argument of the following sort. Any pooling equilibrium supported by beliefs that an out-of-equilibrium message was sent by the type benefiting from the pooling impact, when that message could be a sensible announcement for the type who does not benefit from the pooling arrangement is likely to seem less plausible than the separating equilibrium, under the same sort of arguments raised in refining pooling equilibria of finite signaling games.

8. Market Efficiency

We briefly discuss the efficiency of trading in this model, assuming that behavior is characterized by a subgame-perfect separating equilibrium. Initially consider

the singly-inconsistent case: whether each player is optimistic is his own private information, but the subjective prior odds are consistent: $\lambda_a = \lambda_b = \lambda_s$. Each player is offering a take-it-or-leave-it contract which maximizes his share of gains from exchange, given the behavior of the others and equilibrium continuation. Thus, each has incentives to take efficiency into account. In particular, a social planner faced with the same informational constraints as the players could not suggest ex ante an alternative pattern of behavior that would yield a higher expected payoff for one type of one player without yielding a lower expected payoff for some type of some player (possibly the other type of the same player). In this sense, the singly-inconsistent case reaches an outcome that is informationally constrained, ex ante efficient.

It is difficult to arrive at a satisfactory definition of an efficient outcome for the doubly inconsistent case, the general case analyzed above. Players disagree about the strength of demand for an exhaustible resource at the maturity date of a futures market; neither optimists or pessimists revise their beliefs even if certain that others hold different beliefs. This is essential for rational players to strictly prefer some speculative trades over foregoing any speculative trade which is acceptable to the trading partner. In addition, here, we have allowed players to have inconsistent prior beliefs about the probability that another player is an optimist; $\mathcal{A}$ does not revise his estimate of the odds that $\mathcal{S}$ is an optimist upon learning that $\mathcal{B}$ attaches higher odds.

Let the rationality of each player i evaluating uncertainty over types via λ_i be accepted for the purposes of a welfare analysis that is at least a distant analogue to efficiency. Then the following sort of characterization holds for the separating equilibrium. There is no alternative pattern of offer and acceptance behavior which a social planner, subject to the same informational constraints and to subgame-perfect equilibrium actions guided by the individual λ_i's, could suggest that would yield an outcome in which some type of some player viewed himself as attaining a higher expected payoff, without the other type of the same player or some type of another player viewing himself as attaining a lower expected payoff.

In sum, despite doubly inconsistent incomplete information, a logic of specific rational behavior that was straightforward, step-by-step, emerges. It predicts a nonzero volume of purely speculative activity, roughly akin to ex ante efficiency. Inconsistent priors increase the difficulties of calculating equilibrium strategies by at least an order of magnitude; doubly inconsistent priors add little further difficulty. No "anything can happen" sort of Pandora's box is opened.

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Games of Incomplete Information: The Inconsistent Case*

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1. Introduction

In his well known series of papers Harsanyi [1967-68] develops the theory of noncooperative games of incomplete information, in which such games are replaced by games of complete information involving chance. He emphasizes the *consistent case*, in which the various types of players in a game of incomplete information Γ, as generated from the various beliefs, can be thought as being derived from a *joint probability matrix*. In this case Harsanyi constructs a game G of perfect recall and in extensive form, whose players represent the various types of the players in Γ. They are called *agents* and G itself is called *the agent-form game representing* Γ.

The game G has the following properties:

(i) Each agent, when he comes to play in G, has the same beliefs as the type in Γ that he represents.
(ii) Each agent, when he comes to play in G, has the same choices as the type he represents has in Γ.
(iii) If the players agree in Γ to play a Nash equilibrium of G, no type will find it beneficial to deviate, because deviating alone will not increase his payoff.

It is sometimes erroneously believed that Harsanyi's theory will not work if the beliefs are inconsistent. The purpose of this paper is to dispel this belief. One example which indicates how this can be done already appeared in Aumann and Maschler [1995]. The present paper generalizes that example and supplies a formal proof.

2. The Data

To keep the presentation simple, we restrict ourselves to a game Γ with incomplete information, involving two players I and II. The generalization

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to more players is straightforward. Each player in Γ can be one of several types, denoted $I_1, I_2, \ldots, I_r$ and $II_1, II_2, \ldots, II_s$. For each combination of types (I_ρ, II_σ) there corresponds an $m \times n$ bimatrix game $(a_{ij}^{\rho\sigma}, b_{ij}^{\rho\sigma})_{m\times n}$, which is the payoff bimatrix of the players, if the real types that play are I_ρ and II_σ. The beliefs of the various types of player I on the probabilities that he is facing one of the types of player II are given by a matrix $(p_\sigma^\rho)_{r\times s}$, $\rho = 1, 2, \ldots, r$, $\sigma = 1, 2, \ldots, s$. Thus, player I of type ρ believes that he is playing against player II of type σ with probability p_σ^ρ. Naturally, $\sum_{\sigma=1}^{s} p_\sigma^\rho = 1$, for all ρ, $\rho = 1, 2, \ldots, r$. Similarly, the beliefs of each type of player II on the types of player I are summarized by the matrix $(q_\rho^\sigma)_{r\times s}$, where $\sum_{\rho=1}^{r} q_\rho^\sigma = 1$, for all σ, $\sigma = 1, 2, \ldots, s$.

It is assumed that the matrices $(a_{ij}^{\rho\sigma})$, (p_σ^ρ) and (q_ρ^σ) are common knowledge. Harsanyi shows that many classes of games with incomplete information can be cast into the above framework.

3. An "Extensive Form" Representation of the Data

We now construct *two game trees* — one for each player. The trees will be identical, except for the numbers representing the probabilities assigned to the choices of chance and the payoffs at the endpoints. Briefly, both trees start with chance choices for all possible matching of a type of player I with a type of player II. The continuation yields m moves for player I, and information sets are placed to show that this player knows his type but does not know the type of his rival. The tree continues with n moves for player II and information sets placed to indicate that this player knows his own type, but does not know neither the type of player I nor the move he took.

In more details, each tree will start with $r \times s$ arcs emanating from the root, which is a vertex of *chance*. These correspond to the various possibilities that chance can match an agent for player I and an agent for player II. To be specific, we assume that the s leftmost arcs correspond to a matching of agent I_1 with the agents II_1, II_2, ..., II_s, in this order. The next s arcs will correspond to a matching of I_2 with the s types of player II, again in the order $II_1, II_2, \ldots, II_s$, etc.

We now partition the endpoints of these arcs into r consecutive *information sets*, each containing s vertices, representing the fact that Player I in Γ knows his own type, but does not know the type of player II against whom he is playing.

We now partition the $m \times s \times r$ new endpoints into s information sets each containing $r \times m$ vertices. This should be done in a way that matches the fact that player II does not know neither the actions of player I, nor his type in Γ. Thus the m leftmost arcs emanating from each leftmost vertex in an information set of player I should be grouped together to make an information set for II_1, etc.

To complete the trees we now issue n arcs from every new endpoint. These correspond to the n choices that player II has. We end up with $r \times s \times m \times n$ endpoints, each corresponding to an outcome in which a certain type of player I and a certain type of player II have each made some move. Looking at the corresponding payoff matrix, we can now assign payoffs to the endpoints. In the tree for player I we assign the payoffs of player I and in the other tree we assign the payoffs of player II.

It remains to assign probabilities to the choices of chance at the roots. this is done as follows:

We choose *arbitrary* positive numbers $\alpha_1, \alpha_2, \ldots, \alpha_r$, adding up to 1, and *arbitrary* positive numbers $\beta_1, \beta_2, \ldots, \beta_s$, also adding up to 1. For the tree for player I, we assign the probabilities $\alpha_1 p_1^1, \alpha_1 p_2^1 \ldots, \alpha_1 p_s^1$ in this order, to the s leftmost choices of chance (which end up in the information set of agent I_1). We assign the probabilities $\alpha_2 p_1^2, \alpha_2 p_2^2, \ldots, \alpha_2 p_s^2$ in this order, to the next s choices of chance (which end up in I_2's information set), etc. Note that these probabilities add up to one. For the tree for player II we assign the probabilities $\beta_1 q_1^1, \beta_1 q_2^1, \ldots, \beta_1 q_r^1$ in this order to the choices of chance whose continuations intersects the information set of agent II_1. Similarly, we assign the probabilities $\beta_2 q_1^2, \beta_2 q_2^2, \ldots, \beta_2 q_r^2$ in this order to the choices of chance whose continuations intersect the information set of agent II_2, etc. These probabilities too add up to 1.

Example 3.1. Player I can be one of two types and so can player II be. The bimatrix games that correspond to the various matching of the types are given below:

	Player II_1	
Player I_1	1,0	0,0
	0,0	0,2

	Player II_2	
Player I_1	0,0	1,2
	0,0	0,0

	Player II_1	
Player I_2	0,2	0,0
	0,0	1,0

	Player II_2	
Player I_2	0,0	0,0
	1,2	0,0

The beliefs of the types of players I and II are given by the following tables:[1]

[1] Tables open to the right represent beliefs of player I. Tables open to the bottom represent the beliefs of player II. I owe this convention to Harsanyi (oral communication).

	II_1	II_2
I_1	$\frac{2}{5}$	$\frac{3}{5}$
I_2	$\frac{3}{4}$	$\frac{1}{4}$

	II_1	II_2
I_1	$\frac{3}{7}$	$\frac{1}{2}$
I_2	$\frac{4}{7}$	$\frac{1}{2}$

The corresponding trees are drawn in Figures 1, and 2.

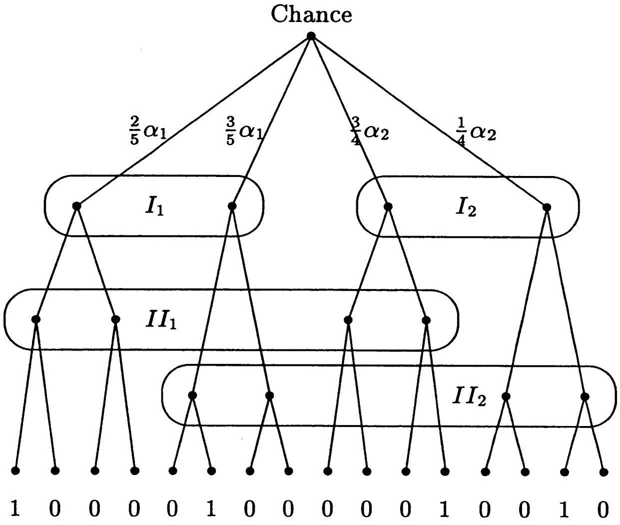

Figure 1: The Tree for Player I.

4. Recommendation

The trees described in the preceding section possess the same pure strategies; therefore, they can be converted into a single bimatrix game — call it G. Note that in general it will be a nonconstant-sum game even if all the games associated with the types happen to be constant sum. The strategies in G are exactly those available to the players in Γ. We say that the pair of trees, as well as the game G represent the game with incomplete information Γ.

The trees have another important property: Each agent, when he comes to play in G, he is actually located in one of the information sets of the tree of the player to whom the agent belong — the one that corresponds to the

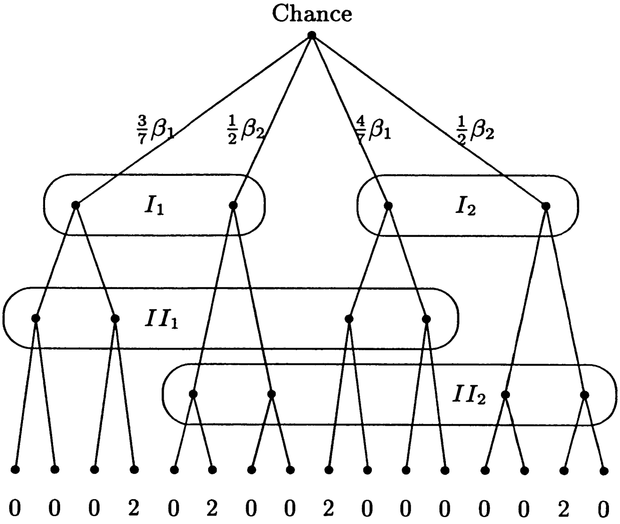

Figure 2: The Tree for Player II.

type in Γ that he represents. At that information set he has exactly the same beliefs about the types of his opponent as the corresponding type has in the game Γ. These facts imply the following theorem:

Theorem 4.1. *If the players agree in Γ to play a Nash equilibrium point of the game G, then none of them, regardless of his type, will find it beneficial to deviate alone in Γ.*

Proof. Let (u, v) be an equilibrium pair of strategies for the game G. Suppose that a certain type t of player I, playing in Γ, finds it profitable to deviate from his part u_t in u, to a strategy $\hat{u}_t$, assuming that his opponent, whatever his type may be, is playing as his part in v. Consider the pair of strategies $(u_{-t}, \hat{u}_t, v)$ played in G. The payoffs to all matchings other than those in which agent agent I_t plays will be the same and the payoffs to the remaining matching will yield a higher payoff to I_t. Since $\alpha_t > 0$, this implies that $(u_{-t}, \hat{u}_t, v)$ yields in G a higher expected payoff to player I. This contradicts the fact that (u, v) was an equilibrium point. A similar contradiction obtains if we assume that a certain type of player II finds it profitable to deviate. ∎

The converse theorem is also true, and it shows that the choice of the α_i's and β_j's is not important, as long as they satisfy the requirements specified in Section 3. (However, some choices may lead to a simpler game G, as can be seen in the next section.)

Theorem 4.2. *If a set of strategies is specified for each type, and together they have the property that no player, regardless of his type, can benefit by deviating, given that all the other types play as specified, then these strategies, when placed at the corresponding information sets in game trees, combine to form behavior strategies which constitute an equilibrium point in the game G.*

Proof. If a player in G can benefit by deviation alone, than this deviation induces deviations in some of his information sets in the tree games. In at least one of them he will benefit, since his expected payoff is a positive linear combination of the expected payoffs of his types (given the strategy of the other player). This means that at least one of his types will benefit by deviations, which is contrary to the data, because the expected payoff of a type does not depend on what other types of the same player do (they do not exist, anyhow). ∎

5. The Consistent Case

Suppose now that the types were derived from a joint probability matrix $(w_{ij})_{r\times s}$. Then

$$p_j^i = \frac{w_{i,j}}{w_{i1} + w_{i2} + \ldots + w_{is}}, \qquad q_i^j = \frac{w_{ij}}{w_{1j} + w_{2j} + \ldots + w_{sj}}$$

and therefore, if we choose $\alpha_i = w_{i1} + w_{i2} + \ldots + w_{is}, i = 1, 2, \ldots, r$ and $\beta_j = w_{1j} + w_{2j} + \ldots + w_{rj}, j = 1, 2, \ldots, s$, we find that $\alpha_i p_j^i = w_{ij} = \beta_j q_i^j$. Thus, in this case, both trees will have the same probabilities assigned to the choices of chance. These trees can therefore be combined into a single tree (with vector payoffs), which is, in fact, Harsanyi's tree.

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Admissibility and Stability

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Abstract. Admissibility is a useful criterion for selecting among equilibria, but I argue that enforcing admissibility dilutes the power of stability criteria to select among equilibrium components, and this accounts for anomalous examples of stable sets. Therefore, admissibility should be invoked only when selecting equilibria within a component that is immune to payoff perturbations.

The purpose of this essay is to examine the role of admissibility in formulating criteria for stability of equilibria. Recall that admissibility excludes an equilibrium that uses a dominated pure strategy. In particular, each pure strategy used (i.e., with positive probability) must be a best response to some completely mixed strategy. Reinhard Selten's (1975) criterion of perfection ensures admissibility by requiring that a pure strategy is used only if it is a best response to a sequence of completely mixed strategies. Admissibility was adopted subsequently by Kohlberg and Mertens (1986) and Mertens (1989) as a primary desideratum in their identification of stable sets of equilibria.

My thesis here is that admissibility plays two roles, only the first of which advances the program of defining stability for games in extensive form. The first role is that in selecting among equilibria within a component, admissibility strengthens the criterion of stability. However, in its second role in selecting among equilibrium components, admissibility weakens the criterion of stability — as examples will illustrate. I argue, therefore, that admissibility criteria should be deleted when selecting among equilibrium components; that is, the selected components should be *essential*, in the sense that they are stable with respect to all perturbations of normal-form payoffs. This lends credence to Kohlberg and Mertens' original definitions of 'hyperstable' and 'fully stable' sets of equilibria, provided they are restricted to lie in essential components. It suggests modifying Mertens' (1989) homological definition of stability to invoke payoff perturbations rather than strategy perturbations.

Section 1 summarizes technical aspects that are the basis in Section 2 for a comparison of stability criteria derived from perturbations of strategies and payoffs. All examples are collected in the Appendix. For a general survey of this topic see van Damme (1991).

1. Background

To simplify, we consider only games in extensive form with two players, called I and II. Each player has a finite number of pure strategies; each has perfect recall; and all the data of the game are common knowledge. Thus the normal form of a game has two matrices with rows labeled by I's pure strategies and columns labeled by II's pure strategies. For each pair of their pure strategies, the corresponding entry in one matrix describes I's payoff, and in the other

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matrix II's payoff, each expressed in terms of that player's von Neumann-Morgenstern utility. We use u^k_{ij} to denote player k's payoff from I's pure strategy i and II's pure strategy j. Recall that each pure strategy assigns a feasible action to each information set of that player; that is, to each event in which that player takes an action. Each player's feasible strategies are his mixed strategies, i.e., the probability distributions over his pure strategies. If I uses the strategy $x = (x_i)$ and II uses the strategy $y = (y_j)$, where $x \geq 0$ and $\sum_i x_i = 1$ and similarly for y, then k's expected payoff is $\sum_i \sum_j x_i u^k_{ij} y_j$. A Nash equilibrium is a pair (x, y) of strategies for the two players such that each player uses only pure strategies that are optimal responses to the other player's strategy; that is,

$$x_i > 0 \quad \text{only if} \quad \sum_j u^I_{ij} y_j = \max_\ell \sum_j u^I_{\ell j} y_j ,$$

and similarly for player II.

Selten (1965, 1975) argued convincingly that only a subset of the Nash equilibria predicts behaviors by rational players. He proposed restricting equilibria to those that are *perfect.* One version of his 1975 (normal-form) definition says that an equilibrium (x, y) is perfect if it is the limit of a sequence of completely mixed strategies (i.e., every pure strategy is used), say $(x^h, y^h) \gg 0$ converging to (x, y), such that

$$x_i > 0 \quad \text{only if} \quad \sum_j u^I_{ij} y^h_j = \max_\ell \sum_j u^I_{\ell j} y^h_j ,$$

and similarly for player II. Because each used pure strategy is an optimal response to a completely mixed strategy, it must be undominated. A perfect equilibrium therefore satisfies admissibility.

One interpretation of Selten's construction is that each strategy used in a perfect equilibrium must remain optimal against some 'tremble' by the other player; that is, small probabilities of using strategies that the equilibrium predicts will otherwise be unused. His other, equivalent definition emphasizes this aspect: an equilibrium (x, y) of a game G is perfect if there is a sequence $(x^h, y^h; G^h)$ converging to $(x, y; G)$, for which each (x^h, y^h) is an equilibrium of the perturbed game G^h. In this version, G^h is the *strategy perturbation* of G obtained by adjoining the constraints $x^h \geq \epsilon^h$ and $y^h \geq \delta^h$, where $(\epsilon^h, \delta^h) \gg 0$ and $\lim_{h \to \infty} (\epsilon^h, \delta^h) = 0$. This interpretation in terms of perturbations of the game had a profound effect on the subsequent development of theories of equilibrium refinements.

The prominent feature of Selten's formulation is that it considers perturbations of mixed strategies. This approach was well-adapted to the specification of Kreps and Wilson's (1982) definition of sequential equilibrium, adapted from Selten's 1965 definition of subgame-perfect equilibrium, to address the games without proper subgames that motivate Selten's 1975 article. This criterion imposes the requirement of sequential rationality; that is, at each information set a player's continuation strategy is optimal with respect to a probability distribution (over possible histories) consistent with the game and Bayes' Rule. A perfect equilibrium meets this requirement, using the limit of the conditional probabilities induced by the sequence of perturbed games. The perfect equilibria are therefore the sequential equilibria satisfying

a strong form of admissibility.[1] Indeed, in generic extensive games the sets of perfect and sequential equilibria coincide (Blume and Zame, 1994). One version of this fact is that allowing all perturbations of payoffs (rather than strategies) produces exact coincidence between the resulting 'weakly perfect' equilibria and the sequential equilibria. The different style of reasoning involved in using payoff perturbations to select equilibria is illustrated in the Appendix via the well-known Beer-Quiche game.

In their study of hyperstability, Kohlberg and Mertens allowed all perturbations of normal-form payoffs. As they recognized, this enlarges the set of allowed perturbations. Each perturbation of strategies has the same net effect as a corresponding perturbation of payoffs. This can be seen by interpreting the trembles as chance moves whose small probabilities perturb the expected payoffs from each pure strategy specifying intended actions at information sets. Kohlberg and Mertens advanced Selten's agenda towards identifying 'essential' or 'strictly perfect' equilibria, those immune to *all* perturbations in a neighborhood of the game, not just a single sequence of strategy perturbations. Because such ideal equilibria need not exist, this step required reconsideration of the unit of analysis.[2] They demonstrated that each game's equilibria are partitioned into a finite number of closed connected components; and for generic extensive-form games, equilibria in the same component have the same outcomes, namely, they differ only off the equilibrium path and therefore their probability distributions over histories are the same. Combining these two results indicates that an equilibrium outcome immune to payoff perturbations is identified generically by a component that is essential; that is, every nearby game has an equilibrium near the component.[3] They also proved that every game has essential components.[4] Moreover, some are *invariant* in the sense that they are essential in every game with the same *reduced* normal form obtained by deleting redundant strategies (those strategies whose payoffs for all players are convex combinations of other strategies' payoffs).

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[1] Admissibility and normal-form perfection of equilibria are equivalent in two-person games. The analogous extensive-form definition of perfection allows use of dominated strategies. In Mertens' (1995) example, the dominant-strategy equilibrium is not extensive-form perfect, and remarkably, every extensive-form perfect equilibrium uses a dominated strategy. Similar examples led van Damme (1984) to propose an alternative definition of quasi-perfection.

[2] Kohlberg and Mertens (1986, Appendix D) show that an equilibrium that is extensive-form perfect in every extensive game with the same reduced normal form is strictly perfect, provided all best responses are used. For many interesting games, however, this proviso is not satisfied.

[3] Genericity is mostly immaterial here because a nongeneric case has a component that is the union of components induced by nearby games; a case where none of these subcomponents is essential would be doubly rare.

[4] That such a component exists follows from their demonstration that the projection of the graph of the Nash equilibrium correspondence is homotopic to a homeomorphism. This is called the 'rubber sphere' theorem, because it shows that when the space of games is mapped onto the surface of a sphere via a one-point compatification, the equilibrium graph is mapped to a deformation of a sphere of larger radius. Consequently, above every game lies a component for which every nearby game has an equilibrium nearby.

Kohlberg and Mertens showed that an invariant essential component has several desirable properties.

Invariance: Because an invariant component depends only on the reduced normal form, it does not depend on which among the many strategically equivalent extensive forms is used.

Backward Induction: It contains perfect and even proper equilibria.[5] Such equilibria satisfy Selten's 1965 criterion (each strategy remains optimal in every subgame) and Kreps and Wilson's generalization to 'extended subgames' in games with imperfect information. Proper equilibria are perfect, and therefore use only admissible strategies. Moreover, a proper equilibrium induces a sequential equilibrium in every extensive form with the same reduced normal form.

Iterated Dominance: Its projection onto the equilibrium graph of a smaller game, obtained by deleting dominated strategies, contains an invariant essential component.

Forward Induction: Its projection onto the equilibrium graph of a smaller game, obtained by deleting a strategy that is suboptimal at all its equilibria, contains an invariant essential component.[6]

Further, within an invariant essential component is a minimal closed set (called hyperstable) with analogous properties; inside that is a minimal closed set (called fully stable) immune to perturbations of any finite set of pure or mixed strategies; and inside that is a minimal closed set (called stable) immune to perturbations of pure strategies. The latter (though omitting minimality) allows a homological definition proposed by Mertens (1989). In the next section we re-examine Kohlberg and Mertens' and Mertens' rejection of all these except stable sets, on the grounds that only stable sets' equilibria exclude inadmissible strategies. We shall see that this has unfortunate consequences, because some stable sets reside in *inessential* components.

This brief review omits a vast literature emanating from Selten's insights, but it includes some main themes. Extensive games require equilibrium refinements. Intuitive criteria such as admissibility, backward induction, and consistent conditional probabilities can be founded on consideration of strategy perturbations. Combining these with iterative elimination of dominated or suboptimal strategies meets further criteria, such as rationalizability (Pearce, 1984) and forward induction.

To convey the scope of extensions not described above, we mention only the models of reputation effects in finitely-repeated and centipede games. These models are derived from Selten's (1978) study of the chain-store game. In repeated games (without or with stopping options), a perturbation of a

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5 To be proper, the sequence justifying a perfect equilibrium must assign a probability of lower order to one pure strategy that is inferior to another (Myerson, 1978).

6 This property is motivated by games involving signaling. These games typically have multiple equilibria reflecting possible inferences from disequilibrium behaviors. The motive for forward induction insists that the conditional probabilities validating a sequential equilibrium should be zero for histories dependent on strategies that are suboptimal at every equilibrium in the component. Thus, one can prune those branches of the game tree that occur only when such suboptimal strategies are used. See Banks and Sobel (1987) and Cho and Kreps (1987).

strategy that repeats a particular behavior can attract imitation, leading to an equilibrium in which the perturbed strategy is the only one used, except in the last stages of the game. These models are relevant to the subsequent discussion because typically (as in the repeated chain-store and prisoners' dilemma games) their striking feature is that it is a strategy that is inadmissible in the stage game that attracts imitation.

2. A Critique of Admissibility

We now examine the role of admissibility in constructing equilibrium refinements such as those described above. First I suggest via examples that the known deficiencies of stable sets derive primarily from the imposition of admissibility criteria. The justification offered for admissibility is that it is a cornerstone of single-person decision theory, but this rationale applies only to selection among equilibria, not to selection among components (e.g., Kohlberg and Mertens, p. 1014; Mertens, 1989, p. 577(c)). Applying admissibility criteria to component selection is therefore misplaced. This still leaves ample latitude to select equilibria *within an invariant essential component* using criteria of admissibility or perfection. I conclude with cautionary remarks: in some contexts admissibility is unduly restrictive even in selecting among equilibria within an essential component.

In the construction of stable sets, admissibility is ensured by allowing only strategy perturbations, rather than all payoff perturbations. This formulation might seem legitimate in view of the fact that differences between weak-perfect or sequential (justified by payoff perturbations) and perfect (justified by strategy perturbations) equilibria are nongeneric. In fact, however, the difference between the existential quantifier ('there exist perturbations yielding nearby equilibria') used for these equilibrium selections, and the universal quantifier ('all perturbations yield nearby equilibria') used for component selections can be substantial. In fact, generic games can have stable sets (immune to strategy perturbations) inside components that are not essential (immune to payoff perturbations). For this reason, I see no justification for restricting the allowable payoff perturbations when selecting among components.

In the Appendix, three examples illustrate how the exclusion of some payoff perturbations allows implausible components to survive the criterion of stability.

- The first is van Damme's (1989) generic example in which there is a stable set that fails to induce an equilibrium in a subgame. Mertens' (1989) revised definition produces a larger stable set that remedies this deficiency. Nevertheless, one sees easily that these stable sets lie in an inessential component, as one can see from a simple payoff perturbation.
- The second example is the nongeneric game Trivial Pursuit in which there is a stable set whose outcome for one player is inferior to his stable outcome in a continuation that he has the option to elect. Again, a simple payoff perturbation suffices to eliminate this stable set lying in an inessential component.
- The third example is Cho and Kreps' (1987) ingenious example of a stable set that motivates their conclusion that "if there is an intuitive story to go

with the full strength of stability, it is beyond our powers to offer it here" (p. 220). But once again, a simple payoff perturbation suffices to show that this stable set resides in an inessential component.

I conclude from such examples that in devising criteria for selecting among components, it is better to use all payoff perturbations. Of necessity, this approach may select essential components with equilibria using inadmissible strategies. The second step, therefore, is to use criteria such as admissibility to select, say, stable sets (or proper or perfect equilibria) within the selected component.[7] This two-step procedure was implicit in Kohlberg and Mertens' original work, but unfortunately they abandoned it when they attempted to construct a definition that obtains all desirable properties in a single step.

Besides the above examples, the deficiency of the single-step approach is evidenced by Mertens' (1995) second example (which he calls "pretty damning as to the behavior of stable sets in at least some non-generic games") of a game with perfect information in which the unique stable set includes all admissible equilibria (which includes a two-dimensional set), of which only one is subgame-perfect. Moreover, this subgame-perfect equilibrium yields the only stable outcome derived from all games in the neighborhood of payoff perturbations except those in a 'slice' satisfying a single equality condition. Such examples indicate that a second step of selection *within* an essential component is inescapable.

It is important to realize, however, that a two-step procedure has its own deficiencies. To illustrate, the Appendix presents a generic extensive-form game Ω with an invariant essential component containing two stable sets (in the strong sense of Mertens' definition) and two proper equilibria, reflecting the fact that its projection map (from its neighborhood in the equilibrium graph to the space of games) has degree 2. Neither of these stable sets is immune to payoff perturbations, even though they reside in a component that is immune.

We turn now to the selection of equilibria within an (invariant) essential component and ask: Are the strategy perturbations the best set to invoke in selecting among equilibria? Clearly, Selten's pure-strategy perturbations are a minimal set sufficient to ensure admissibility and consistency.[8] On these grounds one can say that Selten's construction is exactly right. On the other hand, I think it is prudent to realize that admissibility is justified only if one is quite sure about the validity of some particular extensive form of the game. As Fudenberg, Kreps, and Levine (1988) prove, *every* pure-strategy equilibrium is the limit of strict equilibria of nearby 'elaborations'; that is,

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[7] An example of the effectiveness of this two-step procedure even when admissibility is not an issue is van Damme's (1987, p. 119) generic extensive game with a two proper equilibria, one of which he argues is "not sensible." A simple payoff perturbation suffices to show that this equilibrium's component is inessential.

[8] These comments apply to normal-form perturbations; for extensive-form versions, one can also use the perturbations used by van Damme (1984) to define quasi-perfection. A somewhat larger set that includes mixed-strategy perturbations is apparently a nearly maximal set ensuring admissibility. One subset of mixed-strategy perturbations ensuring admissibility is used by Kohlberg and Mertens (1986, p. 1025) to construct proper equilibria; however, they show by example (p. 1026, Fig. 9) that the full set fails to ensure admissibility.

games in which players may have differing information about which perturbation applies. Fudenberg and Maskin (1986), Myerson (1986), van Damme (1987) illustrate particular examples. Further, Bagwell (1995) provides an example in which perturbations of the observability of actions induce payoff perturbations that justify an inadmissible equilibrium in the unique essential component containing the subgame-perfect equilibrium. Bagwell's Stackelberg game is described in the Appendix. In all these cases, the inadmissible equilibria are justified by payoff perturbations, but not by strategy perturbations.

3. Conclusion

In sum, I see no harm and substantial advantages to selecting invariant essential components based on consideration of all payoff perturbations. Doing so eliminates anomalous stable sets in the known examples. As a second step, one can select a stable set or a proper or perfect equilibrium within the essential component, provided there is assurance that admissibility is an appropriate criterion. This second step is problematic, however, whenever potentially relevant perturbations of the extensive form generate payoff perturbations not induced by strategy perturbations.

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4. Appendix: Examples

4.1 Example 1: The Beer-Quiche Game

The distinction between strategy and payoff perturbations is immaterial in generic cases. To illustrate, Figure 4.1 shows a version of Kreps' Beer-Quiche game (1990, p. 465; Kohlberg and Mertens, 1986, p. 1031). In this game, player I chooses Left or Right and then player II chooses Across or Up; however, only I knows whether chance has chosen the payoffs along the Top or Bottom (with probabilities p and $1-p$, where $0 < p < 1/2$). The stable component contains the pure strategy equilibrium in which I surely chooses Left, and II chooses Up after Left and Across after Right. The unstable component is reversed: I surely chooses Right, and II chooses Across after Left and Up after Right. The latter violates forward induction because I's unused strategy that chooses Left after Top and Right after Bottom is suboptimal in that component; if it is deleted then II's action after Left becomes suboptimal, and if that is revised then I surely prefers Left. In terms of strategy perturbations this conclusion is evident because II's strategy cannot be optimal whenever the conditional probability of a deviation to Left is less after Top than Bottom. In terms of payoff perturbations the reasoning is altered: a perturbation that makes II's unused optimal strategy (i.e., Up invariably)

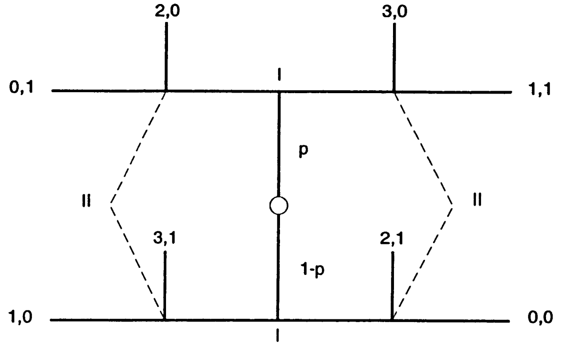


Fig. 4.1. The Beer-Quiche game.

superior requires II to use this strategy exclusively, in which case I prefers Left after Bottom.[9]

4.2 Example 2: van Damme's Game

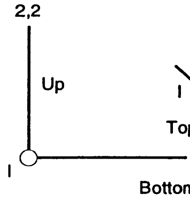
Figure 4.2 depicts the generic extensive-form game studied by van Damme (1989) in which player I initially chooses either Up or to play a simultaneous-move subgame with player II. This game has a stable set with the payoff (2,2) obtained when player I chooses Up. This outcome cannot arise from an essential component: a negative perturbation of II's payoff from (Up, Middle) yields a game in which no equilibrium uses Up. The effect of such a perturbation on II's best-response regions within the simplex of I's strategies is shown in Figure 4.3.

From his study of this game, van Damme argued that the outcome Up violates a plausible interpretation of forward induction; moreover, no equilibrium in a minimal stable set induces an equilibrium in the subgame after I chooses Right. Mertens' (1989) definition provides a larger stable set that does include subgame equilibria. Even so, it is stable with respect to strategy perturbations but not payoff perturbations.

4.3 Example 3: Trivial Pursuit

Figure 4.4 depicts the game Trivial Pursuit. In the continuation after player II initially chooses Right, were this known to I, I's strictly dominant strategy

[9] Other than the implicit corollary seemingly implied by the elaborate proof in Kreps and Wilson (1982, strengthened by the results of Blume and Zame, 1994) of the generic coincidence of perfect and sequential equilibria, I know no direct demonstration of the generic equivalence of these two styles of reasoning.



I \ II	Left	Middle	Right
Top	0,0	3,2	0,3
Bottom	3,3	0,2	3,0

Fig. 4.2. Van Damme's example of a generic game with a stable set in an inessential component.

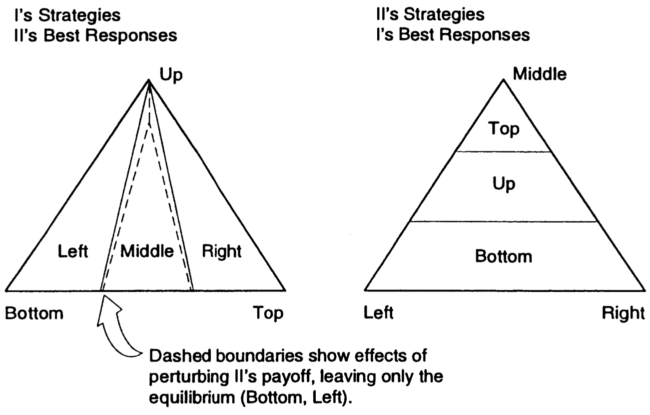


Fig. 4.3. Van Damme's example depicted graphically, showing the effect of perturbing player II's payoff.

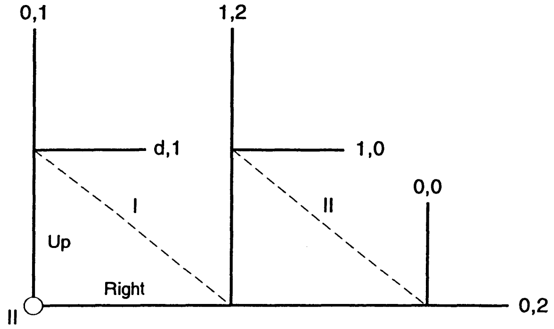


Fig. 4.4. The game Trivial Pursuit with a stable set in an inessential component.

is Up, ensuring a payoff of 1, and the unique stable payoffs are (1,2). Now consider the full game with the parameter satisfying $0 < d < 1$. This adds an initial move (Up) by II from which he gets a certain payoff of 1 and I gets less — both of which are inferior to their payoffs from the stable outcome after Right were it anticipated by I. However, there is a stable set in which all equilibria require both players to randomize their initial choices — including a positive probability of Up initially by II. This stable set is not in the essential component, however, because every negative perturbation of either of II's payoffs from Up yields a game in which II surely chooses Right initially, as shown in Figure 4.5.

This game is nongeneric only to the extent that II's payoffs from Up initially are replicated by a mixture of his payoffs from Right, so Up is a redundant strategy for II. Nevertheless, the normal form of this game cannot be reduced by deleting Up, because I's payoffs are affected.

4.4 Example 4: Cho and Kreps' Signaling Game

Figure 4.6 depicts graphically the relevant data for a signaling game like Example 1 except that there are three types of player I and player II has three actions. They consider an equilibrium outcome in which all types of I choose Right (R). In the figure, the left simplex comprises II's possible beliefs about I's type after observing I's alternative action Left (L), and shows its partition into II's best response regions. The right simplex shows the partition of II's simplex of behavioral strategies after Left into I's best response regions; e.g., at the top the notation (L,R,R) indicates that types 1, 2, and 3 prefer Left, Right, and Right respectively if II is sufficiently likely to choose his Action 1 after Left. Cho and Kreps (1987) show that this configuration enables the outcome from the specified equilibrium [in which I uses the strategy (R,R,R)] to be stable. One sees easily, however, that this equilibrium is in

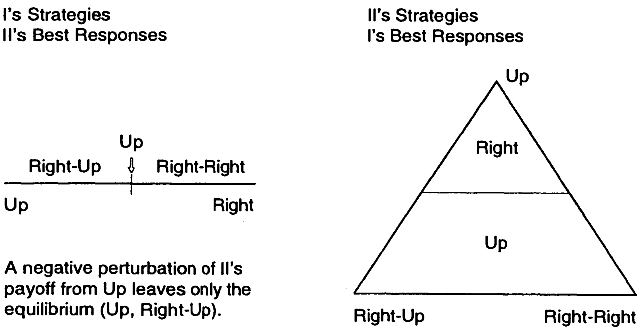


Fig. 4.5. The best-responses for Trivial Pursuit, showing a perturbation of II's payoff from Up that eliminates the inessential component.

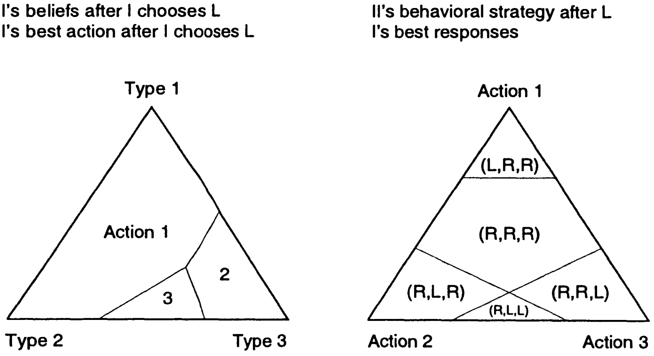


Fig. 4.6. The data for Cho and Kreps' example of a stable set in an inessential component.

an inessential component. A positive payoff perturbation of any one of II's optimal strategies ensures that it must be used exclusively, in which case the corresponding type prefers Left. For instance, a positive payoff perturbation of Action 1 requires that II uses only Action 1 in response to Left, but then I's Type 1 prefers Left.

4.5 Example 5: The Game Ω

Figure 4.7 depicts the generic extensive-form game Ω , and Figure 4.8 shows

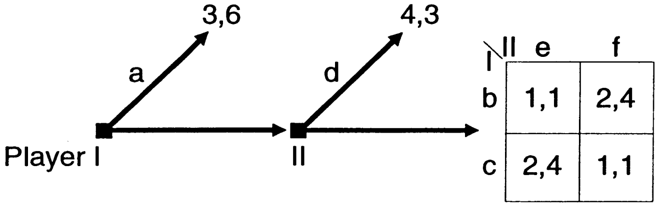


Fig. 4.7. The generic extensive-form game Ω .

the players' best responses. The component in which I uses only strategy **a** is invariant and essential; in fact the projection from its neighborhood has degree 2. The degree is reflected in the presence of the two proper equilibria where II uses **e** or **f**, each of which is contained in a Mertens-stable set: the interval [**e**,**de**] or the interval [**f**,**df**]. Neither of these stable sets is immune to payoff perturbations. For instance, those perturbed games with two equilibria near **f** and **def** have no equilibria near [**e**,**de**].

4.6 Example 6: Bagwell's Stackelberg Game

This example illustrates that even in the games of perfect information that motivate Selten's 1965 criterion of subgame perfection, the sole essential component can contain inadmissible equilibria justified as the limits of equilibria of nearby games with imperfect observability. Figure 4.9 displays the extensive form of Bagwell's (1995) Stackelberg game. First I chooses Up or Down, then chance reveals 'up' or 'down' to II (with a probability p of an erroneous report), and then II chooses Up or Down. If the error probability is zero then this is a game with perfect information: the unique subgame-perfect

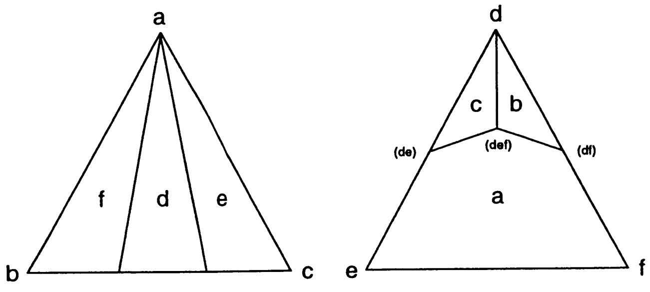


Fig. 4.8. The players' best-response correspondences for the game Ω.

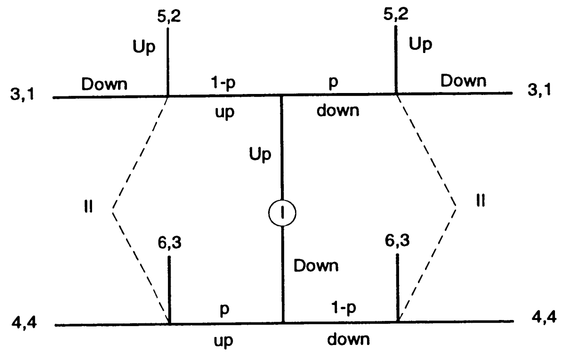


Fig. 4.9. Bagwell's Stackelberg game with imperfect observation by II of I's action.

equilibrium has I choosing Up and II choosing Up after up and Down after down. This subgame equilibrium lies in the unique essential component that requires II to choose Down after down with conditional probability at least one-half. (The inessential component has I choosing Down and then II choosing Down, provided II's conditional probability of Down after up is at least one-half.) However, any perturbation that allows a positive probability of erroneous reports has only one equilibrium close to the essential component, and the equilibrium it is close to is not the perfect equilibrium, but rather the inadmissible equilibrium where II mixes equally between Up and Down after down. This is a robust feature of games with imperfectly observed actions.

The important feature of the 'observability' perturbation above is that it induces a payoff perturbation that cannot be mimicked by a strategy perturbation. Indeed, a perturbation of I's strategies is capable of excluding only the dominated strategies of II that choose Up invariably or Down invariably. In contrast, the payoff perturbation induced by a positive error probability is one that makes these strategies *undominated.* Unlike strategy perturbations, which envision nonrational trembles, payoff perturbations account for slight chances of inaccurate observations. This is similar to the effect of reputational considerations in repeated games, where again the relevant perturbation is one that converts a dominated strategy into an undominated strategy.

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Equilibrium Selection in Team Games

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Abstract. It is shown that in team games, i.e. in games in which all players have the same payoff function, the risk-dominant equilibrium may differ from the Pareto dominant one.

1. Introduction

The general theory of equilibrium selection that has been proposed in Harsanyi and Selten (1988) invokes two completely different selection criteria: risk dominance and payoff dominance. The first is based on *individual* rationality, while the second incorporates *collective* rationality. The latter criterion captures the idea that if one equilibrium E_1 yields all players in the game uniformly higher payoffs than the equilibrium E_2 does, then rational players are more tempted to play the former. The first criterion captures the idea that, in a situation where players do not yet know which of the two equilibria, E_1 or E_2, will be chosen, players will lean towards that equilibrium that appears less risky in the situation at hand. For the case of 2×2 games, Harsanyi and Selten give an axiomatic characterization of their risk-dominance relation. In the special case of a symmetric 2×2 game, the risk-dominance relation can be easily characterized: E_1 risk dominates E_2 if and only if each player finds it optimal to play according to E_1 if he expects the other to play in accordance with E_1 with a probability of at least $1/2$.

The criteria of payoff dominance and of risk dominance may yield conflicting recommendations, and in such cases Harsanyi and Selten give precedence to payoff dominance. An example of such a conflict is illustrated in the stag hunt game from Figure 1 which has been adapted from Aumann (1990). (Also see Harsanyi and Selten (1988, Sect. 10.12)). Each player has two strategies, a safe one and a risky one, and if both play their risky strategy, the unique Pareto efficient outcome results. However, playing this strategy is very risky: If one player chooses it while the other player chooses the safe strategy, then the payoff to the first is only zero. In contrast, the safe strategy guarantees a payoff of 7, and it might even yield more. In this stag hunt game (R, R) is the payoff dominant equilibrium, while (S, S) is the risk-dominant equilibrium. (Indeed each player finds it optimal to play S as long as his opponent does not choose R with a probability more than $7/8$.)

The payoff dominance requirement is based on collective rationality, i.e. on the assumption that rational individuals will cooperate in pursuing their common interests if the conditions of the game permit them to do so. Harsanyi

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	R	S
R	9,9	0,8
S	8,0	7,7

Figure 1: stag hunt

and Selten argue that risk dominance is only important in those cases where there is some uncertainty about which equilibrium "should" be selected. If one equilibrium gives all players a strictly higher payoff than any other equilibrium (and if this equilibrium satisfies all other desirable properties that the selection theory imposes) such uncertainty will not exist – each player can be reasonably certain that all other players will opt for this equilibrium – and this makes risk-dominance comparisons irrelevant. It is this argument that leads Harsanyi and Selten to give precedence to payoff dominance.

Yet, relying on collective rationality is somewhat unsatisfactory. For one, it implies that the final theory is not ordinal, that is, two games with the same best reply structure need not have the same solutions. For example, the game from Figure 1 is best-reply-equivalent to one in which the off-diagonal payoffs are zero and in which the payoffs to (R, R) and (S, S) are (1,1) and (7,7) respectively, and, in the latter, payoff dominance selects (7,7) as the outcome. Secondly, one feels that it should be possible to obtain collective rationality as an outcome of individual rationality: If one equilibrium is uniformly better than another, then the players' individual deliberations should bring them to play this equilibrium. The reason that this does not happen in a game as that from Figure 1 – at least if one views the risk-dominant equilibrium as the outcome of the individual deliberation process – is that indeed a player is not sufficiently certain *ex ante* that his "partner" will play the risky equilibrium. In fact, he cannot be sure of this exactly because of the fact that his partner cannot be sure that *he* will play it: one only needs a "grain of doubt" in order for it to be the unique rationalizable strategy to play safe. (See Carlsson and Van Damme (1993ab), and, for an informal argument to that extent, Schelling (1960, Chapter 9).)

The above raises the question of whether, in games in which players can indeed be quite certain that the opponents will play the payoff dominant equilibrium, or at least in games in which this equilibrium is uniquely focal, risk-dominance considerations will induce players to play this equilibrium. This note aims to address this issue, and it provides a negative answer. We consider *team games*, that is, games in which all players have the same payoff function. Any maximum of this function is trivially an equilibrium and one might argue that, in those cases in which the maximum is unique, this maximum provides the unique focal equilibrium of the game. In other words, under those situations the conditions are most favorable for individual rationality to be in agreement with collective rationality. We provide examples

to illustrate that, even in these cases, risk-dominance considerations do not necessarily lead to the playing of the unique payoff dominant equilibrium. Specifically, we show that the modified Harsanyi/Selten theory, that does not invoke payoff comparisons, may select a Pareto dominated equilibrium in a team game.[1]

2. Notation and Definitions

In this section we introduce notation, and define team games and the risk dominance relation. Readers already familiar with these concepts can immediately turn to Section 3.

Let $\Gamma = < A_i, u_i >_{i \in I}$ be a strategic form game. I is the player set, A_i is the set of pure strategies of player i and $u_i : A \to \mathbb{R}$ is the payoff function of this player ($A = \Pi_{i \in I} A_i$). We write S_i for the set of mixed strategies of player i, $S = \Pi_{i \in I} S_i$ for the set of mixed strategy profiles and $u_i(s)$ for the expected payoff to player i for when $s \in S$ is played. A strategy profile s^* is a (Nash) *equilibrium* of Γ if no player can improve his payoff by a unilateral change in strategy, i.e.

$$u_i(s^*) = \max_{s_i \in S_i} u_i(s^*_{-i}, s_i), \tag{2.1}$$

where (s^*_{-i}, s_i) is shorthand notation for $(s^*_1, \ldots, s^*_{i-1}, s_i, s^*_{i+1}, \ldots, s^*_n)$. We write $E(\Gamma)$ for the set of equilibria of Γ. The (linear) *tracing procedure* is a map T from S into $E(\Gamma)$, hence, it converts each mixed strategy profile into an equilibrium of the game. Formally the map T is associated with a homotopy. For $s \in S$ and a homotopy parameter $t \in [0,1]$ write $\Gamma^{t,s}$ for the game $< A_i, u_i^{t,s} >_{i \in I}$ in which the payoff function of player i is given by

$$u_i^{t,s}(a) = t u_i(a) + (1-t) u_i(s \backslash a_i). \tag{2.2}$$

Hence, for $t = 1$ we have the original game, while for $t = 0$ each player faces a (trivial) one-person problem. In nondegenerate cases, the game $\Gamma^{0,s}$ contains exactly one equilibrium $e(0,s)$ and this will remain an equilibrium of $\Gamma^{t,s}$ as long as t is sufficiently small. Now it can be shown that the equilibrium graph

$$E = \{(t, \bar{s}) : t \in [0,1], \quad \bar{s} \in E(\Gamma^{t,s})\} \tag{2.3}$$

[1] We will not spell out the full details of the Harsanyi and Selten (1988) theory. The reader is referred to the flowchart on p. 222 of their book for a quick overview of the theory. In the examples we take certain shortcuts through this flowchart. We leave it to the reader to prove that these shortcuts are justified.

contains a unique distinguished curve $\{e(t,s) : t \in [0,1]\}$ that connects $e(0,s)$ with an equilibrium $e(1,s)$ of Γ. (See Harsanyi and Selten (1988) and Schanuel et al. (1992) for the technical details. In particular, the latter paper points out how $e(1,s)$ can be found by applying the logarithmic tracing procedure.) The endpoint $e(1,s)$ of this path is the linear trace $T(s)$ of s. This tracing map T is used to define the risk-dominance relation.

Imagine that the players are uncertain about which of two equilibria, s^* or s^{**}, should be considered as the solution of the game. Player i assumes that his opponents already know it and he himself attaches probability z_i to the solution being s^* and the complementary probability $1-z_i$ to the solution being s^{**}. Obviously, in this case he will play his best response against the correlated strategy $z_i s^*_{-i}+(1-z_i)s^{**}_{-i}$ of his opponents. Assume that if player i has multiple best responses, he plays each of them with equal probability and denote the resulting centroid best reply by $b_i(z_i; s^*, s^{**})$. Now, an opponent j of i does not know i's beliefs z_i. Assume that, according to the principle of insufficient reason, such a player considers z_i to be uniformly distributed on $[0,1]$. Clearly, an opponent will then predict i to play the mixed strategy

$$s_i(s^*, s^{**}) = \int_0^1 b_i(z_i; s^*, s^{**}) dz_i. \tag{2.4}$$

Let $s(s^*, s^{**})$ be the mixed strategy vector determined by (2.4), the so called bicentric prior associated with s^* and s^{**}. We now say that:

(i) s^* risk dominates s^{**} if $T(s(s^*, s^{**})) = s^*$,
(ii) s^{**} risk dominates s^* if $T(s(s^*, s^{**})) = s^{**}$, and
(iii) there is no dominance relation between s^* and s^{**} if $T(s(s^*, s^{**})) \notin \{s^*, s^{**}\}$.

We note that the risk-dominance relation need not be transitive, nor complete. We also note that a simple characterization of this relation can be given for 2-person 2×2 games with two strict equilibria, say s^* and s^{**}. Write z_i^* for the critical belief of player i where he is indifferent between both pure strategies, that is, $b_i(z_i^*; s^*, s^{**}) = (1/2, 1/2)$. Then, s^* risk dominates s^{**} if and only if $z_1^* + z_2^* < 1$.

The Harsanyi/Selten solution of a game is found by applying an iterative elimination procedure. Starting from an initial candidate set (consisting of all so called primitive equilibria), candidates that are payoff dominated or risk dominated are successively eliminated until exactly one candidate is left. We will consider the modification of that theory that only invokes risk dominance. In both our examples the initial candidate set will simply be the set of all pure equilibria of the game. It will be clear from the above definition that it can easily happen that there is no risk-dominance relation between pure equilibria, say s^* and s^{**}, and that both are "maximally stable". In this case

Harsanyi and Selten propose to replace the pair by one substitute equilibrium, viz. by the equilibrium s^{***} that results when the tracing procedure is applied to the mixed strategy in which each player i chooses $1/2s_i^* + 1/2s_i^{**}$. (Note that this, so called centroid strategy, typically differs from the bicentric prior as determined by (2.4).) Hence, in case of a deadlock with two equally strong candidates s^* and s^{**}, these equilibria are eliminated from the initial candidate set. They are replaced by the equilibrium s^{***} and the process is restarted with this new candidate set. As in both our examples, a single equilibrium remains after at most one substitution step has been performed, there is no need to go into further details of the process.

We conclude this section by giving the definition of a *team game*. $\Gamma = < A_i, u_i >_{i \in I}$ is said to be a team game if $u_i = u_j$ for $i, j \in I$, hence, all players always have the same payoff. Writing u for this common payoff function, we say that the team game is *generic* if there exists a unique $a^* \in A$ at which u attains its maximum. Attention will be confined to symmetric games, i.e. the payoff to a player depends only on which actions are chosen and not on the identities of the players choosing them. Because of this symmetry we can confine ourselves to analyzing the situation from the standpoint of player 1. In the next two sections we investigate risk dominance in generic symmetric team games and compute the associated (modified) solutions.

3. A Two-Person Example

The discussion in this section is based on the 2-person game from Figure 2. (The right hand side of the picture displays the best reply structure associated with this game.)

	T	M	B
T	3	0	2
M	0	3-ϵ	2
B	2	2	2 + ϵ

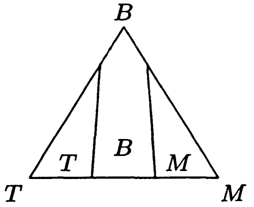

Figure 2: A Symmetric Two-Person Team Game ($0 < \varepsilon < 1$)

The game has three (pure) equilibria with payoffs 3, 3-ϵ and 2 + ϵ. We first investigate the risk dominance relationship between T and B. Note that when a player is uncertain whether the opponent will play T or B, but is certain that this player will not choose M, then this player will never be tempted to choose M since M is never a best response against a mixture $zT + (1-z)B$. This shows that M is irrelevant for the risk-dominance relationship between T and B and that this relationship can be determined simply in the 2×2

reduced game spanned by T and B. Now, as is shown by the RHS of Figure 2, the bicentric prior relevant for this risk-dominance relation assigns almost all weight to T, hence, the tracing path starts at T and it stays there: T risk dominates B. A similar argument establishes that M risk dominates B.

It remains to investigate the risk-dominance relation between T and M. As is shown by the RHS of Figure 2 the bicentric prior relevant for the comparison of these equilibria, is approximately equal to $(1/3, 1/3, 1/3)$. It follows that the unique equilibrium of the game $\Gamma^{0,s}$ is (B, B). Since (B, B) is a strict equilibrium of $\Gamma^{1,s}$ it is a strict equilibrium of $\Gamma^{t,s}$ for all $t \in [0, 1]$, hence, the distinguished curve in the graph E is constantly equal to (B, B). Therefore

$$T(s(T, M)) = B \tag{3.1}$$

and there is no risk-dominance relation between T and M. Hence, there is a deadlock: Both T and M dominate B, but T and M are equally strong. To resolve the deadlock we apply the substitution step, hence, we start the tracing procedure with the prior $1/2T + 1/2M$. Again, as the RHS of Figure 2 makes clear, the unique best response against this prior is B, so that the tracing path starts at B and remains there. The substitution set eliminates the pair $\{T, M\}$ and replaces it with the equilibrium B. Hence, if we modify the theory of Harsanyi and Selten, by not imposing the payoff dominance requirement, then the equilibrium B is selected in the game of Figure 2. Individual rationality, as incorporated into risk dominance, does not lead to collectively efficient outcomes, not even in generic symmetric team games.

At the intuitive level, one may explain the phenomenon as follows. Risk considerations favor the selection of equilibria that give "reasonably good" payoffs against a set of diffuse priors: A player does not know what the others will do and he investigates what action gives good outcomes no matter what the others do. These considerations favor actions that have large stability sets, i.e. that are best responses against many mixed strategies of the opponents. In Figure 2, B is such a good and safe strategy. In fact, we could make the stability set of B to cover almost the entire strategy simplex without losing the fact that T is the unique payoff dominant equilibrium: just replace 2 by $3\text{-}2\epsilon$ everywhere in the payoff matrix. By increasing the payoff associated to B, one makes B more attractive, hence, at the same time T is made less attractive. What this makes clear is that there is nothing special about team games. Either one assumes that the logic of common payoffs and collective rationality is so strong that players do not have any doubt to start with about what to play, or one allows for prior doubt and then one does not see how the common payoff assumption helps to reduce it.

The doubt concerning what equilibrium to play may, for example, arise out of slight payoff uncertainty as in Carlsson and Van Damme (1993a,b). One may imagine that it is common knowledge among the players that they

are playing a team game, but each player may have a tiny bit of private information about what the actual payoffs are. If the uncertainty is small, then, if the actual game is generic, the strategy combination that attains the maximum will be mutually known to a high degree. However, around a nongeneric game the latter will not hold. As Carlsson and Van Damme show, such nongeneric games exert an influence on "far removed" generic games: since players choose safe strategies in non-generic games, they are forced to choose safe strategies also in generic games, in order to avoid coordination failures.

To illustrate this argument, consider the modification from Figure 2 as in Figure 3. This is a nongeneric game. As Schelling (1960, Appendix C) already argued, the unique focal equilibrium in this game is B: If players cannot communicate, then, if they aim to coordinate on T or M, they will actually succeed only with 50% probability and, hence, it is better to play B ($5/2 > 1/2 \cdot 3 + 1/2 \cdot 0$). Now, consider a game that is close to the one from Figure 3, but that is generic and that has a unique maximum associated with T. Should a player play T? Well, if he is not exactly sure that he observed the correct payoffs the answer is: maybe not. In that case, the actual payoffs may be such that the maximum is at M, or that his opponent thinks that the maximum is there. In such a case, it might still be better to choose B and thereby avoid the coordination problem.

	T	M	B
T	3	0	2
M	0	3	2
B	2	2	5/2

Figure 3: A Coordination Game

To conclude this section, we give one individualistic rationality argument that, for 2-person team games, distinguishes the payoff maximal equilibrium from any other pure equilibrium. Is it true that in such games, the payoff maximal equilibrium ***pairwise risk dominates*** any other pure equilibrium. Specifically, if s^* is the Pareto dominant equilibrium and s^{**} is any other pure equilibrium, then in the 2×2 game in which the players only have $\{s^*, s^{**}\}$ available, s^* risk dominates s^{**}. Hence, the Pareto dominant equilibrium may be said to be the pairwise risk-dominant one. A proof is simple and uses the alternative characterization of risk dominance for 2×2 games given in the previous section: Since the sum of the off-diagonal payoffs is smaller than the sum of the diagonal (equilibrium) payoffs, the sum of the players' critical probabilities for switching away from the payoff maximal equilibrium is less than one. An illustration is provided by the reduced games associated with

the equilibrium T from Figure 2, which are displayed in Figure 4. In the game on the LHS, T dominates M, while in the game on the RHS, T dominates B.

	T	M
T	3	0
M	0	3-ϵ

	T	B
T	3	2
B	0	2-ϵ

Figure 4: Reduced Games Associated with Figure 2

It should be noted that, in general, the concept of pairwise risk dominance captures the overall risk situation rather badly, see Carlsson and Van Damme (1993b). This is also evident from the LHS of Figure 4: when a player believes that his opponent may play T or M, then he has an incentive to play B, however, B is not present in the reduced game. In this respect it is also interesting to refer to the relationship between risk dominance and the stochastic stability of equilibria in an evolutionary context (Kandori et al. (1993), Young (1993)). In symmetric 2×2 games only the risk-dominant equilibrium is stochastically stable, but Peyton Young already provided an example of a 3×3 game in which the stochastically stable equilibrium differs from the pairwise risk-dominant one. Recently, Kandori and Rob (1993) have shown that for 2 player symmetric games that satisfy the Total Bandwagon Property (TBP) and the Monotone Share Property (MSP) only the pairwise risk-dominant equilibrium is stochastically stable. (TBP says that any best response against a mixture is an element of the mixture; MSP says that if a pure strategy is eliminated, the shares of all other pure strategies increase in the completely mixed equilibrium; the game from Figure 2 violates the Total Bandwagon Property.)

4. A Three-Person Example

The above two-person example is somewhat unsatisfactory since the solution process involves using the tie-breaking procedure and the latter might be considered ad hoc. The aim of this section is to provide a three-person team game that has a non-payoff maximal equilibrium that strictly risk dominates any other pure equilibrium. In fact, the example has the additional (desirable) feature that the details of the tracing procedure do not matter: since the game is symmetric, the risk-dominant solution is determined directly from the bicentric prior (2.4). Finally, the example shows that for 3-player games the Pareto dominant equilibrium need not be even pairwise risk dominant.

The example in question is the game given in Figure 5. (Here x is a real number in the interval $[0, 1]$, player 1 chooses a row, 2 a column and 3 a matrix. Obviously, the payoff dominant equilibrium is L if $x > 1/3$, while it is R if $x < 1/3$).

	L	R
L	$3x$	$2x$
R	$2x$	x

L

	L	R
L	$2x$	x
R	x	1

R

Figure 5: The Three-Person Team Game $\Gamma(x)$ $(0 < x < 1)$

It is easily seen that $\Gamma(x)$ has two strict Nash equilibria, viz. (L, L, L) and (R, R, R). To determine the risk-dominance relationship between these two equilibria, we compute the bicentric prior as in (2.4). If the players 2 and 3 play (L, L) with probability z and (R, R) with probability $1 - z$, then player 1 strictly prefers to play L if and only if

$$zx > (1 - z)(1 - x),$$

or, equivalently,

$$z > 1 - x. \tag{4.1}$$

Applying the principle of insufficient reason, the players 2 and 3 attach a probability x to the inequality (4.1) being satisfied. Hence, the prior beliefs of the players 2 and 3 are described by player 1's mixed strategy

$$xL + (1 - x)R \tag{4.2}$$

Since the game $\Gamma(x)$ is symmetric with respect to the players, the mixed strategy (4.2) actually is the prior of each player $i \in \{1, 2, 3\}$. To determine the risk-dominance relationship between (L, L, L) and (R, R, R), we have to determine the best response of player i when his opponents j and k independently randomize according to (4.2). The reader easily verifies that L is the unique best response if and only if

$$x(1 - (1 - x)^2) > (1 - x)^3$$

or, equivalently,

$$x^2 - 3x + 1 < 0 \tag{4.3}$$

Hence, if $x \in \left[\frac{3-\sqrt{5}}{2}, 1\right]$ then (L, L, L) is the risk-dominant equilibrium of the game. Now $\frac{3-\sqrt{5}}{2} > 1/3$, so that, there exists a range where the risk-dominant solution is $(R, R, R,)$ even though the payoff dominant equilibrium is (L, L, L).

We conclude this section by noting that also the equilibrium selection theory that has recently been proposed in Harsanyi (1995) and that involves a multilateral risk comparison of equilibria that is not based on the tracing procedure, does not always select the payoff dominant equilibrium in team games. Harsanyi proposes to select that equilibrium that has the largest stability region. As was already indicated in Section 3, the payoff dominant equilibrium may have a rather small stability region. Also for the example discussed in this section, the reader may verify that the equilibrium with the largest stability region, need not be payoff dominant. (Harsanyi defines the stability region of a strategy as the set of correlated strategies of the opponents against which the strategy is a best response, hence, the stability region of L is the set in the three-dimensional unit simplex where $p(R, R) \leq x$. He does not just take the Lebesgue measure of this set but first applies a transformation of the simplex.)

5. Conclusion

From one point of view one might argue that a generic team game is simple to play: Firstly, the payoffs of the players coincide so that there is no conflict of interest; secondly, the payoff function admits a unique maximum so that there is no risk of confusion. Hence, one might say that the unique payoff dominant equilibrium is the unique focal point: Players might view the game just as a one-person decision problem and solve it accordingly. These arguments are even more compelling for symmetric games. Upon closer inspection it turns out, however, that the above argument is not entirely convincing: The Pareto dominant focal point is not robust and this is reflected in the fact that risk-dominance considerations need not select it. Hence, even in symmetric team games, collective rationality need not be implied by individual rationality. With Harsanyi and Selten (1988, p. 359) we may conclude that "if one feels that payoff dominance is an essential aspect of game theoretic rationality, then one must *explicitly* incorporate it into one's concept of rationality."

Nevertheless, there is a sense in which team games are different from games in which the players' payoff functions do not coincide. Aumann (1990)

argued that, in the stag hunt game of Figure 1, even communication cannot help to bring about the equilibrium (R, R) in case players are convinced *a priori* that (S, S) is the solution of the game without communication. When communication is possible each player will always (i.e. no matter how he intends to play) suggest the other to play R since he can only benefit by having the other do so. Consequently, no new information is revealed by communication, hence, communication cannot influence the outcome. One may argue that things are somewhat different if it is common knowledge that the game is a team game: In this case a player has no incentive whatever to suggest an outcome different from the payoff maximal one. It remains to be investigated whether risk dominance selects the payoff dominant equilibrium if one, or more, rounds of preplay communication are added to the game.

At a somewhat more abstract level one may raise the question of *why* payoff dominance and risk dominance may be in conflict even in team games. I conjecture that this is because of the ordinality property of the risk-dominance concept. Hence, the conjecture is that there exist two team games with the same best reply correspondence that have different payoff-dominant equilibria. Thus far, I have not been able to formally prove this conjecture.

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Sustainable Equilibria in Culturally Familiar Games

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It is a pleasure to write for a volume in honor of Reinhard Selten, who has been to me both a mentor and a friend. Like many others, I am indebted to him for guidance and inspiration in many domains, from the logical study of rational behavior, to the pleasures of walking in the German countryside.

The definition of equilibrium was offered by Nash (1951) as the basic characterization of rational behavior in games, but Selten (1965, 1975) showed that the set of Nash equilibria may be too weak to characterize rational behavior in many games. That is, there may be strategic scenarios which pass the test of being Nash equilibria, but which do not fit our intuitive sense of what rational behavior should be in a game. So Selten launched the literature on refinements of Nash equilibrium, one of the great central strands of the literature of noncooperative game theory.

To evaluate a refined equilibrium concept, we may apply four criteria. First, the definition of the refined equilibrium concept should have some intuitive appeal as a characterization of what is rational behavior. Second, in games where we have strong intuitions about what are the reasonable equilibria, the refined solution concept should coincide with our intuitions, eliminating the unreasonable equilibria but not eliminating the reasonable ones. Third, the set of refined equilibria should be nonempty for all games to which this solution concept is supposed to be applied. Fourth, our refined solution concept should be invariant under transformations of the game that seem intuitively irrelevant. No solution concept in the literature has yet fully satisfied all of these criteria (even as far as we can reach a consensus about what is "intuitively reasonable"), and so the search for new refinements of the equilibrium concept continues. In this essay, I want to sketch one problem area in which more work on refinements of Nash equilibrium seems particularly needed.

My intuition about this problem comes from consideration of the well-known "Battle of the Sexes" game:

		Player 2	
		a_2	b_2
Player 1	a_1	3,2	0,0
	b_1	0,0	2,3

Table 1. The Battle of the Sexes Game

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This game has three equilibria: (a_1,a_2) which gives payoffs (3,2), (b_1,b_2) which gives payoffs (2,3), and a mixed equilibrium $(.6[a_1]+ .4[b_1], .4[a_2] + .6[b_2])$ which gives expected payoffs (1.2, 1.2). I believe that we need a refinement of Nash equilibrium that excludes the mixed equilibrium and includes only the two pure-strategy equilibria.

If this game were played only once, by a pair of subjects in an experimental laboratory setting, then I would consider the mixed equilibrium to be a very reasonable characterization of how rational players might behave. Indeed, given the obvious symmetry of the game, one might plausibly argue that it would be the only reasonable characterization of rational behavior in such a situation, where the players have no history to guide them.

But most games that interest us are not played in such isolation. More commonly, players are aware of a history of similar games that have been played by similar players, and this shared cultural history may crucially influence the players' expectations about how the game will be played in the future. For this Battle of the Sexes game, if the distinct roles of "player 1" and "player 2" are identified in the historical record of past outcomes, then the history of previous play can break the symmetry of the situation in favor of either player 1 or player 2, and thus can focus the players on one of the two pure-strategy equilibria.

This point about cultural history is implicit in the story from which this game derives its name. We may imagine that this game is played by couples on their first date, where player 1 is the man, player 2 is the woman, a_1 and a_2 denote the strategy of going somewhere that the man prefers, and b_1 and b_2 denote the strategy of going somewhere that the woman prefers; but both want most to be together. In a world where such couples really must make their decisions simultaneously and separately, I very much doubt that they would persist in the kind of uncoordinated confusion that we find in the mixed equilibrium. Surely a social expectation would evolve that steers such couples to one of the two pure-strategy equilibria. Cultural expectations favoring the woman (b_1,b_2) might evolve in one society, while expectations favoring the man (a_1,a_2) might equally well evolve in another cultural context. Once cultural expectations have focused on either pure-strategy equilibrium, however, Schelling's (1960) focal-point effect would operate, leading all future players to reproduce the expected pattern of behavior, because each player would understand that deviation from the cultural norms would reduce his or her payoff to zero.

It is hard to see how cultural expectations of the mixed equilibrium could be sustained, however. For example, if the outcome (b_1,b_2) occurred 100 times in a row (as must eventually happen in a sufficiently long history of plays of the mixed equilibrium), then surely each player would increase his or her subjective probability of the other player choosing b_1 or b_2 at the next play of the game. But when this subjective probability is increased above the probability in the mixed equilibrium, then choosing b_2 or b_1 would become optimal for each player, and so the repetition of the (b_1,b_2) outcome should continue. Thus any evidence that pushes the players slightly away from the mixed equilibrium can ultimately move them all the way to one or the other of the pure-strategy equilibria.

When we say that a game is played in a culturally familiar context, we are not assuming that anybody plays this game more than once. We are assuming that the game will be played in an infinite sequence of times, and the history of previous plays will be known by the players each time the game is repeated. But to guarantee that the payoffs in the matrix fully describe the effect that each player's strategy can have on his own payoff, we must also assume that each individual plays the game only once, so there are two new players in each repetition of the game. That is, we are assuming that, at each repetition of the game, no player has any incentive to develop a reputation or to otherwise affect the expectations that will be held in future plays of the game.

Another version of such cultural evolution has been suggested by Woodford (1990). Woodford's model is posed in a price-theoretic framework, but we can describe here a simplified game-theoretic version of the same story. Suppose that a payoff-irrelevant random variable (say, with finitely many possible values) is drawn independently from some fixed distribution before each play of the game, and the value of this random variable is announced to the players and is recorded along with their strategy choices in the published history. Then it is possible that equilibrium expectations could be based on a shared perception that the only relevant part of past history is in the cases where the value of the random variable was the same as its current value. One possible equilibrium scenario is that, for each possible value of the random variable, the players follow the mixed equilibrium until (a_1,a_2) or (b_1,b_2) occurs in a round with this random value, and thereafter they play the same pure-strategy equilibrium every time this random value is observed. So in this story the cultural expectations will lead to behavior that might appear random to an outside observer who ignored the payoff-irrelevant random variable; but the outcomes will eventually all be either (a_1,a_2) or (b_1,b_2), which is certainly not the pattern of outcomes for the mixed equilibrium $(.6[a_1]+.4[b_1],.4[a_2]+.6[b_2])$ (where (a_1,b_2) happens 36% of the time, for example).

Thus, when the Battle of the Sexes game is played as a culturally familiar game (with the roles of players 1 and 2 being distinctly identifiable across plays), we may expect that the players should have evolved a pattern of expectations which rules out the mixed equilibrium but admits either of the pure-strategy equilibria. For now, let us say informally that the two pure-strategy equilibria seem to be "sustainable" in a way the mixed equilibrium does not. This description begs the question of what kind of formal solution concept can we define to identify the "sustainable" equilibria that might be expected to persist in any strategic-form game, when it is played in a culturally familiar context. That is, how should we extend this intuitive concept of "sustainability" from the Battle of the Sexes game to a general refinement of Nash equilibrium that can be applied to any strategic-form game?

One naive approach would be to view this as a question of distinguishing between pure-strategy equilibria and randomized-strategy equilibria. However, some games have no pure-strategy equilibria, and so the set of pure strategy equilibria cannot be the refinement of Nash equilibrium that we are seeking. Even

the rule "eliminate properly randomized or mixed equilibria when pure-strategy equilibria exist" does not seem right, as the following example illustrates.

		Player 2		
		a_2	b_2	c_2
Player 1	a_1	4,0	0,4	3,0
	b_1	0,4	4,0	3,0
	c_1	0,3	0,3	3,3

Table 2. A game where the pure-strategy equilibrium seems wrong.

The unique pure-strategy equilibrium is (c_1,c_2), but this equilibrium is in weakly dominated strategies and so seems unlikely to persist. If the players perceive that there is any positive probability of either player using another strategy, even by accident, then neither player can rationally choose c_1 or c_2.

Another approach would be to focus on the fact that, in the Battle of the Sexes game, the two "sustainable" equilibria are both Pareto-superior to the unsustainable mixed equilibrium. But the unsustainable (c_1,c_2) equilibrium in Table 2 is actually Pareto-superior to the more reasonable mixed equilibrium $(.5[a_1]+.5[b_1], .5[a_2]+.5[b_2])$ for this game. (The game may be viewed as a modified Prisoners' Dilemma game.) Furthermore, consider the following "Stag Hunt" game (adapted from a similar game discussed by Harsanyi and Selten (1988) in Section 10.12).

		Player 2	
		a_2	b_2
Player 1	a_1	6,6	0,4
	b_1	4,0	3,3

Table 3. The Stag Hunt Game

This game has three equilibria: a pure-strategy equilibrium (a_1,a_2) which gives payoffs (6,6), a pure-strategy equilibrium (b_1,b_2) which gives payoffs (3,3), and a mixed equilibrium $(.6[a_1]+.4[b_1],.6[a_2]+.4[b_2])$ which gives expected payoffs (3.6, 3.6). In this game, I would argue that the two pure-strategy equilibria are both sustainable, even though one of them (b_1,b_2) is Pareto-inferior to both of the other two equilibria. In a culture where everyone expects others to choose b_1 or b_2 with high probability, the introduction of a small probability of deviation to a_1 or a_2 would not cause players to rationally switch away from b_1 or b_2. Of course, the same could be said about the Pareto-superior (a_1,a_2) equilibrium: In a culture where everyone expects others to choose a_1 or a_2 with high probability, the introduction of a small probability of deviation to b_1 or b_2 would not cause players to rationally switch away from a_1 or a_2. On the other hand, the mixed equilibrium does seem unstable. In a dynamic environment where people initially expect 60% a_i and 40% b_i choices, any small increase in the perceived rate of choice of either action would lead others to further increase their rational selection of that action (because a_1 is best

for player 1 if the probability of a_2 is anything more than 60%, whereas b_1 is best for player 1 if the probability of a_2 is anything less than 60%). So any perceived deviation from the mixed equilibrium should induce larger deviations in the same direction until there was a general convergence to one of the two pure-strategy equilibria. Thus, my intuition is that both the (b_1,b_2) equilibrium and the (a_1,a_2) are "sustainable" equilibria in Table 3, but the mixed equilibrium is not.

In the terminology of Harsanyi and Selten (1988), the (b_1,b_2) equilibrium in Table 3 risk dominates the (a_1,a_2) equilibrium. Indeed, each player would prefer to choose his b-strategy whenever the other player's probability of choosing the b-strategy is anything more than 40%. The model of Kandori, Mailath, and Rob (1993) predicts that the (b_1,b_2) equilibrium will be played infinitely more often than the (a_1,a_2) equilibrium, in the long run of a dynamic society where different pairs play this game in an infinite sequence. However, I would argue that our "sustainability" solution concept should include both (a_1,a_2) and (b_1,b_2) for the game in Table 3, without distinguishing either pure-strategy equilibrium as more sustainable than the other, but the concept should exclude the mixed equilibrium of this game. I believe that, for example, if we are looking for simple game-theoretic models to help us better understand the problems of economic development that separate rich nations from poor nations, then there may be no better place to start than with this Stag Hunt game, and with the recognition that a culture could persist in either the Pareto-superior (a_1,a_2) equilibrium or the Pareto-inferior (b_1,b_2) equilibrium.

Kohlberg and Mertens's (1986) concept of stability does not exclude any of the three equilibria in either the Battle of the Sexes or the Stag Hunt game. For each of these two games, any sufficiently small perturbation of the payoffs would still leave a game that has a mixed equilibrium close to the mixed equilibrium of the given game. Both Selten's (1975) concept of trembling-hand perfect equilibrium and Myerson's (1978) concept of proper equilibrium also admit all three equilibria of these games. In particular, the mixed equilibrium is perfect and proper for each of these two games, because any randomized equilibrium that assigns positive probability to all pure strategies is perfect and proper.

There is at least one well-known refinement of Nash equilibrium in our published literature which does what we want for both the Battle of the Sexes and the Stag Hunt games: Kalai and Samet's (1984) concept of persistent equilibrium. For both of these games, the two pure-strategy equilibria are persistent equilibria but the mixed equilibrium is not. Furthermore, Kalai and Samet have proven the general existence of persistent equilibria for all finite strategic-form games. Other variations on persistence have been discussed in the literature, such as Harsanyi and Selten's (1988) concept of equilibria in primitive formations, and Basu and Weibull's (1991) concept of equilibrium in minimal curb sets, which (as Van Damme, 1994, has remarked) is equivalent to persistence for generic strategic-form games.

Let us denote a general finite strategic form game by

$$\Gamma = (N, (C_i)_{i\in N}, (u_i)_{i\in N})$$

where N is the set of players, C_i is the set of pure strategies for player i, and u_i is the utility function for player i (mapping strategy profiles in $\times_{j\in N} C_j$ to real numbers). Given any finite game in strategic form, let us say that a block is any profile of nonempty subsets of all players' pure strategy sets. That is, $B = (B_i)_{i\in N}$ is a block if each B_i is a nonempty subset of C_i. A block is a curb set if, for every profile of randomized strategies that assign zero probability to all strategies outside the block, the best responses of each player are all within the block. ("Curb" is an abbreviation for "closed under rational behavior.") In the analysis of simple strategic form games, a formation is the game restricted to a curb set that is also closed under best responses when correlated strategies are taken into account. A block is a retract if there exists some positive number ε such that, for every profile of randomized strategies that assign less than ε probability to all strategies outside the block, each player has a best response that stays in the block. We may say that a retract or curb set or formation is minimal if it is does not contain any subsets that share this property. A minimal formation has been called primitive by Harsanyi and Selten (1988), and a minimal retract has been called persistent by Kalai and Samet (1984). A persistent equilibrium is an equilibrium that assigns positive probability only to strategies that are in one minimal retract.

Our interest in persistent retracts and primitive formations can be motivated by the following evolutionary story. When by chance the strategies outside of some formation or retract happen to be very rarely used during a long interval, then the players should begin to believe that these outside strategies would be unlikely in the future, and this assumption (by the definition of a formation or retract) would tend to confirm itself. So, during the long history of repetitions, the set of strategies that the players consider likely should eventually shrink until it coincides with some minimal set that is self-confirming (in some appropriate sense).

For both the Battle of the Sexes game and the Stag Hunt game, the retracts (and the curb sets, and the formations) are $(\{a_1,b_1\}, \{a_2,b_2\})$, $(\{a_1\},\{a_2\})$, and $(\{b_1\},\{b_2\})$. So $(\{a_1\},\{a_2\})$ and $(\{b_1\},\{b_2\})$ are the persistent retracts (and the primitive formations). Thus, (a_1,a_2) and (b_1,b_2) are persistent equilibria. But support of the mixed equilibrium is not contained in either persistent retract and so the mixed equilibrium is not a persistent equilibrium.

Following an argument of Kohlberg and Mertens (1986), Van Damme (1994) has emphasized that the concept of persistent equilibrium is not invariant under substitution of equivalent subgames, however. For example, consider the following simple game.

		Player 2	
		x_2	y_2
Player 1	x_1	4,-1	-4,1
	y_1	-4,1	4,-1

Table 4. A Game with Unique Equilibrium Payoffs (0,0)

This game has an equilibrium $(.5[x_1] + .5[y_1], .5[x_2] + .5[y_2])$ which gives expected payoffs (0,0), and there are no other Nash or correlated equilibria of this game. Now consider again the Battle of the Sexes game, but suppose that the (0,0) payoffs in that game really represent situations in which the players must go on to play the game in Table 4. Then we get an elaborated version of the Battle of the Sexes game with can be represented in strategic form as shown in Table 5.

		Player 2			
		ax_2	ay_2	bx_2	by_2
Player 1	ax_1	3,2	3,2	4,-1	-4,1
	ay_1	3,2	3,2	-4,1	4,-1
	bx_1	4,-1	-4,1	2,3	2,3
	by_1	-4,1	4,-1	2,3	2,3

Table 5. An Elaborated Battle of the Sexes Game

In this elaborated Battle of the Sexes game, there are no retracts, formations, or curb sets other than the block of all strategies in the game. Thus, all equilibria are persistent, including the elaborated version of the original mixed equilibrium

$$(.3[ax_1] + .3[ay_1] + .2[bx_1] + .2[by_1], .2[ax_2] + .2[ay_2] + .3[bx_2] + .3[by_2]).$$

So the substitution of subgames with unique equilibria, a seemingly irrelevant transformation of the game, has transformed and trivialized the set of persistent equilibria. Nonetheless, I would argue that the elaborated versions of the (former) pure-strategy equilibria

$$(.5[ax_1] + .5[ay_1], .5[ax_2] + .5[ay_2]) \text{ and}$$
$$(.5[bx_1] + .5[by_1], .5[bx_2] + .5[by_2])$$

still deserve to be called "sustainable" in a way that the full-support mixed equilibrium does not.

(One possible way of rescuing the primitive-formation or persistent-retract approach has just appeared in a new paper on dual reduction; see Myerson, 1995. Using this new technique, the game in Table 5 can be reduced to the Battle of the Sexes game in Table 1. So the possibility of applying the persistence refinement after dual reduction may deserve further attention in future research.)

Maynard-Smith's (1982) concept of evolutionary stable strategies has also been viewed as a way of distinguishing the two pure-strategy equilibria of the Stag Hunt game from the mixed equilibrium. For any symmetric game two-player game, an evolutionary strategy is a pure or randomized strategy σ such that, for any other pure or randomized strategy τ, there is some mixture of σ and τ against which the strategy σ is better than the strategy τ. In the Stag Hunt game, the strategy $\sigma = .6[a_i] + .4[b_i]$ fails to be an evolutionary stable strategy because, when player 2 randomizes between this strategy and $\tau = [a_2]$, player 1 finds that using a_1 is never worse than using σ.

		Player 2		
		a_2	b_2	c_2
Player 1	a_1	4,4	5,0	0,5
	b_1	0,5	4,4	5,0
	c_1	5,0	0,5	4,4

Table 6. A game with no ESS.

Evolutionary stable strategies sometimes do not exist, however. For example, the game in Table 6 has a unique equilibrium, in which each player randomizes uniformly over his or her three strategies, choosing each with probability 1/3. This equilibrium gives each player an expected payoff of 3. But when the number of players using the a_i strategy is increased above 1/3, then using a_i becomes better than randomizing uniformly over all three strategies.

Recently there has been wide interest in the study of evolution and adaptive dynamics in games (see Mailath, 1992, and Crawford, 1993, for example). It may seem obvious that the solution being sought in this paper should be definable as the set of limits of some dynamic process of learning and adaptation in repeated plays of the game. Unfortunately, no simple adaptive-dynamics models are known for which convergence to equilibrium can be proven from generic initial conditions. Without such a convergence result, we cannot prove general existence for a refinement that is defined as the set of equilibria that can be (robustly) achieved as limits of such adaptive dynamics. Furthermore, if the limit points of the distribution of outcomes are not at least contained within the set of correlated equilibria, then there must exist some simple transformations of behavior (of the form "when the adaptive process cues you to strategy a_i, do strategy b_i instead") that would strictly improve some players' average payoffs over arbitrarily long periods of time.

The tracing procedure (see Harsanyi, 1975, and Harsanyi and Selten, 1988) offers another possible approach to the characterization of "sustainability." Given any strategic-form game Γ as above, if $\rho = (\rho_i)_{i\in N}$ is any profile of randomized strategies and t is any number between 0 and 1, then $\Gamma'(t,\rho)$ may be defined as the game in which there is probability t that all the players are playing Γ together, but there is probability 1-t that each player i is playing separately against machines that use the strategies ρ_{-i}. So $\Gamma'(1,\rho)$ is the same as the original game Γ; but $\Gamma'(0,\rho)$ is the game in which each player separately faces opponents who are behaving according to ρ. We may say that an equilibrium σ can be reached from ρ if there exists a continuous function $\theta(\bullet)$ such that $\theta(t)$ is an equilibrium of $\Gamma'(t,\rho)$, for all t, and $\theta(1) = \sigma$.

It is easy to see that any equilibrium of Γ is reachable from itself. However, the mixed equilibrium of the Battle of Sexes game can be reached from only a one-dimensional set of alternative conjectures ρ, whereas each of the two pure-strategy equilibria can be reached from a full two-dimensional set of alternative conjectures ρ. Thus, the size of the set of alternative conjectures that can reach an equilibrium may offer a way to systematically distinguish "sustainable" from

"unsustainable" equilibria. However, the computational difficulties of the tracing procedure have made it difficult for me to check the general properties of this approach.

I can think of another approach to our problem which is more purely mathematical. The generic oddness of the number of Nash equilibria (see Harsanyi, 1973) is derived from the fixed-point theorems of algebraic topology. To understand the intuition behind these theorems, it may be helpful to think about the following example. Let $y{:}R \to R$ be any continuous function such that

$$\lim_{x\to+\infty} y(x) = +\infty, \text{ and } \lim_{x\to-\infty} y(x) = -\infty.$$

(For example, think of $y(x) = x^3 - x$.) The graph of $y(\bullet)$ must cross the x-axis (where $y = 0$) an odd number of times because, as we move from negative x to positive x, there must be one more crossing up from below than crossings down from above. That is, each crossing of the x-axis can be assigned an index value, +1 for each crossing up from below (where y(x) is locally increasing in x) and -1 for each crossing down from above (where y(x) is locally decreasing), and then the sum of the indexes of all crossings must be +1. (Tangencies where the graph does not cross the x-axis, as for $y(x) = x^3 - x^2$ at $x = 0$, are not counted as crossings, or get index 0.)

This mathematical heuristic suggests that we might look for some way of assigning index numbers to Nash equilibria such that, for all generic games, each equilibrium can be assigned the index +1 or -1, and the sum of the indexes of all equilibria is +1. If any such theory were applied to the Battle of Sexes game, the symmetry of the two pure strategy equilibria should guarantee that they must have the same index number, and so the pure-strategy equilibria of this game would have index +1 and the mixed equilibrium must have index -1. Thus, generalizing this heuristic, we may wonder whether the mathematics of differential topology might be able to give us some sophisticated index function such that the sustainable equilibria have index +1 and the unsustainable equilibria have index -1.

We can probe this intuition somewhat further, however, without waiting for some magic formula or theorem to drop from the realm of higher mathematics. If such an index-number theory existed, then we can also predict how it would apply to the following game.

		Player 2			
		a_2	b_2	c_2	d_2
Player 1	a_1	0,6	1,4	0,3	1,0
	b_1	1,0	0,3	1,4	0,6

Table 7. A game with three equilibria.

The optimal strategy for player 2 depends on the probability of player 1 choosing a_1 as follows: a_2 is optimal if the probability of a_1 is greater than .6, b_2 is optimal if the probability is between .5 and .6, c_2 is optimal if the probability is between .4 and .5, and d_2 is optimal if the probability is less than .4. So this game has just three randomized equilibria:

$(.6[a_1] + .4[b_1], .5[a_2] + .5[b_2])$,
$(.5[a_1] + .5[b_1], .5[b_2] + .5[c_2])$, and
$(.4[a_1] + .6[b_1], .5[c_2] + .5[d_2])$.

There is an obvious symmetry between the first and third of these equilibria, and so they must get index +1, and the middle equilibrium must get index -1. Thus we should ask, is there any reasonable sense in which the middle equilibrium of Table 7 is less "sustainable" than the other two equilibria? The approach of looking for primitive formations and persistent retracts would not suggest any distinction among these three equilibria, but perhaps this game deserves some further consideration.

In this paper, I have discussed the importance of trying to define some formal concept of "sustainability" of equilibria, to distinguish equilibria that are likely to be played in culturally familiar settings. I have not offered any precise formal definition of "sustainability," however. Instead, I have tried to describe an open problem, which I have pondered for long time. I have occasionally found promising new approaches to this problem, but so far none have led to the kind of simple and appealing formulation that I feel intuitively must be out there somewhere. Still I have hope that tomorrow someone may have the conceptual breakthrough that can help to fill this important gap in the game-theory literature.

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Evolutionary Conflict and the Design of Life

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1. Introduction

During biological evolution there have been many circumstances under which game-like conflict must have occurred among entities that could reproduce, such as self-replicating molecules, genes, chromosomes, and organisms. These „games of real life" resulted from the Darwinian struggle for long-term existence. They had a major influence on the course of evolution and left their impressive traces in the design of organisms. For example, the existence of two different sexes appears to be an evolutionary consequence of the severe Darwinian conflict among those parts of the genetic material that are not located in the cell's nucleus (cytoplasmic DNA). Without identifying this „cytoplasmic war", we would not have reached a deep understanding of why there are two sexes and why cytoplasmic DNA is usually transmitted by one of the sexes only. Similarly, it is impossible to understand the behavioural and morphological differences between males and females without studying their intra- and intersexual „games". Conflict analysis turns out to be generally one of the most powerful tools for revealing the evolutionary logic of organismic design. This explains the similarity in how evolutionary biologists and game-theorists approach their problems.

In the following I describe some cases of evolutionary conflict with intriguing consequences for the organisation of genetic material, cells, and organisms. All examples can be understood intuitively and no formal mathematical models are presented. I do not even claim that models from classical game theory can be used for analysing these cases of conflict. However, it is the „flavour of a game" which certainly can be felt when looking at the biological problems discussed in this essay.

2. Conflict in the Early Days of Life

We all remember Miller's famous experiments in which he created artificial „lightning" that struck his laboratory version of the „primitive soup". He demonstrated the formation of proteinogenic amino acids in an experimental setting which mimicked conditions on the primitive Earth that might have caused the origin of life. However, the relevant context for the origin of life is not well enough described by the picture of a „soup exposed to thunderstorms". Many important molecules - especially those needed for membrane formation - have not

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been synthesized in such electric discharge reactions. It seems more convincing that the first important steps towards life were made close to hypothermal vents in the deep sea, where they probably took place on mineral surfaces, such as that of pyrite (Wächtershäuser, 1988). The origin of membranogenic lipids can be explained for a two-dimensional chemical world on such a surface. An appropriate metaphor would be to consider the first forms of life not as the „garnish of the primitive soup“ but as the „topping of the primitive pizza“. The expression „primitive pizza“ was recently suggested by two biologists (Maynard Smith & Szathmáry, 1995) with strong culinary inclinations. In their delicate description of the origin of life, the pizza's cheese layer stands for the first lipid membrane and bubbles of this layer can perhaps be viewed as primitive predecessors of what later in evolution became the cell.

After this first stage of primitive life, many steps must have occurred before the cheese bubbles could evolve into a true cell with all its sophisticated features. Szathmáry & Demeter (1987) discuss an intermediate stage in cell evolution with the following properties. Their „protocells“ consist of a membrane that forms a compartment in which a simple metabolism can take place. In the early protocell there are several kinds of molecules, say A, B, and C, which replicate separately within the compartment. These replicators are not linked and chromosomes have not yet evolved in Szathmáry & Demeter's view of this stage of cell evolution. A protocell produces daughter cells by fission, whereby random samples of the existing molecules A, B, and C are passed on to each member of the new cell generation. The rate of division of a protocell depends on the types and proportions of replicating molecules contained.

Under this assumption, there is conflict among replicators within a protocell regarding the speed of their replication. By replicating faster than others, a molecule of a given type can increase the average proportion of its own type in the daughter cells. This would be true even if the faster replicator gained its speed by deteriorating chemical resources of the protocell. Such a replicator would be more successful than others at the expense of the overall protocell performance. This really looks like a conflict situation known as the „tragedy of the commons“. In view of this potential conflict, the protocell stage of evolution must have been very fragile. It is also interesting to observe that the tragedy of the commons seems to have threatened living matter already at such an early stage.

Why is it the case that the protocells have not become extinct before more advanced forms of life could evolve? Due to the assumed absence of horizontal replicator transmission and due to the lack of recombination and of cell fusion we find in the protocell world an ideal situation for group selection to operate. A line of descendants containing a very fast replicator will simply die out without „infecting“ any other lineage by transmission of the non-cooperative molecule. But one problem remains: due to the basic properties of chemistry there will always be small differences in replication rates between different types of replicators. This could have lead to a dramatic long-term change in all replicator proportions and it could have entailed the extinction of all protocells.

Their complete extinction apparently did not happen. Had it happened, this might have prevented further evolution of life and, perhaps, this article could not have been written. In order to understand the long-term survival of protocells, note that the sampling effect during their fission produces daughter cells with new proportions of replicators. Due to this sampling effect, some of them will not have an excessive amount of the faster replicators. As Szathmáry has shown, the joint effect of this „stochastic correction“ and of group selection can be strong enough to keep a population of protocells alive and to favour the evolution of some cooperation between replicators of the same protocell. However, life on Earth would not have gone very far with this basic design of the cell. The reason is that stochastic correction only works efficiently if the number of molecules per compartment is very small. Under such conditions evolution would have taken place in a chemical world with relatively few dimensions.

3. Chromosomes and the Drunken Walk of the Diploid

As we saw in the previous section, early protocell evolution could not generate cells containing many replicators. Undoubtedly, this limitation has been finally overcome by the evolution of linkage between replicators, that is by the origin of chromosomes. Once there is linkage between two or more of the replicators, their replication rates are equal and this enhances the scope for efficient stochastic correction. Furthermore, since linkage ensures that a group of cooperating replicators will not be separated during protocell fission, there is also an enhanced potential for the evolution of cooperation within the cell.

All these advantages of chromosomes do not immediately explain why they have evolved in the first place. One reason is that chromosome replication is slow as compared to the separate replication of its constituents. For the case of two linked replicators, Maynard Smith & Szathmáry (1993) consider a twofold disadvantage in replication speed as a conservative estimate of the "cost of linkage". They are able to show that such a disadvantage can be offset by selective advantages if the number of "genes" in the original protocell is small and if the particular combination of the linked genes is needed for efficient cell reproduction. Under these circumstances, linkage evolves because genes with complementary effects now benefit strongly from their cooperation and gene competition due to different replication rates is reduced.

Although the evolution of chromosomes was perhaps the most important step towards a limitation of reproductive competition in the genome, this has not been sufficient to overcome all intragenomic conflict and to completely „tame the genes". As soon as there is more than one chromosome in the same cell, genes can achieve a replication advantage if they cause the chromosome on which they "travel" to gain a disproportional share in the next generation of chromosomes. Such potential gains become a particular threat to intragenomic cooperation when the additional features of cell fusion and genetic recombination also come into play. Both new features strongly undermine the strength of group selection. This

means that little power is left to the selective force that seems to have maintained "genetic order" in early protocell evolution. Other mechanisms must have replaced group selection as the "policeman“ that holds the genes in check and ensures their cooperation.

To cut a long story short, evolution has finally produced very elaborate patterns of how chromosomes are passed on to new generations. In order to discuss the sophistication of genetic transmission at later stages of evolution, let us now turn our attention to sexually reproducing organisms with diploid genetic material in the cell's nucleus. The nuclear genetic material then consists of pairs of homologous chromosomes. A parent transmits only one of its two homologous chromosomes via gametes (eggs or sperm) to an offspring. This requires the segregation of chromosomes during gamete production. In principle, such a segregation could be achieved by a single „reduction division“ which transforms one diploid cell into two haploid daughter cells. But evolution did not favour this simple design of reproductive physiology. Instead of producing an elegant solution, evolution has invented a complicated process, called meiosis, during which chromosome numbers are first doubled and the cell then undergoes two consecutive reduction divisions.

Hurst (1993) calls this puzzling aspect of meiosis "the drunken walk of the diploid". In order to make one step forward, the diploid makes first one step backward and then two steps forward. How can we explain this strange design found in many forms of life? Remember that gametes are the devices that carry chromosomes into the next generation. In a thought experiment, suppose that only a single reduction division takes place which directly produces two haploid "sister cells". Consider now a mutant gene that disturbs the process of reduction division by killing the sister cell as the reduction division takes place. Chromosomes with this mutant gene would end up in more than half of the gametes. Such an effect is called meiotic drive and the driving mutant may initially invade the population even if it has otherwise negative effects on organismic performance.

This resembles vaguely the reproductive conflict among replicators in early protocells. But we can no longer invoke group selection as the ordering force that reduces the damaging consequences of this conflict. It may please all alcoholics to learn that it is the „drunken behaviour of the diploid“ which acts as an institution against meiotic drive (Haigh and Grafen, 1991). The step backward, i.e. the apparently useless first duplication of chromosomes, creates a copy of the meiotic drive gene and therefore opens the possibility that this gene "shoots itself in the foot" when it kills a sister cell. This is an advanced way of forcing the nuclear genes into cooperation. We do not know, however, whether this effect of the drunken walk has really caused its long-term maintenance in evolution. The origin of this phenomenon probably relates to the properties of mitotic cell division which seems to be the historical predecessor of meiosis.

4. Cytoplasmic Conflict and its Resolution: The Origin of the Sexes

The story of intragenomic conflict continues outside the cell's nucleus. After the nucleus had been tamed, a new evolutionary battlefield probably arose in the world of cytoplasmic DNA. It is very likely that reproductive competition now occurred among organelles, such as mitochondria or chloroplasts. There seems to be no obvious evolutionary force that would have restricted this competition with high efficiency. However, evolution has resolved most of the cytoplasmic conflict by improving the design of life in an „ingenious" way. This improvement was achieved by the origin of two different mating types, only one of which transmitted its organelles to the next generation. The Darwinian creation of these mating types initiated the evolution of the two sexes. Who would have thought that the sexes were „born" in a cytoplasmic war?

This idea about the origin of binary sexes was first published by two psychologists (Cosmides & Tooby, 1981) and later elaborated by several biologists (Hoekstra, 1987; Hurst & Hamilton, 1992). In order to understand this, suppose that there are no mating types yet. Gametes are then all alike and there is no egg-sperm dimorphism. When two such isogametes fuse, both contribute organelles to the new cell that originates from their fusion. During this fusion , the organelles of the two gametes find themselves in a competitive situation regarding their prevalence in the new cell. Hurst and Hamilton argue that this probably has led to the evolution of active attempts to harm organelles of the other gamete. Such evolved warfare must have had adverse effects on cell performance after fusion. This in turn should have created a selective advantage for those nuclear genes that are capable of undermining cytoplasmic warfare. Hamilton and Hurst imagine, for example, a mutation in the nuclear DNA which suppresses the harmful activities of all those organelles with which it travels in the gamete. Even if the fusion occurs with another gamete that does not carry the suppressor gene, the cytoplasmic war will be short in view of the unilateral disarmement caused by the suppressor gene. At the population level, a polymorphic equilibrium can be reached containing both the suppressor and the non-suppressor gene.

In order to complete the evolution of the sexes, two more steps are required. First of all, we need the origin of two mating types, each of which fuses only with the other. Taking the equilibium of suppressor and non-suppressor as the new evolutionary starting point, assortative mating between suppressor and non-suppresor can easily evolve. By this process, the two mating types „plus" and „minus" come into existence. The symmetry of gametes is now entirely broken and, in a second step, evolution can attach further attributes to plus and minus. This must have largely facilitated the differentiation into egg and sperm, i.e. the evolution of the „true" sexes. Once males and females existed, their reproductive

competition at the organismic level must have generated various kinds of intra- and intersexual conflict. For example, there is quite a potential for evolutionary conflict between the sexes regarding mating, paternity, and parental investment. At this point, the Darwinian tale begins to describe phenomena that we are all very familiar with.

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Evolutionary Selection Dynamics and Irrational Survivors*

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Abstract. We consider certain classes of evolutionary selection dynamics in discrete and continuous time and investigate whether strictly dominated strategies can survive in the long run. Two types of results are presented in connection with a class of games containing the game introduced by Dekel and Scotchmer [5]. First, we use an overlapping-generations version of the discrete-time replicator dynamics and establish conditions on the degree of generational overlap for the survival and extinction, respectively, of the strictly dominated strategy in such games. We illustrate these results by means of computer simulations. Second, we show that the strictly dominated strategy may survive in certain evolutionary selection dynamics in continuous time. *Journal of Economic Literature* Classification Numbers: C72, C73.

1. Introduction

A basic rationality postulate underlying much of non-cooperative game theory is that players never use strategies that are strictly dominated. Hence, in so far as one is interested in establishing evolutionary foundations for solutions concepts in non-cooperative game theory, an important question is whether evolutionary selection processes do eliminate such strategies.

We here consider a few classes of evolutionary selection processes in discrete and continuous time, and investigate conditions under which strictly dominated strategies are eliminated, or, alternatively, can survive in the long run. The set-up is standard for evolutionary game theory: we imagine a large (infinite) population of individuals who are randomly drawn to play a symmetric and finite two-player game in normal form. The classes of selection dynamics here considered include the discrete-time and continuous-time versions of the replicator dynamics used in biology, but are wide enough to

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also include a range of social and cultural transmission processes based on replication by way of imitation (cf. Björnerstedt and Weibull [2], Weibull [12]). In the case of biological evolution, the payoffs in the game in question represent reproductive fitness while in the case of social evolution the payoffs represent preferences - shared by all individuals.

Our study relates to the following established results:

Fact 1 The continuous-time replicator dynamics eliminates any strictly dominated pure strategy (Akin [1]).

Fact 2 The discrete-time replicator dynamics eliminates any pure strategy strictly dominated by a pure strategy (Samuelson and Zhang [10]), but may fail to eliminate a pure strategy strictly dominated only by a mixed strategy (Dekel and Scotchmer [5]).

Fact 3 All continuous-time payoff-monotone evolutionary selection dynamics eliminate any pure strategy strictly dominated by a pure strategy (Samuelson and Zhang [10]).

Below we present two groups of results. *First*, we devise a class of overlapping generations models in discrete time, a class that contains the usual discrete-time and continuous-time replicator dynamics as a special and limiting case, respectively. The model we construct is different from that devised by Cabrales and Sobel [4], see Section 3.6. The discrepancy between facts 1 and 2 above is explained theoretically in this framework with reference to the degree of intergenerational overlap: Akin's [1] result is extended to a range of high degrees of overlap and Dekel's and Scotchmer's [5] result is extended to a range of low degrees of overlap. We also report a series of computer simulations that indicate how the intergenerational overlap affects the nature of the dynamic evolutionary solution paths in a game studied by Dekel and Scotchmer. For low degrees of overlap the paths converge to the unique Nash equilibrium of the game, for an intermediate range of overlaps the paths form an interior limit cycle, and for low degrees of overlap the paths diverge toward the boundary. In the last two cases the game's strictly dominated pure strategy (that is, however, not dominated by any pure strategy) survives in the long run.

Second, we investigate to what extent the extinction of strictly dominated strategies can be guaranteed in a wide range of evolutionary selection processes modelled in continuous time. More exactly, we consider two such classes: the class of "payoff monotone dynamics" (Samuelson and Zhang [10]) with the property that pure strategies with higher payoffs have higher growth rates, and the wider class of "weakly payoff monotone dynamics" (Friedman [6]) which only requires that the population composition moves in the general direction of increasing payoffs. We provide a game and a weakly payoff monotone dynamics such that a pure strategy that is strictly dominated by

all other pure strategies in the game survives in the long run, in a sizable population share and for a wide range of initial conditions. Dekel and Scotchmer [5] construct a game that has a strictly dominated strategy that is not strictly dominated by any pure strategy. We provide a payoff monotone dynamics under which this strategy survives, again in a sizable population share and for a wide range of initial conditions.[1]

The rest of the paper is organized as follows. Section 2 introduces basic notation and terminology. Section 3 contains our discrete-time analysis, centred on the mentioned overlapping-generations replicator dynamics. Section 4 contains our continuous-time analysis, and Section 5 concludes.

2. Notation and Preliminaries

We consider finite symmetric two-player games in normal form. Let $I = \{1, 2..., k\}$ be the set of pure strategies (the same for both players). A mixed strategy is then a point x on the unit simplex $\Delta = \left\{x \in R_+^k : \sum_{i=1}^k x_i = 1\right\}$. Let A denote the $k \times k$ payoff matrix, so that a_{ij} is the payoff to strategy i when played against strategy j. We represent a pure strategy i as the (degenerate) mixed strategy which assigns probability one to the i'th pure strategy. Geometrically, this is the i'th vertex of the mixed strategy space, which we will denote $e^i \in \Delta$. Then $u\left(e^i, x\right) = e^i Ax$ is the expected payoff of a pure strategy i when played against a mixed strategy $x \in \Delta$. Accordingly, $u(x, y)$ is the expected payoff of a mixed strategy $x \in \Delta$ when played against a mixed strategy $y \in \Delta$.

A strategy $x \in \Delta$ is *strictly dominated* if there exists a strategy $y \in \Delta$ which always earns a higher payoff, i.e. if $u(y, z) > u(x, z)$ for all strategies $z \in \Delta$. A strategy pair (x, y) is a *Nash equilibrium* if x is a best reply to y and y is a best reply to x.

Much of the subsequent analysis will refer to modifications and extensions of the so-called ROCK-SCISSORS-PAPER game, the payoff matrix of which is the special case $\alpha = 0$ of

$$A = \begin{pmatrix} 1 & 2+\alpha & 0 \\ 0 & 1 & 2+\alpha \\ 2+\alpha & 0 & 1 \end{pmatrix} \qquad (\text{for } \alpha > -1). \tag{1}$$

Strategy 1 (ROCK) "beats" strategy 2 (SCISSORS), which "beats" strategy 3 (PAPER), which in its turn beats strategy 1. The mixed strategy $\bar{x} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is clearly a best reply to itself, so $(\bar{x}, \bar{x})$ constitutes a Nash equilibrium. Indeed, this is the only Nash equilibrium (granted $\alpha > -1$). A game with payoff matrix 1 will be referred to as a *RSP*(α) *game*.

[1] Nachbar [9] shows that if an interior solution trajectory to a payoff monotone selection dynamics converges, then the limiting state is a Nash equilibrium strategy. Hence, survival of a strictly dominated strategy requires that the solution in question diverges.

We here consider a large population of individuals who every now and then are randomly drawn to play some finite and symmetric two-person game as defined above, each individual using some *pure* strategy whenever playing the game, and each individual having the same probability to be drawn to play the game. In this context, a point $x \in \Delta$ represents a *population state*, i.e., a vector of population shares $x^i \in [0,1]$ of individuals playing the different pure strategies $i \in I$. Accordingly, $u(x,x)$ is the *average payoff* in the population.

The most well-known evolutionary selection dynamics for this setting is the *replicator dynamics*. Modelled in continuous time it is defined by the following system of ordinary first-order differential equations:

$$\dot{x}_t^i = \left[u(e^i, x_t) - u(x_t, x_t)\right] x_t^i \qquad \text{(for all } i \in I \text{ and } t \in R). \tag{2}$$

Here the growth rate of the population share associated with a pure strategy i equals the difference between its own payoff $u(e^i, x_t)$ and the average payoff $u(x_t, x_t)$.[2]

Applied to an $RSP(\alpha)$ game with positive α, this dynamics is known to induce solution orbits all of which converge in the interior of the simplex to the unique Nash equilibrium strategy $\bar{x}$. For negative α, all interior solution orbits instead diverge toward the boundary of the simplex, while in the benchmark case $\alpha = 0$ all interior solutions form periodic orbits around $\bar{x}$.

Modelled in discrete time, the replicator dynamics takes the form of the system of first-order difference equations:

$$x_{t+1}^i = \frac{u(e^i, x_t)}{u(x_t, x_t)} x_t^i \qquad \text{(for all } i \in I \text{ and } t \in N). \tag{3}$$

Here the population share associated with a pure strategy i increases from one period to the next by a factor which equals the ratio between its own payoff $u(e^i, x_t)$ and the average payoff $u(x_t, x_t)$.[3] Applied to an $RSP(\alpha)$ game, this dynamics induces solution orbits all of which (except when starting at the Nash equilibrium strategy $\bar{x}$) diverge toward the boundary of the simplex, irrespective of the parameter value α (Weissing [13], see Theorem 1 below).

For references to the continuous and discrete replicator dynamics, and for discussions and analyses of these, see Hofbauer and Sigmund [8].

3. Evolutionary Selection in Discrete Time

In this section we develop an overlapping-generations replicator dynamics which subsumes the above discrete and continuous replicator dynamics as a special and limiting case, respectively. The present model is used to illustrate how three qualitatively different long run outcome patterns are possible in

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[2]Here and in the sequel all random variables are approximated by their mean values. A careful investigation of the appropriateness of this approximation is called for.

[3]Unlike in the continuous-time version, the discrete-time version presumes that the average payoff $u(x,x)$ is always positive.

Dekel's and Scotchmer's game, dependent on the overlap between generations. It turns out that a certain result by Weissing [13] for the discrete replicator dynamics in a generalized class of ROCK-SCISSORS-PAPER games is useful for our theoretical investigation. The results are illustrated by means of computer simulations.

3.1 An Overlapping-Generations Dynamics

One may derive the discrete replicator dynamics (3) as follows. Let the time unit be the lifetime of a generation, suppose pure strategies "breed true," i.e., that each offspring inherits its single parent's pure strategy. Let the payoff $u\left(e^i, x\right)$ to pure strategy i against the mixed strategy x represent the number of (surviving) offspring to an individual using strategy i in population state $x \in \Delta$. Let n_t denote the number of individuals in generation t, and let n_t^i be the number of these who use strategy i. Then

$$n_{t+1}^i = u\left(e^i, x_t\right) n_t^i \qquad (\text{for } t = 0, 1, 2, ..), \tag{4}$$

and hence, by linearity of u in its first argument,

$$n_{t+1} = \sum_{i \in I} n_{t+1}^i = u\left(x_t, x_t\right) n_t \qquad (\text{for } t = 0, 1, 2, ..). \tag{5}$$

Dividing (4) by (5) gives (3).

In this scenario, all individuals are simultaneously replaced by their offspring, at times $t = 1, 2, 3....$ Suppose instead that generations overlap so that replacements take place r times per time unit and only involve the fraction $\delta = 1/r$ of the total population. If each individual in the population has the same probability of being drawn each time, and $u\left(e^i, x\right)$ is the number of offspring to an individual using strategy i in population state x, then the resulting dynamics is

$$n_{t+\delta}^i = (1 - \delta) n_t^i + \delta u\left(e^i, x_t\right) n_t^i \qquad (\text{for } t = 0, \delta, 2\delta, ..). \tag{6}$$

Summing across all strategies $i \in I$ and dividing through (analogously with the derivation of (3)) one obtains the following *overlapping generations (OLG) dynamics*:

$$x_{t+\delta}^i = \frac{1 - \delta + \delta u\left(e^i, x_t\right)}{1 - \delta + \delta u\left(x_t, x_t\right)} x_t^i \qquad (\text{for } t = 0, \delta, 2\delta, ..). \tag{7}$$

As expected, the discrete replicator dynamics (3) appears as the special case $\delta = 1$.

Manipulating (7), and taking limits as $\delta \to 0$, one obtains the continuous replicator dynamics (2):

$$\begin{aligned} \dot{x}^i &= \lim_{\delta \to 0} \frac{x_{t+\delta}^i - x_t^i}{\delta} = \lim_{\delta \to 0} \frac{u(e^i, x_t) - u(x_t, x_t)}{1 - \delta + \delta u\left(x_t, x_t\right)} x_t^i \\ &= \left[u(e^i, x_t) - u(x_t, x_t)\right] x_t^i \end{aligned} \tag{8}$$

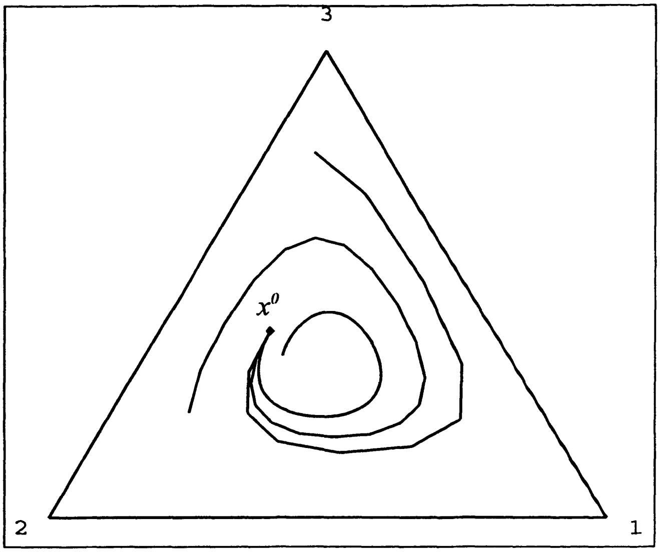

Figure 1: Divergence/convergence in $RSP(0.35)$ depending on the number of overlapping generations ($\delta = 0.05$, $\delta = 0.6$ and $\delta = 1$).

The OLG dynamics (7) thus indeed spans the whole range between the discrete and the continuous replicator dynamics. (For a comparison with the discrete-time dynamics in Cabrales and Sobel see section .

Figure 1 shows how the solutions to the OLG dynamics (7) may, depending on the share δ of simultaneously reproducing individuals, converge or diverge from the unique Nash equilibrium state $\bar{x}$ in a $RSP(\alpha)$ game with $\alpha > 0$. The smooth and convergent path in the diagram approximates the limiting case when $\delta \longrightarrow 0$ - for which we have already noted that the continuous replicator dynamics (2) converges from all interior initial states.

If we rewrite equation (7), for an arbitrary $r = \frac{1}{\delta}$, as

$$\frac{x_{t+\delta}^i}{x_t^i} = \frac{r - 1 + u\left(e^i, x_t\right)}{r - 1 + u\left(x_t, x_t\right)}, \tag{9}$$

then it is apparent that having $r > 1$ generations is equivalent with adding a positive constant $c = r - 1$ to each entry of the payoff-matrix A in the discrete-time dynamics (3). Note that c increases with r from zero to plus infinity as r increases from one to plus infinity. Letting $c \longrightarrow \infty$ is thus equivalent with letting $r \longrightarrow \infty$ (which in its turn is equivalent with letting $\delta \longrightarrow 0$), suggesting that the discrete replicator dynamics (7) approaches the continuous replicator dynamics (2) as c increases, an observation made by Hofbauer and Sigmund [8] concerning the discrete replicator dynamics (3).

A general property of OLG dynamics (7) is that a strategy's population share increases (decreases) over time whenever the strategy earns above (below) the population average: $x^i_{t+\delta} > x^i_t$ if $u(e^i, x_t) > u(x_t, x_t)$, and likewise for the reversed inequality.

3.2 Circulant Rock-Scissors-Paper Games

Weissing [13] calls a symmetric 3×3 game with payoff matrix

$$A = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \qquad (\text{for } c \leq a < b) \tag{10}$$

a *circulant* ROCK-SCISSORS-PAPER game. Any such game has a unique interior Nash equilibrium strategy $\bar{x}$ (Shapley [11], Weissing [13]). Observe that when $a = 1$, $b = 2 + \alpha$ and $c = 0$ the payoff matrix (10) corresponds to a $RSP(\alpha)$ game (for $\alpha > -1$), and then $\bar{x}$ is the mid-point $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Games with payoff matrix A as in (10) will here be termed $RSP(a, b, c)$ *games.*

Weissing [13] shows that under the discrete replicator dynamics (3), as applied to a $RSP(a, b, c)$ game, the unique interior Nash equilibrium strategy $\bar{x}$ is either a global attractor, a global repellor, or a global center (Weissing [13] Theorem 6.8). It turns out that part of his result and proof is important for the subsequent analysis. More exactly, we will use:

Theorem 1 (Weissing) *Let $\bar{x}$ be the unique Nash equilibrium strategy in a $RSP(a, b, c)$ game, for $c \leq a < b$. Then the following holds under the dynamics (3):*
(i) $\bar{x}$ is a global attractor if $a^2 < bc$.
(ii) $\bar{x}$ is a global repellor if $a^2 > bc$.

To prove this result Weissing [13] uses the function W defined on the interior of the unit simplex Δ by

$$W(x) = \frac{xAx}{x_1 x_2 x_3} . \tag{11}$$

He shows that $\bar{x}$ is the unique critical point of this function (which tends to plus infinity near the boundary of Δ), and, moreover, that along any solution to (3) starting from any other interior initial state x_0, $W(x_{t+1}) - W(x_t)$ has the same sign as $a^2 - bc$. Hence W is monotonically decreasing (increasing) over time if $a^2 < bc$ ($a^2 > bc$). Consequently, all interior solution trajectories to (3) converge to $\bar{x}$ if $a^2 < bc$ and diverge from $\bar{x}$ toward the boundary of the simplex if $a^2 > bc$.

Applied to an $RSP(\alpha)$ game, this result says that the discrete replicator dynamics diverges from the unique Nash equilibrium strategy $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ towards the boundary of the simplex, for *any* parameter value $\alpha > -1$. More generally, we noted above that having $r \geq 1$ overlapping generations in the

OLG dynamics (7) results in the same orbits in state space as when adding the constant $r-1$ to each payoff in the discrete-time dynamics (3). Consequently, the OLG dynamics with r overlapping generations in an $RSP(\alpha)$ game induces the same orbits as the discrete replicator dynamics in an $RSP(a,b,c)$ game with $a=r$, $b=1+\alpha+r$ and $c=r-1$. By Theorem 1, this tells us that the OLG orbits in a $RSP(\alpha)$ game converge (diverge) if $\alpha(r-1)>1$ ($\alpha(r-1)<1$). Hence, the orbits diverge for negative α, while for a positive α they converge if there is sufficient overlap, $r>\frac{1}{\alpha}+1$, and diverge if the overlap is small, $r<\frac{1}{\alpha}+1$.[4]

3.3 Rock-Scissors-Paper-Dumb Games

To show the possibility for strictly dominated strategies to survive the discrete replicator dynamics (3), Dekel and Scotchmer [5] added a fourth pure strategy, baptized "DUMB" by Cabrales and Sobel [4] to a particular $RSP(\alpha)$ game. The game they used corresponds to the special case $\alpha=0.35$ and $\beta=0.1$ of the following 4×4 payoff matrix:

$$A=\begin{pmatrix} 1 & 2+\alpha & 0 & \beta \\ 0 & 1 & 2+\alpha & \beta \\ 2+\alpha & 0 & 1 & \beta \\ 1+\beta & 1+\beta & 1+\beta & 0 \end{pmatrix} \qquad (\text{for } 0<3\beta<\alpha<4\beta). \tag{12}$$

We will refer to any such game as a $RSPD(\alpha,\beta)$ *game*. The unique Nash equilibrium of such a game is $(\bar{\bar{x}};\bar{\bar{x}})$, where $\bar{\bar{x}}=(\bar{x},0)=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3},0\right)$. The fourth pure strategy, DUMB, is strictly dominated by $\bar{\bar{x}}$ since $u\left(\bar{\bar{x}},e^i\right)=1+\frac{\alpha}{3}>1+\beta=u\left(e^4,e^i\right)$ for all $i<4$, and $u\left(\bar{\bar{x}},e^4\right)=\beta>0=u\left(e^4,e^4\right)$.

3.4 Irrational Survivors in OLG Dynamics: Theory

Recall that under any OLG dynamics (7) a strategy increases its population share wherever it earns a payoff above average. By straightforward manipulations the relevant inequality $u(x,x)<u\left(e^4,x\right)$ can be re-written as

$$\left(1-x^4\right)\left(\left(1-x^4\right)-\frac{4\beta}{\alpha}\left(\frac{1}{2}-x^4\right)\right)<\left(x^1\right)^2+\left(x^2\right)^2+\left(x^3\right)^2 \tag{13}$$

It is easy to see that this inequality holds along the three edges of the unit simplex where DUMB is extinct. For instance, if $x^3=x^4=0$, then the left-hand side equals $1-2\beta/\alpha$ while the right-hand side is not less than $1/2$ for any combination of x^1 and x^2. Since $\alpha<4\beta$ by assumption, (13) is met. The region where condition (13) is met is shown in Figure 2. This region consists of the points in the unit simplex which lie outside the egg-shaped set. Inside

[4] Expressed in terms of δ, the condition for convergence (divergence) is $\delta<\frac{\alpha}{1-\alpha}$ ($\delta>\frac{\alpha}{1-\alpha}$).

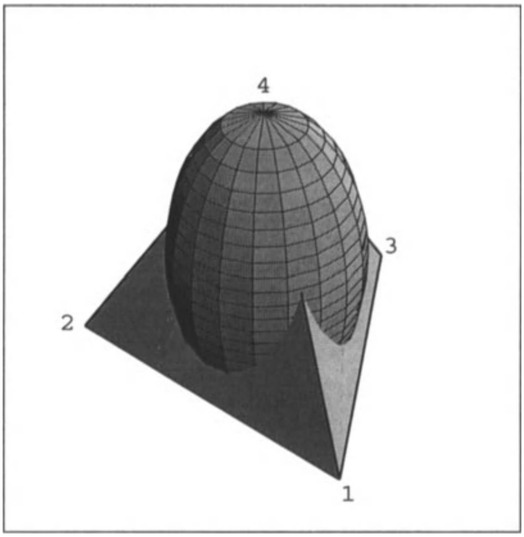

Figure 2: Region of simplex where u^4 is less than average

the egg-shaped set the reversed inequality holds. Thus DUMB decreases over time inside the egg and grows outside it. For a $RSPD(\alpha,\beta)$ game, in any population state $x \in \Delta$ with $x^4 < 1$, let $y = (y^1, y^2, y^3)$ denote the *relative* population shares of the non-DUMB strategies: $y^i = \frac{x^i}{1-x_4}$ for $i = 1,2,3$. Such a vector y is formally identical with a population state in the corresponding $RSP(\alpha)$ game (1). Let $v\left(e^i, y\right)$ denote the payoff to strategy i in population state y in the latter game. Inspection of the payoff matrix (12) shows that the payoffs in the $RSPD(\alpha,\beta)$ game in any population state x are related to their payoff in the associated population state y in the $RSP(\alpha)$ game as follows:

$$u\left(e^i, x\right) = \left(1 - x^4\right) v\left(e^i, y\right) + x^4\beta \tag{14}$$

for all strategies $i < 4$, and for strategy DUMB

$$u\left(e^4, x\right) = \left(1 - x^4\right)(1 + \beta) . \tag{15}$$

After some algebraic manipulations of these expressions one finds that the OLG dynamics (7), as applied to the full $RSDP(\alpha,\beta)$ game, induces the following dynamics for the vector y:

$$\frac{y^i_{t+\delta}}{y^i_t} = \frac{\gamma_t + v\left(e^i, y\right)}{\gamma_t + v\left(y, y\right)} \qquad \text{(for } i = 1,2,3 \text{ and } t = 0, \delta, 2\delta, ...) \tag{16}$$

where $\gamma_t = \frac{1-\delta+x^4_t\beta}{\delta\left(1-x^4_t\right)}$. Thus the dynamics of y is identical with the discrete replicator dynamics (3) in a circulant ROCK-SCISSORS-PAPER game (10) with

time-dependent parameters $a_t = 1 + \gamma_t, b_t = 2 + \alpha + \gamma_t$, and $c_t = \gamma_t$.

Recall that $\bar{\bar{x}} = (\bar{x}, 0) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right)$ is the unique Nash equilibrium strategy in any $RSPD(\alpha, \beta)$ game, and let L denote the line segment in Δ where all non-DUMB strategies appear in equal shares: $L = \left\{x \in \Delta : x^1 = x^2 = x^3\right\}$. Using Weissing's [13] technique for the discrete replicator dynamics (3) in circulant ROCK-SCISSORS-PAPER games, we obtain

Proposition 2 *Consider the OLG dynamics (7) applied to a $RSPD(\alpha, \beta)$ game (for $0 < 3\beta < \alpha < 4\beta$).*

(a) The solution through any interior initial state x_0 converges to $\bar{\bar{x}}$ if $\delta < \frac{\alpha}{1+\alpha}$.

(b) The solution through any interior initial state $x_0 \notin L$ diverges toward the boundary of Δ if $\delta > \frac{\alpha\beta+2\alpha}{1+2\alpha}$. In this case the population share x^4 does not converge to zero.

Proof. As mentioned in the connection of Theorem 1 above, Weissing [13] showed that the function W defined in (11) satisfies the condition that $W(x_{t+1}) - W(x_t)$ has the same sign as $a^2 - bc$ along any interior solution (starting in a state $x_0 \neq \bar{x}$) to the discrete replicator dynamics (3) in a $RSP(a, b, c)$ game. Generalized to such a game with time-dependent parameters, the condition becomes that $W(x_{t+1}) - W(x_t)$ has the same sign as $a_t^2 - b_t c_t$.

In the y-dynamics (16) we have $a_t^2 - b_t c_t = 1 - \alpha\gamma_t$, so in that dynamics $W(y_{t+1}) - W(y_t)$ has the same sign as $1 - \alpha\gamma_t$.

First suppose $\delta < \frac{\alpha}{1+\alpha}$. It is easy to show that then $\alpha\gamma_t > 1$ for all $x_t^4 \in (0, 1)$. Hence $W(y_t)$ is strictly decreasing along the interior solutions to the y-dynamics (16) (with initial state $y_0 \neq \bar{x}$), so $y_t \to \bar{x} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. In the full state space of the $RSPD(\alpha, \beta)$ game, this means that the population state x_t converges to the line segment L. But all states x near L lie inside the egg-shaped region where DUMB earns below average, so $x_t^4 \to 0$ and thus $x_t \to \bar{\bar{x}}$.

Second, suppose $\delta > \frac{\alpha\beta+2\alpha}{1+2\alpha}$. One can show that $x_{t+\delta}^4 < x_t^4$ whenever $x_t^4 > \frac{1}{2}$, and $x_{t+\delta}^4 \leq \frac{1}{2}$ if $x_t^4 \leq \frac{1}{2}$, along any interior solution to the OLG dynamics (7). Hence, we may without loss of generality presume $x_t^4 \leq \frac{1}{2}$. It is easy to show that $\alpha\gamma_t < 1$ for all such x_t^4. Hence (for t sufficiently large) $W(y_t)$ is strictly increasing along any interior solution to the y-dynamics (16) (with initial state $y_0 \neq \bar{x}$), so y_t diverges towards the boundary of the unit simplex in y-space. In the full state space of the $RSPD(\alpha, \beta)$ game this means that the population state x_t diverges toward the boundary $D = \left\{x \in \Delta : x^i = 0 \text{ for some } i < 4\right\}$, from any interior initial state $x_0 \notin L$. All states $x \in \Delta$ near the edge $D \cap \left\{x \in \Delta : x^4 = 0\right\}$ lie outside the egg-shaped region where x_t^4 earns below average, so x_t^4 increases over time near D. In particular, x_t^4 does not converge to zero along any interior solution trajectory starting at a state $x_0 \notin L$. ■

The proof of the second claim in this proposition establishes that for suf-

ficiently little overlap in the OLG dynamics, its solution trajectories diverge toward the boundary of the simplex where at least one of the non-DUMB strategies is extinct. The overlap condition is payoff dependent, $\delta > \frac{\alpha\beta+2\alpha}{1+2\alpha}$, where the right-hand side expression is approximately 0.43 in the game studied in Dekel and Scotchmer [5]. They considered the discrete replicator dynamics (3), so $\delta = 1$. Hence, the dynamic path will sooner or later reach outside the egg-shaped set, and DUMB then increases.

In fact, as the population state approaches this boundary its movement slows down and it spends longer and longer time intervals close to the edges of the simplex where two of the three non-DUMB strategies are close to extinction. If one such strategy, say PAPER, would be completely extinct, ROCK performs better than SCISSORS, and SCISSORS will converge to zero. With only DUMB and ROCK left it is easy to see that the dynamics converges towards $(1/2, 0, 0, 1/2)$. One may thus conjecture that for all δ sufficiently large the set $\Upsilon = \left\{\left(\frac{1}{2}, 0, 0, \frac{1}{2}\right), \left(0, \frac{1}{2}, 0, \frac{1}{2}\right), \left(0, 0, \frac{1}{2}, \frac{1}{2}\right)\right\}$ of "subsimplex Nash equilibrium" points constitutes an attractor in the statistical sense of the time spent close to points in Υ.[5]

3.5 Irrational Survivors in OLG Dynamics: Computer Simulations

We have used the OLG dynamics (7) for numerical computer simulations of $RSPD(\alpha, \beta)$ games. The simulations support the above results and provide some insights into the nature of the dynamics for intermediate degrees of overlap, i.e., for the range of δ-values for which Proposition 1 neither ensures convergence nor divergence. In most of the simulations reported here we have used the parameter values of Dekel and Scotchmer [5]: $\alpha = 0.35$ and $\beta = 0.1$. Below we refer to these as the *DS-payoffs*.

The Convergent Case: As expected from Proposition 1, the population state converges to $\overline{\overline{x}} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right)$, the unique Nash equilibrium strategy, for δ sufficiently small, more exactly for $\delta < \frac{\alpha}{1+\alpha}$. With DS-payoffs, $\frac{\alpha}{1+\alpha} \approx 0.26$. Our simulations indicate convergence for all δ below some critical value somewhat below 0.28, see Figure 3.

The Divergent Case: As expected from Proposition 1, the population state diverges toward the boundary where at least one non-DUMB strategy is extinct for δ sufficiently large, more exactly for $\delta > \frac{\alpha\beta+2\alpha}{1+2\alpha}$. With DS-payoffs, $\frac{\alpha\beta+2\alpha}{1+2\alpha} \approx 0.43$. The computer simulations exhibit such divergence for all δ exceeding approximately 0.40, see Figure 4. As indicated above, the set Υ of "subsimplex Nash equilibrium points" seems to be a global attractor in terms of sojourn times.

Limit Cycles: With DS-payoffs and values of δ between 0.28 and 0.4 our computer simulations indicate the existence of a limit cycle, see Figure 5. Note that initial conditions are irrelevant for the limiting outcome also in

[5] In the topological sense, the attractor is larger: it will also contain some connecting curves between these three subsimplex Nash equilibria.

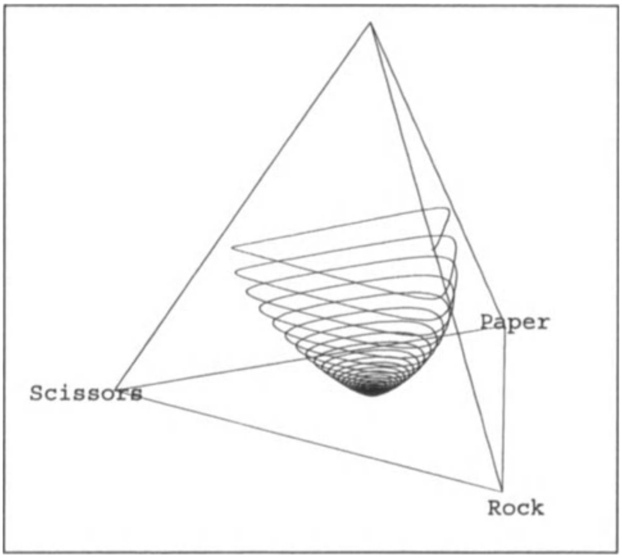

Figure 3: Simulated replicator evolution in the RSPD game: Convergence with $\delta = 0.2$, initial state $(0.3, 0.01, 0.01, 0.49)$.

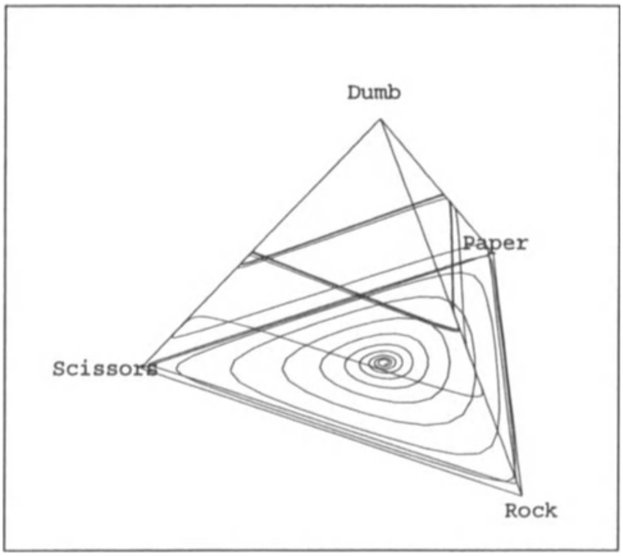

Figure 4: Simulated RSPD replicator evolution: Divergent case with $\delta = 0.5$ and initial state $(0.34, 0.33, 0.32, 0.01)$.

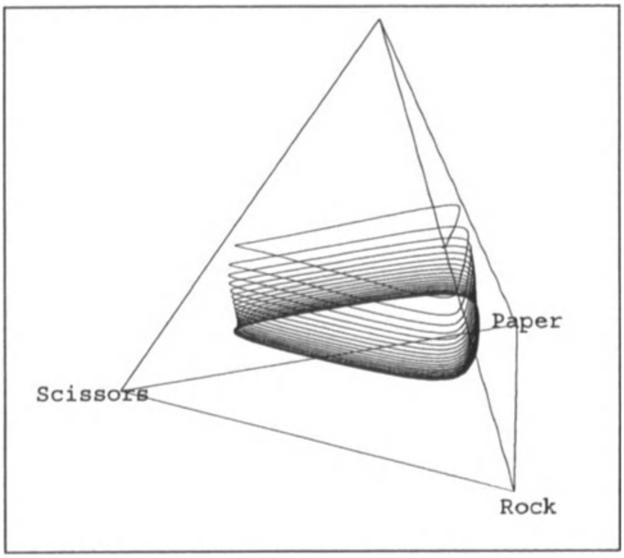

Figure 5: Simulated replicator evolution in the RSPD game with limit cycle when $\delta = 0.3$ and initial state (0.49, 0.01, 0.01, 0.49).

this intermediate case. To check the numerical robustness of the limit cycle performance we have run one thousand simulations with randomly generated, uniformly distributed, initial population states for a variety of parameter values. Figure 6 shows the population state after 2000 iterations, each point corresponding to a different initial state, and all runs were made with the DS-payoffs and the overlap $\delta = 0.3$.

In terms of irrational survivors, our computer simulation results suggest that the strictly dominated strategy 4 in $RSPD(\alpha, \beta)$ games may survive in the long run for a wide range of initial conditions and in a sizable population share (about 0.5). This survival in fact occurs for a wider range of payoff values (α,β) and overlaps (δ) than the theoretical prediction in Proposition1: The dominated strategy appears to survive also in the intermediate range of δ-values.

3.6 The Cabrales-Sobel Dynamics

The following discrete-time dynamics was suggested in Cabrales and Sobel [4] (op.cit. p.414):

$$x_{t+\delta}^i = (1-\delta)\, x_t^i + \delta \frac{u\left(e^i, x_t\right)}{u\left(x_t, x_t\right)} x_t^i \qquad (\forall i \in I \text{ and } t = 0, \delta, 2\delta, ..). \qquad (17)$$

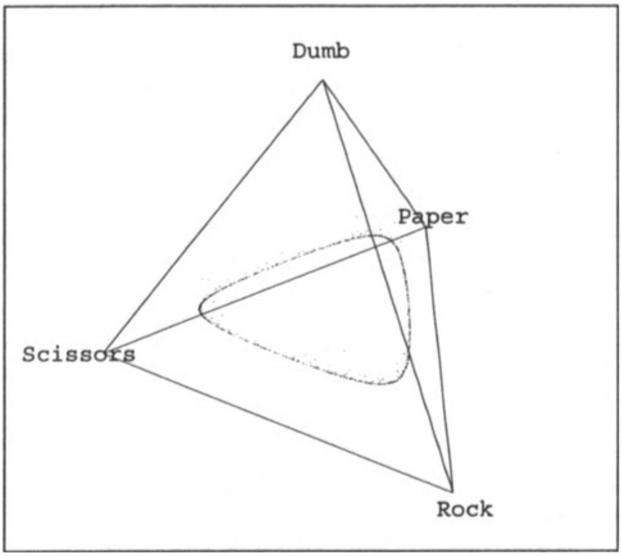

Figure 6: RSPD population states after 2000 iterations with $\delta = 0.3$.

One can give an interpretation of this dynamics in terms of boundedly rational dynamic adaptation. For this purpose, consider a population consisting of n_t individuals, n_t^i of whom use strategy i at time t. The payoff earned by strategy i in this population then is $u\left(e^i, x_t\right) n_t^i$, and the strategy's share of the total payoff earned in the population is (with the time index dropped)

$$\frac{u\left(e^i, x\right) n^i}{\sum_{j \in I} u\left(e^j, x\right) n^j} = \frac{u\left(e^i, x\right) x^i}{u\left(x, x\right)} . \tag{18}$$

Assume that at each time $t = 0, \delta, 2\delta, ...$ the share δ of individuals review their strategy choice in the light of current payoffs. If reviewing individuals switch to each strategy in proportion to its share of the total payoff earned, one obtains:

$$n_{t+\delta}^i = \left(1 - \delta\right) n_t^i + \frac{u\left(e^i, x_t\right) x_t^i}{u\left(x_t, x_t\right)} \delta n_t \qquad (\forall i \in I \text{ and } t = 0, \delta, 2\delta, ..). \tag{19}$$

Dividing by the size of the total population n_t one obtains (17).

Just as in the OLG model, if δ equals unity, (19) reduces to the discrete-time dynamics (3). However, taking the limit as $\delta \longrightarrow 0$ gives

$$\dot{x}_t^i = \frac{u(e^i, x_t) - u(x_t, x_t)}{u\left(x_t, x_t\right)} x_t^i , \tag{20}$$

a system of differential equations that is *not* the same as in the continuous replicator dynamics (2). Nevertheless, the orbits of (20) and (2) coincide since the denominator in (20) only influences the speed at which the solutions to (20) travel along these orbits. (In multi-population dynamics this makes a difference also in terms of orbits.)

4. Evolutionary Selection in Continuous Time

4.1 Classes of Selection Dynamics

All continuous-time selection dynamics to be studied here can be written as a system of differential equations in the form

$$\dot{x}_t^i = \varphi^i(x_t)\, x_t^i, \tag{21}$$

where φ^i, for each pure strategy i, is a Lipschitz continuous "growth rate" function such that the *orthogonality condition* $\sum_{i \in I} x^i \varphi^i(x) = 0$ is met in all states x. The system (21) then has a unique solution through any initial population state and this solution stays forever in the simplex. The most well-known evolutionary selection dynamics in this form is the continuous replicator dynamics (2), where each growth rate $\varphi^i(x)$ equals the strategy's "excess" payoff $u(e^i, x) - u(x, x)$.

Following Weibull [12] we call a continuous-time dynamics (21) *payoff monotone* (*relative monotone* in Nachbar [9] and *order compatible predynamics* in Friedman [6]) if

$$\varphi^i(x) > \varphi^j(x) \iff u(e^i, x) > u(e^j, x). \tag{22}$$

Payoff monotonicity hence requires that the population share associated with a pure strategy with a higher payoff increases at a higher rate than the population share associated with a pure strategy with a lower payoff. Clearly the replicator dynamics (2) is payoff monotone in this sense.

Likewise, a dynamics (21) will be called *weakly payoff monotone* (*weakly compatible* in Friedman [6]) if it moves in the general direction of increasing payoffs. More exactly, the requirement is that, unless all non-extinct pure strategies earn the same payoff,

$$\sum_{i \in I} \varphi^i(x)\, x^i u(e^i, x) > 0, \tag{23}$$

and, if all non-extinct pure strategies do earn the same payoff their growth rates are zero.

Let *PM* and *WPM* refer to these two classes of evolutionary selection dynamics. They are related as follows: $PM \subset WPM$. To see this, consider any payoff monotone dynamics, and first note that if all non-extinct strategies (those with $x^i > 0$) earn the same payoff, they must all have the same growth rate by monotonicity, and, by orthogonality, this has to be zero. Secondly,

suppose that the payoffs of non-extinct pure strategies are not all equal. Note that the left hand side of (23) is unchanged if some constant c is added to all payoffs in the game (using orthogonality). So is the payoff ranking between pure strategies. Hence, without loss of generality we may assume that each payoff $u\left(e^i, x\right)$ has the same sign as the corresponding growth rate $\varphi^i(x)$. Then all terms on the left hand side of (23) are non-negative and at least one is positive, hence their sum is positive.

Samuelson and Zhang [10] show that if a pure strategy is strictly dominated by another *pure* strategy then the population share of the dominated strategy will go to zero along any interior solution trajectory to any *payoff monotone* dynamics. One may thus wonder: (i) Is extinction of such strategies guaranteed also under weakly payoff monotone dynamics? (ii) Is extinction of pure strategies that are strictly dominated by some *mixed* strategy guaranteed in any of these two classes of evolutionary selection dynamics? By way of counter-examples we will here show that the answer to both questions is negative.

4.2 Irrational Survivors in Weakly Payoff Monotone Dynamics

Proposition 3 *A pure strategy that is strictly dominated by a pure strategy may survive under weakly payoff monotone dynamics.*

In order to prove this claim, consider the symmetric 4×4 game given by the payoff matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 1 & 0 \\ -\beta & -\beta & -\beta & -\beta \end{pmatrix} \quad \text{(for } \beta > 0\text{)} \qquad (24)$$

The first three pure strategies together constitute the bench-mark $RSP(0)$ game. Recall that the solutions to the continuous replicator dynamics (2) in this game constitute closed orbits around the unique Nash equilibrium point (see Section 2). The fourth pure strategy, DUMB, is strictly dominated by all the other three pure strategies.

Consider first the dynamics

$$\dot{x}^i = \begin{cases} \varphi_R^i(x)\, x^i & \text{for } i < 4 \\ 0 & \text{for } i = 4 \end{cases} \qquad (25)$$

where φ_R is the replicator growth-rate function of the $RSP(0)$ "subgame," formally defined by $\varphi_R^i(x) = e^i Bx - x'Bx$, where B is as in (24) with $\beta = 0$. Since the replicator dynamics is weakly payoff monotone (indeed payoff monotone), so is the dynamics (25), except at the line segment L where all three strategies $i < 4$ appear in equal shares. All population states on this line segment are stationary in (25) while weak payoff monotonicity requires

a negative growth rate for strategy DUMB (whenever $x^4 > 0$). Other solution trajectories only rotate in closed orbits around L, so the population share of DUMB remains constant over time along all solution trajectories.

In order to establish the claim in Proposition 2 it suffices to modify the growth rates in (25) in a small region containing L so that the dynamics becomes weakly payoff monotone. For then all initial states sufficiently far outside this small region will have closed orbit solutions lying outside the region, with x^4 constant, forever.

One way to obtain this is to let strategy 4 decline in a cylinder of arbitrary radius $\delta > 0$ containing L. For this purpose, let $d(x)$ be the Euclidean distance from any population state $x \in \Delta$ to L, and define

$$\tilde{\varphi}^i(x) = \begin{cases} \varphi_R^i(x) + x^4 \max\{0, \delta - d(x)\} & \text{for } i < 4 \\ -(1 - x^4) \max\{0, \delta - d(x)\} & \text{for } i = 4 \end{cases} \tag{26}$$

The resulting dynamics (21) is Lipschitz continuous and meets the orthogonality condition. At all population states at distance δ or more from L, $\tilde{\varphi}$ coincides with φ, while at closer population states $\tilde{\varphi}$ adds a slight drift upwards in all strategies $i < 4$ and a slight drift downwards in strategy 4. To see that the resulting dynamics is weakly payoff monotone, just note that

$$\sum_{i \in I} \tilde{\varphi}^i(x)\, x^i u(e^i, x) =$$

$$= \sum_{i=1}^{3} \varphi_R^i(x)\, x^i u(e^i, x) + (1 + \beta)(1 - x^4)x^4 \max\{0, \delta - d(x)\}, \tag{27}$$

where, in the δ-cylinder, the first term is non-negative, actually positive when $x \notin L$, by weak payoff monotonicity of the replicator dynamics, and the second term is positive except where $x^4 = 0$ and $x^4 = 1$.

4.3 Irrational Survivors in Payoff Monotone Dynamics

We finally establish that a pure strategy that is strictly dominated by a mixed strategy may survive a payoff monotone dynamics, again for a large region of initial states. This result is due to Björnerstedt [3], and we here only sketch his proof.

Proposition 4 *A pure strategy that is strictly dominated by a mixed strategy may survive under payoff monotone dynamics.*

In order to prove this claim, we re-consider a $RSPD(\alpha, \beta)$ game and construct a particular payoff monotone dynamics that comes with a story. First imagine a large but finite population of individuals spending their time playing this game, each individual using some pure strategy all the time. However, every now and then an individual randomly reviews his strategy choice,

with equal probability for each individual. If the reviewing individual finds that his strategy is among the currently worst performing, i.e., earns payoff $\min_j u(e^j, x)$, then the individual imitates "the first man in the street," i.e., selects a pure strategy in proportion to the current population shares. Now add some noise to the individual's observation of the other pure strategies.

Formally, suppose that the observed payoff to strategy j equals its expected payoff $u\left(e^j, x\right)$ plus some noise term ε_j. Assume that all noise terms are i.i.d. with c.d.f. F. Thus the probability that the individual's own pure state, i, is perceived as worse performing than j is $F\left[u\left(e^j, x\right) - u\left(e^i, x\right)\right]$. The probability $P_i(x)$ that i is perceived to be among the very worst is accordingly

$$P_i(x) = \prod_{j \neq i} F\left[u\left(e^j, x\right) - u\left(e^i, x\right)\right] \ . \tag{28}$$

In terms of expected values, the dynamics that arises when the review rate is constant over time is (up to a constant rescaling of time)

$$\dot{x}^i = \sum_{j \neq i} x^i P_j(x) x^j - \sum_{j \neq i} x^j P_i(x) x^i =$$

$$= \left(\sum_{j \in I} \left[P_j(x) - P_i(x)\right] x^j\right) x^i = \left[\sum_{j \in I} P_j(x) x^j - P_i(x)\right] x^i \tag{29}$$

If F has everywhere positive density, this dynamics is payoff monotone: The "inflow" sum $\sum_j P_j(x) x^j$ is the same for all strategies i and the "outflow" term, $P_i(x)$, is by (28) a decreasing function of $u(e^i, x)$. There is a smaller chance that a better performing strategy is weeded out by mistake than that a worse performing strategy is weeded out by mistake, and hence the growth rate of the former will exceed that of the latter.

We can study the qualitative properties of this dynamics for small observational errors by reducing the variance of the distribution F toward zero. In the limit we have that $P_i(x) = 1$ if i is among the worst-performing strategies and otherwise $P_i(x) = 0$.[6] In a $RSPD(\alpha, \beta)$ game there are four geometrically well-shaped regions within each of which exactly one of the four pure strategies is worst, see Figure 7.

For instance, near the vertex of pure strategy 1 (ROCK) pure strategy 2 (SCISSORS) is worst. Accordingly, in the absence of observational errors only SCISSORS is decreasing:

$$\frac{\dot{x}^2}{x^2} = x^2 - 1 \qquad \text{and} \qquad \frac{\dot{x}^i}{x^i} = x^2 \quad \text{for all } i \neq 2, \tag{30}$$

and likewise for the vertices $i = 2, 3$. Not surprisingly, pure strategy 4 (DUMB) is worst when the other three strategies appear in approximately their Nash

[6] The resulting limiting vector field in (29) is thus discontinuous. However, the discontinuities only appear on certain hyperplanes, see Figure 8.

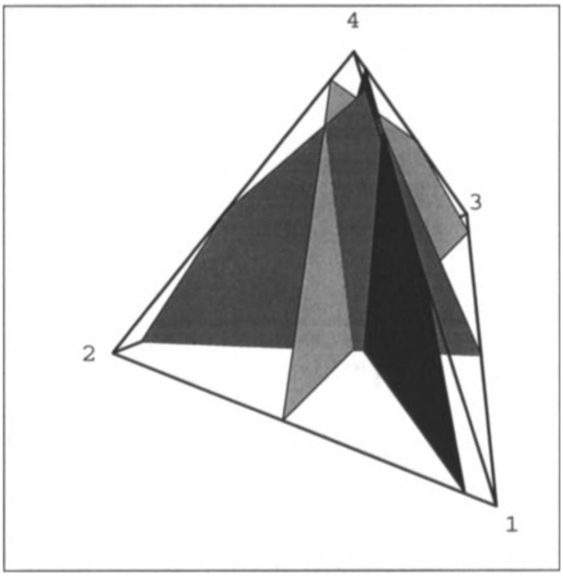

Figure 7: Region of simplex where e^4 is the worst strategy.

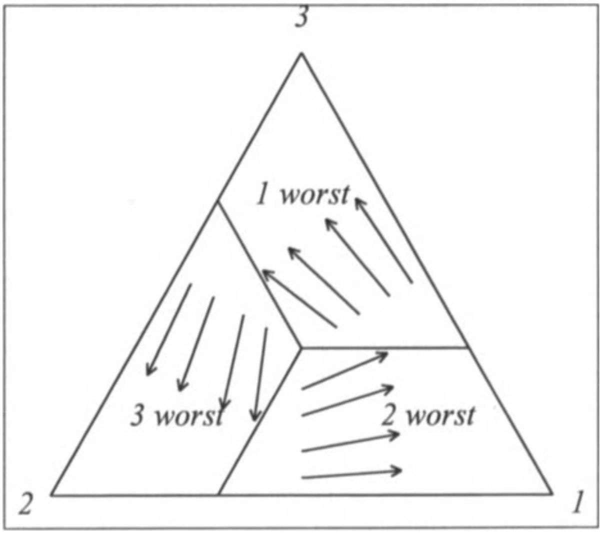

Figure 8: Direction of movement in (29) without observational error.

equilibrium shares $(\frac{1}{3})$. This region is the indicated "conical cylinder" C around the line segment L where all three strategies $i < 4$ appear in equal shares (see Figure 7).

In the absence of observational errors, the solutions to (29) are well-defined straight line segments inside each of these four regions. Inside the set C the movement makes a ray from the vertex where $x^4 = 1$ toward the boundary face where $x^4 = 0$. Having left the set C, the solution orbit will not enter again, as the vector field points away from C. Outside the set C the movement is directed linearly out toward one of the three boundary faces where each of the other pure strategies $i < 4$ is extinct, see Figure 8 which describes the dynamics outside the set C on the boundary face where DUMB is extinct.

It is not difficult to find a cylinder D (parallel to the x^4-axis in R^4) containing the above mentioned set C (in its relative interior) such that the vector field of the dynamics (29) in the limiting case with no observational errors points outward on the surface of D. Then DUMB increases outside D. By continuity and compactness the set D has these two properties also for sufficiently small observational errors. Hence, for initial states outside D and for sufficiently small observational errors, the population state will remain forever in D and DUMB will increase forever. Computer simulations suggest that all interior solutions outside D swirl around forever, spending longer and longer times near the set Υ of "subsimplex Nash equilibrium" points, cf. Section and see Figure 4.[7]

5. Concluding Remarks

We have studied the possibility that "irrational behavior," in the sense of strictly dominated pure strategies, may survive in the long run under evolutionary selection dynamics. For this purpose we constructed an overlapping generations replicator model in discrete time and showed how a strictly dominated strategy necessarily dies out when the degree of intergenerational overlap is large but survives when this overlap is small. We then investigated whether strictly dominated strategies can survive in continuous-time evolutionary dynamics. We showed that strictly dominated strategies that are not strictly dominated by any pure strategy may survive in payoff monotone evolutionary dynamics, and that even a pure strategy that is strictly dominated by all other *pure* strategies in a game may survive in weakly payoff monotone evolutionary dynamics.

[7] We know, however, that the solutions do not converge to any one of the points in Υ, since the limit point to an interior solution under any payoff monotone dynamics is necessarily a Nash equilibrium strategy (Nachbar [9]).

Since elimination of strictly dominated strategies is a relatively weak rationality assumption underlying much of non-cooperative game theory, these findings appear potentially disturbing for the attempts that are currently being made to provide evolutionary foundations for non-cooperative game theory.

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Strict and Symmetric Correlated Equilibria are the Distributions of the ESS's of Biological Conflicts with Asymmetric Roles*

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Abstract. We investigate the ESS's of payoff-irrelevant asymmetric animal conflicts in Selten's (1980) model. We show that these are determined by the symmetric and strict correlated equilibria of the underlying (symmetric) two-person game. More precisely, the set of distributions (on the strategy space) of ESS's coincides with the set of strict and symmetric correlated equilibria (described as distributions). Our result enables us to predict all possible stable payoffs in payoff-irrelevant asymmetric animal conflicts using Aumann's correlated equilibria. It also enables us to interpret correlated euqilibria as solutions to biological conflicts: Nature supplies the correlation device as a phenotypic conditional behavior.

1. Introduction

Evolutionary game-theory studies conflict situations in which individuals adopt actions (strategies) in order to maximize their Darwinian fitness (Maynard-Smith, 1982; Reichert and Hammerstein, 1983). In contrast to game theory which assumes rationality in human behavior, evolutionary ecology bases its "rationality" on the natural selection mechanism of the Darwinian process: Nature selects genotypes (which represent morphological and/or behavioral characters) that give higher fitness (Parker and Hammerstein, 1985; Hammerstein and Selten, 1991). Thus, in evolutionary ecology we assume that animals' and plants' traits are 'winning strategies' that have been selected through evolution because of their higher fitness.

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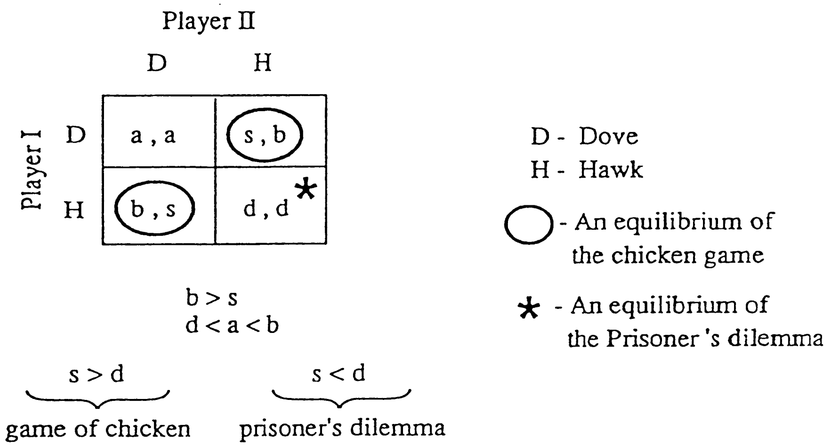


Figure 1. The "Hawk-Dove" game: An example of a symmetric 2 × 2 bimatrix game. Each player has two pure strategies: "Hawk" and "Dove". If $s > d$ then we obtain the "game of chicken". It has two asymmetric equilibria in pure strategies. (It also has a symmetric mixed equilibrium). When $d > s$ we have the "prisoner's dilemma". It has exactly one equilibrium which is symmetric and pure.

On this background Maynard-Smith established the concept of *Evolutionarily Stable Strategy (ESS)*. This is a strategy such that if all the members of a population adopt it, then no mutant strategy can invade (Maynard-Smith, 1982, p.204). It can be proved that an ESS is a Nash equilibrium which satisfies an additional stability condition (see, e.g., Van Damme, 1987).

Only *symmetrical equilibria* in *symmetric games* are relevant to biological conflicts of animals and plants (Reichert and Hammerstein, 1983; Parker and Hammerstein, 1985; Hammerstein and Selten, 1991). Fig. 2.a. represents an example of the game of chicken with Hawk and Dove strategies[1]. This game has three Nash equilibrium points. Two equilibrium points with pure strategies exist: $[(H, D); (D, H)]$. (If player one chooses strategy H and player two chooses strategy D, then we have the equilibrium (H, D). (D, H) is interpreted similarly). These two equilibria are not symmetric because the players use different strategies. The third equilibrium point which is mixed $[p\,(D) + (1 - p)(H)$ where $p = 1/2]$ is symmetric, and it is chosen by the two players.

Contests between animals within the same species, in which one opponent behaves aggressively ("Hawk" strategy) and the other opponent behaves peacefully ("Dove" strategy) have attracted ethologists for many years: The animal which chooses "Dove" behaves as if it was an altruist.

Maynard-Smith solved this debate in a game theoretical approach by proving that playing "Dove" against "Hawk"—is an equilibrium. That is, "Dove" is the best reply to "Hawk". The problem is that the two pure equilibria are not symmetrical and therefore cannot be realized without further

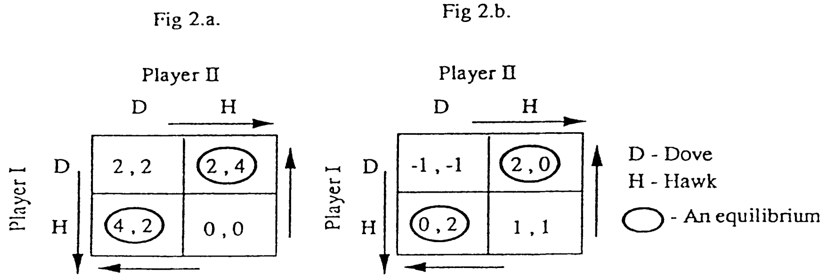


Figure 2. Two examples of the "game of chicken".
Figure 2.a. The two asymmetric pure equilibria cannot be realized in any biological contest (based on this game). In a bimorphic population with two distinct phenotypes evolution may lead to an ESS in behavior strategies which yields the payoff $(3,3)$ $(= 1/2(4,2) + 1/2(2,4))$. This is the largest symmetric payoff; hence, it is the maximum payoff in any (biolgoical) contest based on this game. Also notice that $(3,3)$ is the payoff of the (symmetric and strict) correlated equilibrium $1/2(H,D) + 1/2(D,H)$.
Figure 2.b. The original example of Maynard-Smith of the "Hawk-Dove" game (1982, p. 17).
Each arrow points in the direction of the strategy that yields a higher payoff against a given opposing strategy.

structure which was introduced by Selten (1980). Maynard-Smith (1982, p. 17,18,22) suggests at least three different biological situations which make possible equilibrium behavior in the "Hawk-Dove game":

a) A normal form game as in Fig. 2.a. which has only a mixed symmetric equilibrium ($p = 1/2$). Some ecologists do not consider the mixed strategy as a completely specified biological situation (Selten per comm.).
b) A game with an additional strategy named "Retaliator" which is executed as follows: play "Dove" if your opponent plays "Hawk" and "Hawk" if your opponent plays "Dove". In such a situation the retaliation is an ESS. This situation is not consistent with the basic assumption of game theory that strategies are chosen simultaneously. From the biological setup, the retaliator strategy is unacceptable because player one can play retaliator only if he plays after player two; similarly for player two. Therefore, the two players cannot both be instructed to play retaliator.
c) A game with an additional strategy—"Bourgeois" which says: If you are an "owner" play "Hawk" and if you are an "intruder" play "Dove". In such situations the bourgeois strategy is an ESS.

Situation c) can be developed to a semi-extensive[2] game that corresponds to a *correlated equilibrium distribution* (Aumann 1974, 1987)—this is the goal of this study. Parker (1982, 1984) studied further situation c) and coined the term "phenotype-limited ESS". Unlike others (see, for example, Dawkins' conditional behavior (1976, p.74)) he correctly explained the equilibrium sit-

uation of the "conditional pure strategies" and suggested that it be called "Conditional ESS".

Selten (1980) built a game model for situation c) and showed that for *asymmetric animal conflicts*[3] with asymmetric information, all the ESS's are pure. He allows only contests between animals with non-identical roles, which he obtains by choosing appropriately the probabilities of the possible encounters. Given a biological game by a fitness matrix, our theorem completely specifies all the payoffs that are achievable by some uncorrelated asymmetry of nature.

The biological bases of understanding "Hawk-Dove encounters" in nature were presented by the fundamental work of Maynard-Smith and Parker (1976). They emphasized that (p. 171) "in asymmetric contests some asymmetric features will be taken as a cue by which the contest will be settled conventionally". They also emphasize the importance of "*roles*" (e.g.—owner vs. intruder; juvenile vs. adult; different phenotyphic individuals of the same genotype) and the fact that they may be uncorrelated[4] with the outcome of the game (*payoff irrelevant*—Hammerstein, 1981). Their models already contain analysis of contests with payoff irrelevant asymmetry and incomplete information. A basically similar framework is the model of Hammerstein (1981) who deals only with the complete information situation.

Selten (1980) observed, in particular, that the foregoing intraspecific biological conflict cannot be solved in pure strategies (in the normal form game), because in equilibrium the two opponents should have the same strategies (and in a "game of chicken" each of the two pure equilibria is asymmetrical (see Fig. 2)). He built a semi-extensive model with incomplete information and "Role asymmetry"[5]. Selten's model allows only encounters between non-identical roles. We will show that this requirement can be relaxed and pure Nash equilibrium in the semi-extensive game (which corresponds to Aumann's correlated equilibrium) can be achieved also when we allow identical roles to meet. We will also show that the pure equilibrium strategies might be derived, at least formally, from Aumann's correlated equilibrium when it is strict and symmetric.

The reader is advised to look into figures 3,4,6, and 7 for the graphical demonstration of the foregoing arguments.

A concrete example with which the reader may be familiar is the following one: In intraspecific biological conflicts with a payoff matrix of a "game of chicken" (Fig. 1) the pure equilibria cannot be realized if the biological situation cannot be considered as a semi-extensive game with "role asymmetry". Aumann's lottery device of correlated equilibria is realized in nature by evolution of phenotypic roles which reflect both role asymmetry and incomplete information. In such situations not only pure equilibria which are totally correlated[6] can be achieved (Fig. 3), but it is also possible to obtain partially correlated equilibria[6] which yield very large payoffs for both opponents (see

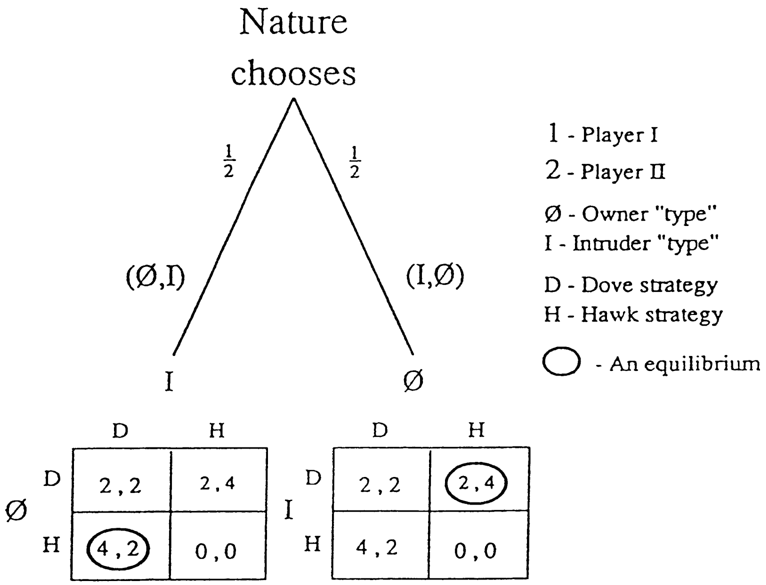


Figure 3 and **Figure 4**. An ESS in behavior strategies which yields the payoff of a (symmetric and strict) totally correlated equilibrium.
Figure 3. A representation of an asymmetric conflict (bimorphic population with owner and intruder phenotypes), in a semi-extensive form. Each of the two players may be of any one of these two phenotypes (with equal probability). The phenotype "owner" always plays H and "intruder" always plays D. $(\emptyset, I)$ means that nature has chosen Player I as owner and Player II as intruder. $(I, \emptyset)$ is interpreted in a similar way. The lottery (of nature) $1/2(\emptyset, I) + 1/2(I, \emptyset)$ is identical to Selten's basic probability distribution. The possible roles of animals (i.e., owner or intruder) are chosen randomly with equal probability.

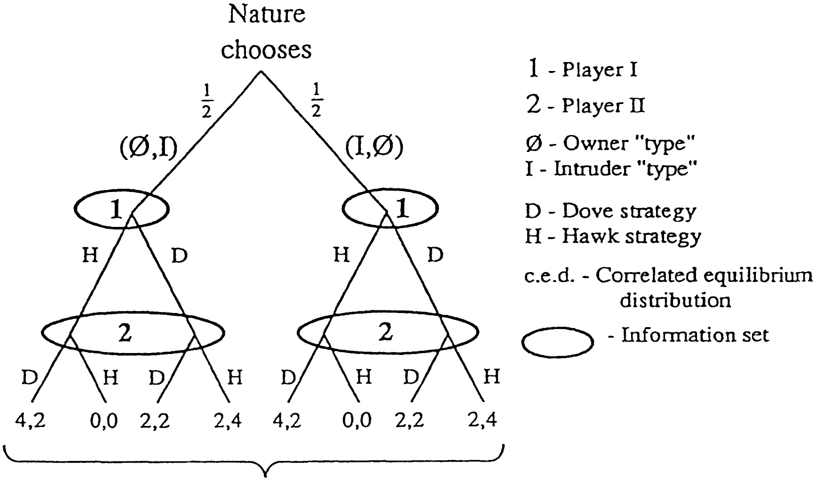
Fig. 6). Such efficient equilibria, in situations of incomplete information, have not been suggested before in evolutionary ecology.

2. Correlated Equilibria

Let A be an $m \times m$ fitness matrix, let $S^1 = S^2 = \{1, \ldots, m\}$, and let $S = S^1 \times S^2$. Thus, the payoff functions of the game specified by A are given by:

$$h(j,k) = (h^1(j,k), h^2(j,k)) = (a_{jk}, a_{kj}), \text{ all } (j,k) \in S.$$

A *correlated strategy pair* (c.s.p.) is a function f from a finite probability space Γ into S (see Aumann, 1987). Let f be a c.s.p. . The two components of f are denoted by f^1 and f^2. Thus, $f = (f^1, f^2)$. If $i \in \{1,2\}$ then we denote $-i = \{1,2\} \setminus \{i\}$. Also, throughout the sequel, E denotes "expectation".



Player 2

Player 1	D	H
D	2 , 2	2 , 4
H	4 , 2	0 , 0

c.e.d.

0	$\frac{1}{2}$
$\frac{1}{2}$	0

Figure 4. A representation of an asymmetric conflict by an extensive game. The c.e.d. matrix is the distribution of the correlated equilibrium $1/2(H, D)+1/2(D, H)$. An information set of a player is a set of decision points with the following properties: a) In all decision points, the player has the same information. (According to his information, the player cannot distinguish between the different points in the set.) b) In all decision points, the player has the same number of alternatives. Figures 3 and 4 represent the same (asymmetric) conflict which is payoff-irrelevant.

DEFINITION 2.1. A c.s.p. f is a *correlated equilibrium* (c.e.) if for every $i \in \{1, 2\}$ and every $g^i : S^i \to S^i$

$$Eh^i(f) \geq Eh^i(f^{-i}, g^i \circ f^i)$$

This definition is a simple expression of "incentive compatibility" (Bayesian rationality sensu Aumann, 1987); if 'nature' tells a player to play a certain strategy, then it is to the benefit of the player to follow nature's instruction.

DEFINITION 2.2. A c.e. f is *strict* if for every $i \in \{1, 2\}$ and for every $g^i : S^i \to S^i$ such that $P_r\{g^i \circ f^i \neq f^i\} > 0$

$$Eh^i(f) > Eh^i(f^{-i}, g^i \circ f^i)$$

Let f be a c.s.p. and let $(j, k) \in S$. We denote $q_{jk} = Pr\{f^{-1}(j, k)\}$. Thus, (q_{jk}) is the *distribution* of f. If f is a c.e. then its distribution is called a c.e. *distribution* (c.e.d.).

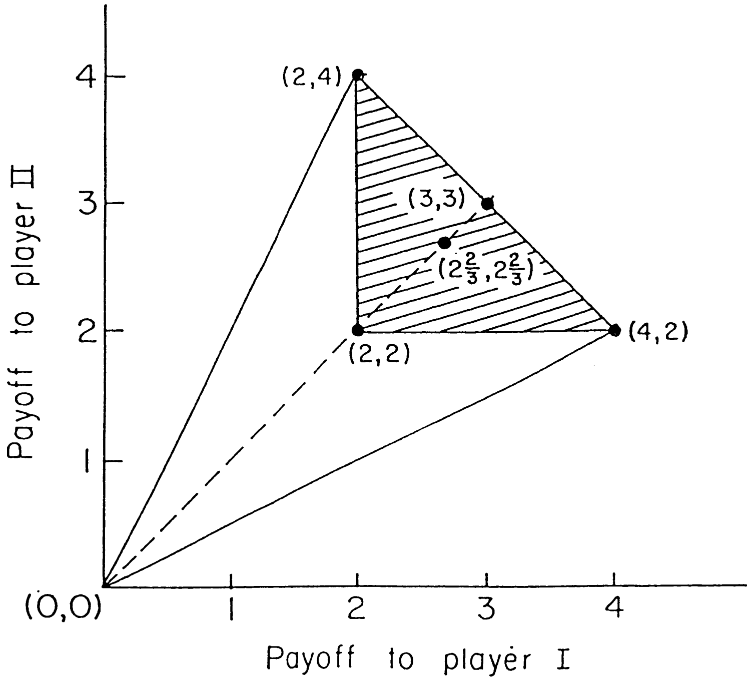


Figure 5. Various regions in the set of feasible payoffs of the "game of chicken" of Figures 2.a, 3, and 4.

(1) The feasible set itself is the convex hull of $(0,0)$, $(2,4)$, and $(4,2)$.

(2) The hatched area is the convex hull of the three Nash equilibrium payoffs $(2,2)$, $(2,4)$, and $(4,2)$. $(2,2)$ is the outcome of the mixed equilibrium and $(2,4)$ and $(4,2)$ correspond to pure equilibria.

(3) $(2\frac{2}{3}, 2\frac{2}{3})$ is the outcome of a partially correlated equilibrium.

(4) The interval $[(0,0),(3,3)]$ on the 45° line is the set of possible payoffs in biological contests (based on this game). The half-open interval $((2,2),(3,3)]$ is the set of payoffs to strict and symmetric correlated equilibria. By the equivalence theorem each point in this interval is an outcome of an ESS of some asymmetric conflict based on our game. The payoff vector $(3,3)$, which is Pareto optimal, is therefore an outcome of an ESS.

DEFINITION 2.3. A c.s.p. is *symmetric* if its distribution (q_{jk}) is symmetric, that is $q_{jk} = q_{kj}$ for $(j,k) \in S$.

EXAMPLE 2.4. Let h be given by the following bimatrix:

	D	H
D	6,6	2,7
H	7,2	0,0

This is the "game of chicken" (see Figures 6 and 7).

Then

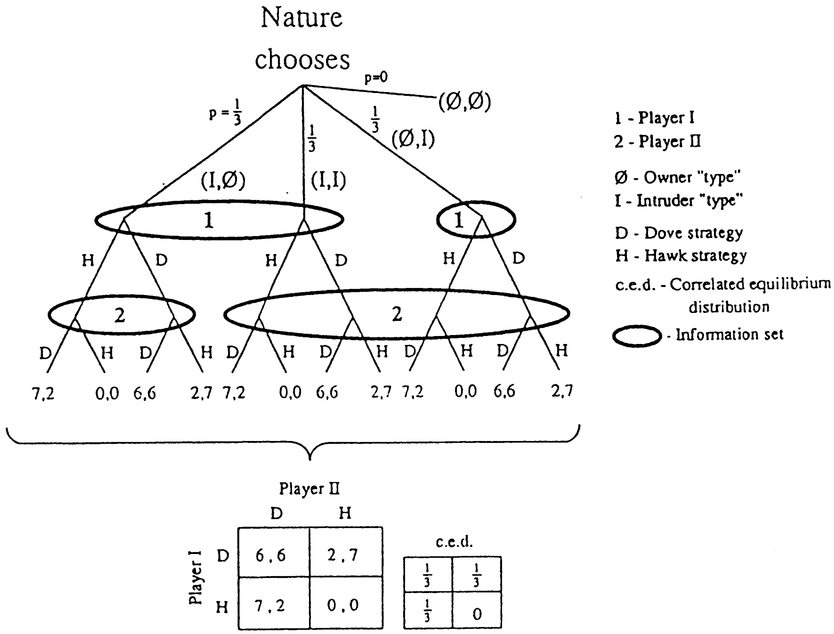


Figure 6. A representation of an asymmetric conflict with incomplete information (and without "role asymmetry"), by an extensive game—the case of Davies' butterflies (a detailed exposition of the example is given in Section 5).

1/3	1/3
1/3	0

is a symmetric and strict c.e.d. which cannot be obtained by mixed strategies (see Aumann, 1987).

3. Asymmetric Animal Conflicts with Payoff-Irrelevant Types and Role Asymmetry[5]

An asymmetric animal conflict is a system (U, p, α) where $U = \{u_1, \ldots, u_n\}$, $n \geq 2$, is the set of possible types of the animals; p is a probability distribution on $U \times U$ that satisfies $p(u, v) = p(v, u)$ for $u, v \in U$; and α is a function which assigns to each pair of types (u, v) a payoff matrix $\alpha(u, v)$ (of the first player). Formally, (U, p, α) is a two-person game with incomplete information. U is the common set of types of the players; p determines the (consistent) beliefs of the two players; and α is the payoff function. (U, p, α) satisfies *role asymmetry* if $p(u, u) = 0$ for all $u \in U$. (U, p, α) has *payoff-irrelevant types* if there exists an $m \times m$ (fitness) matrix A such that $\alpha(u, v) = A$ for all

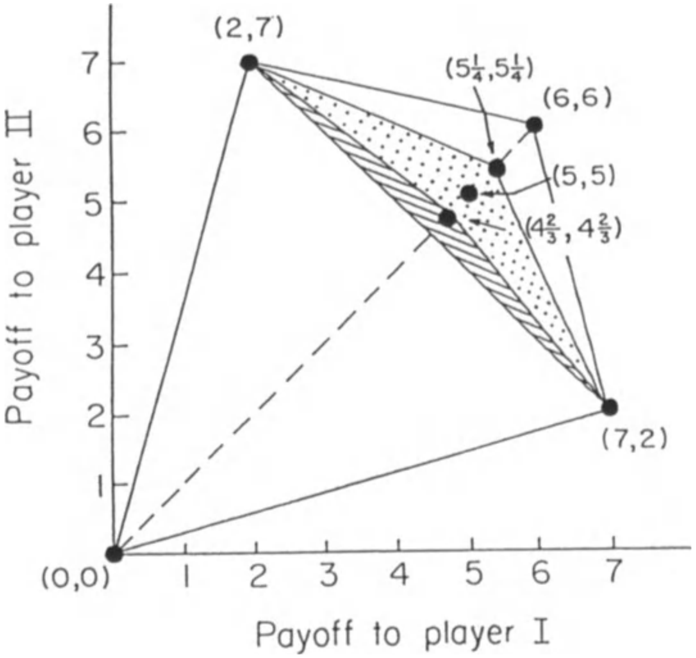


Figure 7. Various regions within the set of feasible payoffs of the "game of chicken" of Davies' butterflies (see the game of Figure 6).

(1) The feasible set itself is the convex hull of the vectors $(0,0)$, $(2,7)$, $(6,6)$, and $(7,2)$.

(2) The hatched area is the convex hull of the three Nash equilibrium payoffs $(2,7)$, $(7,2)$ and $(4\frac{2}{3},4\frac{2}{3})$.

(3) The dotted region is the set of payoff vectors of correlated equilibria, which are "above" the convex hull of payoffs of Nash equilibria.

(4) The interval $[(0,0,(6,6)]$ on the 45° line is the set of possible payoffs in biological contests (based on this game). The set of payoffs of strict and symmetric correlated equilibria that dominate the Nash payoff is the open interval $((4\frac{2}{3},4\frac{2}{3}),(5\frac{1}{4},5\frac{1}{4}))$. Each of these payoff vectors may be obtained as the outcome of an ESS (by the equivalence theorem).

Finally, we remark that payoff vectors of correlated equilibria also exist *below* the line through $(7,2)$ and $(2,7)$. For example, each point in the interval $[(3\frac{3}{5},3\frac{3}{5}),(4\frac{1}{2},4\frac{1}{2})]$ is the outcome of a correlated equilibrium.

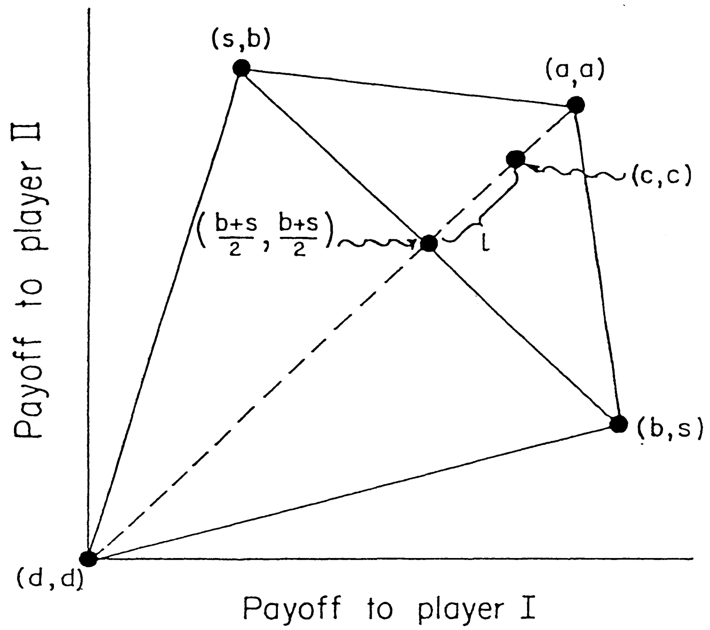


Figure 8. A graphic representation of the "game of chicken" of Figure 1. We assume here that $a > (b+s)/2$ and $b > s$ (as in the figure).

(1) The feasible region is the convex hull of the vectors (d,d), (s,b), (a,a), and (b,s).

(2) $((b+s)/2,(b+s)/2)$ is the outcome of a strict symmetric and totally correlated equilibrium. Hence, it is also the payoff vector of a (pure) ESS of an asymmetric conflict with two (complementary) roles and "role asymmetry".

(3) The open interval $(((b+s)/2,(b+s)/2),(c,c))$ on the 45° line is the set of payoff vectors to strict, symmetric, and partially correlated equilibria, which (Pareto) dominate the vector $((b+s)/2,(b+s)/2)$. c is given by the following formula.

$$c = \frac{2(b-a)(b+s)/2}{2(b-a)+s-d} + \frac{(s-d)a}{2(b-a)+s-d}$$

Thus, $c = \alpha(b+s)/2 + (1-\alpha)a$ where $\alpha = \frac{2(b-a)}{2(b-a)+s-d}$.

The details of the computation of c are given in the Appendix B.

$u, v \in U$. An asymmetric conflict with payoff-irrelevant types is denoted in the sequel by (U, p, A), where A is the payoff matrix.

Let (U, p, A) be an asymmetric conflict. A (behavior) strategy is a function $\varphi : U \to S_m$, where S_m is the (standard) $(m-1)$-dimensional simplex. Thus, φ is a function which assigns to each type $u \in U$ a mixed strategy in the (matrix) game A. If σ^i, $i = 1,2$, is a strategy of i, then the payoff to i in (U, p, A) is $Eh^i(\sigma^1, \sigma^2)$, $i = 1,2$. σ is a *symmetric Bayesian equilibrium* if σ is a best reply to σ. σ is an *evolutionarily stable strategy* (ESS) if:

$$\sigma \text{ is a symmetric Bayesian equilibrium;} \tag{3.1}$$

$$\begin{gathered}\text{if } \mu \text{ is a best reply to } \sigma, \text{ that is, } Eh^1(\sigma,\sigma) = Eh^1(\mu,\sigma), \\ \text{and } \mu \neq \sigma, \text{ then } Eh^1(\sigma,\mu) > Eh^1(\mu,\mu).\end{gathered} \tag{3.2}$$

A behavior strategy σ is pure if $\sigma(u)$ is a pure strategy (in the matrix game A), for every $u \in U$. The following result is a corollary of Selten (1980).

PROPOSITION 3.1. Let (U, p, A) be an asymmetric animal conflict with payoff-irrelevant types and role asymmetry. If σ is an ESS then

$$\sigma \text{ is pure;} \tag{3.3}$$

$$\text{if } \mu \neq \sigma \text{ then } Eh^1(\mu,\sigma) < Eh^1(\sigma,\sigma). \tag{3.4}$$

4. The Main Result

4.a. An Equivalence Theorem

THEOREM 4.1: Let A be an $m \times m$ fitness matrix. Then (q_{jk}) is the probability distribution of a strict and symmetric c.e. iff there exist an asymmetric animal conflict with role asymmetry and payoff-irrelevant roles (U, p, A), and an ESS σ, such that (q_{jk}) is the distribution of (σ, σ).

PROOF: SUFFICIENCY. Let (U, p, A) be an asymmetric conflict (considered as a 2-person game with incomplete information), and let σ be an ESS. Because σ is a (symmetric) pure Nash Equilibrium in (U, p, A), the distribution of $(\sigma, \sigma) : U \times U \to S$ is a c.e.d. (see Aumann, 1987, Sec.3). As p is symmetric, (q_{jk}), the distribution of (σ, σ) is symmetric. Also because of (3.4), (σ, σ) is strict ((σ, σ) itself may be considered as a c.e.; see Aumann, 1987, Sec. 4(g)).

NECESSITY. Let (q_{jk}) be a strict and symmetric c.e.d. . Because (q_{jk}) is strict, it satisfies the following inequalities:

$$\sum_{k=1}^{m}(h^1(j,k) - h^1(l,k))q_{jk} > 0 \text{ if } l \neq j \text{ and } \sum_{k=1}^{m} q_{jk} > 0 \tag{4.1}$$

Now we have to find an asymmetric animal conflict (U, p, A) and an ESS σ with the foregoing properties. Let $W = \{e_1, \ldots, e_m\}$ be a set with m members and let

$$U = W \times \{I, O\} = \{(e_1, I), \ldots, (e_m, I),)(e_1, O), \ldots, (e_m, O)\}$$

$(e_1, \ldots, e_m$ may be considered, e.g., as degrees of fighting experience, O—as ownership, and I as intrusion.) Define a probability distribution p on $U \times U$ by $p((e_j, I), (e_k, O)) = p((e_k, O)(e_j, I)) = q_{jk/2}$, and $p(u, v) = O$ otherwise, for all u, $v \in U$. Finally, define $\sigma(e_j, I) = \sigma(e_j, O) = j$ for $j = 1, \ldots, m$. Then (U, p, A) is an asymmetric animal conflict with role asymmetry. Also, because (q_{jk}) is a symmetric c.e.d, σ is a symmetric Nash equilibrium. By (4.1) σ is the unique best reply to σ. Hence, σ is an ESS. Q.E.D.

4.b. An Example

Consider again Example 2.4. We now show how to apply Theorem 4.1 in this particular example in order to construct an asymmetric animal conflict (U, p, A) and an ESS σ whose distribution coincides with the given c.e.d. Accordng to the proof of the necessity part, let $U = \{(e_1, I)$, (e_2, I), (e_1, O), $(e_2, O)\}$ and let p be given by the matrix

	(e_1, I)	(e_2, I)	(e_1, O)	(e_2, O)
(e_1, I)	0	0	1/6	1/6
(e_2, I)	0	0	1/6	0
(e_1, O)	1/6	1/6	0	0
(e_2, O)	1/6	0	0	0

Then (U, p, A), where A is the fitness matrix of Example 2.4, is an asymmetric animal conflict with role asymmetry. The behavior strategy σ which is defined by $\sigma(e_1, I) = \sigma(e_1, O) = D$ and $\sigma(e_2, I) = \sigma(e_2, O) = H$, is an ESS with the distribution of Example 2.4.

REMARK 4.2. Let A be a fitness matrix and let (q_{jk}) be a strict and symmetric c.e.d. By Theorem 4.1 we may construct an asymmetric animal conflict (U, p, A) with role asymmetry and an ESS σ with the distribution (q_{jk}). However, the construction is *not* unique. Indeed, our particular method is quite arbitrary (except that it seems to minimize the number of roles). Therefore, we cannot predict the structure of asymmetric animal conflicts that might arise in nature.

5. A Generalization and Examples

5.a. Asymmetric Animal Conflicts When Contestants Can Have Identical Roles

Let A be an $m \times m$ fitness matrix and let (q_{jk}) be a symmetric and strict c.e.d. We can always find an asymmetric animal conflict (U, p, A) with m roles, and perhaps without role asymmetry, that has a pure ESS σ such that the distribution of (σ, σ) is (q_{jk}). Indeed, let $U = \{r_1, \ldots, r_m\}$ be a set of m possible roles. Define a probability p on $U \times U$ by $p(r_j, r_k) = q_{jk}$. Then (U, p, A) is an asymmetric animal conflict with payoff-irrelevant types (but perhaps without role asymmetry). Define now a (pure) behavior strategy σ in (U, p, A) by $\sigma(r_j) = j$, $j = 1, \ldots, m$. Because (q_{jk}) is a symmetric and strict c.e.d., σ is an ESS. Thus, we can reduce the number of roles to m if we do not insist on role asymmetry.

In 5.b.2 we shall show how to construct an asymmetric animal conflict, with two roles and without role asymmetry, and an ESS σ that represent Example 2.4 (see also Figure 6).

5.b. Degrees of Seniority as a means to obtain correlated equilibria in the "game of chicken": The field case of Davies' Butterflies.

5.b.1. A case of total correlation

One of the most interesting field studies investigating the "Dove-Hawk conflict" in biology is that of Davies (1978) (for other examples see Maynard-Smith, 1982). Male butterflies (speckled wood species) search for sunspots which are clearings in forests. Males compete for sunspots because these are the best place to find females. When two males meet in the same sunspot a spiral upward flight takes place that ends always in the following fashion: One butterfly leaves the sunspot and the other becomes a resident. Davies found that in such encounters, the "owner" of the sunspot, that is, the first to find it, always wins the clearing. To test whether ownership rather than strength or other payoff-relevant traits determine the outcome of the contest, Davies caught "owners" and introduced them as "intruders" into other sunspots. In all the experiments in which "role asymmetry" was created, Davies observed short spiral flights with the same result, confirming Tinbergen's statement "the resident always wins".

When Davies simultaneously introduced into the same sunspot two male butterflies, a spiral flight occurred which was 10 times longer than typical ones. Davies interpreted these long spiral flights as escalated encounters, where both contestants play "Hawk". Maynard-Smith (1982; p.99) emphasizes that Davies' result "fits beautifully with the predictions of the Hawk-Dove Bourgeois model". Figures 3, 4, and 5 describe in detail how the Davies' butterflies reach correlated equilibria.

This case, as well as all other biological contests in which only complementary roles meet, are called *totally correlated*[6] cases (Forges, 1986). This

is exactly the result of the seminal model of Selten (1980) for the "game of chicken". In such biological games nature selects through evolution such contests in which only non-identical roles meet.

Fig. 1 and Fig. 4. demonstrate how one can solve pairwise interaction of "games of chicken" by getting only pure strategies encounters of (H, D) or (D, H) with payoffs' pairs (b, s) (s, b), and avoiding encounters of the pair (H, H) with low payoff (d, d).

5.b.2. A case of partial correlation

Totally correlated[6] cases are very special examples of correlation (Forges 1986, 1990). In the formal model (Sec. 4) partial correlation is achieved by having in a row j of the probability distribution matrix, more than one entry with $q_{jk} > O$, where q_{jk} is the probability of the encounter between strategy j and strategy k. Aumann (1974, 1987) emphasized that such partially correlated[6] strategies may yield equilibria which have higher payoffs than other Nash equilibria and totally correlated equilibria (see the examples of Figures 6, 7 and, for the general case, see Fig. 8). Can we find a biological situation in which such partially correlated cases can be materialized? Selten (1980) already considered situations in which a row can have more than one entry with basic probability greater than zero (in Selten's model for more than one entry, v, $W_{uv} > 0$). Such a mathematical condition corresponds to a situation of *incomplete information*. Below, we use the Davies' butterflies example as the basis for a model in which incomplete information leads to partially correlated equilibrium. The resident (or owner) role will be called a *senior role* and the intruder role will be called a *junior role.*

Male butterflies compete for sunspot clearings in a forest in order to fertilize females. Females mate only with males that occupy a clearing. The first male to reach a clearing checks the opening and realizes that no other males are there. Before time T is reached, the first male in the clearing has a "junior" role. If no other male has detected the clearing during the time interval T, the first male switches to the "senior" role.

The second male reaching the same opening immediately will observe the first male and will assume a "junior role".

Our hypothetical "Junior-senior example" does not allow the possibility of hidden butterflies and assumes that males have excellent ability to detect other males. In such situations the probability that two males arriving at the same opening will acquire the same "senior role" is zero[7]. We assume that the density of male butterflies in the forest is high enough so that the probability of a pairwise encounter of two butterflies which reach the same clearing before the time T is one-third. Then we will get a situation in which 2/3 of the encounters are between different roles (half Senior-Junior and half Junior-Senior) and one-third of the encounters are between "Junior-Junior". There are no encounters of "Senior-Senior". In such circumstances, if the "Senior role" will play the "Hawk" strategy and the "Junior role" will play the "Dove" strategy, the payoff of a strict and symmetrical, correlated

equilibrium can be obtained (Figures 6 and 7). This is to say that none of the individuals which have such "conditional pure strategy" (sensu Parker, 1984) "can choose a strategy" by which he can get a higher fitness. We have shown formally (Sections 4, 5) that the distribution of a strict symmetric correlated equilibrium is always the distribution of an ESS.

Have such partially correlated biological situations been observed in Nature? In the study of the speckled wood butterfly Davies did not report the percentage of pairwise contests between intruders (the analog to our pairwise Junior encounters). Jacques and Dill (1980) studied contests of spiders and report on many pairwise contests between wandering individuals (analog in our model to intruder and Junior roles), that end by one of the contestants winning without escalation. We hope that the presentation of the idea of correlated equilibrium to ecologists will open the investigation of the question: Are contests with partially correlated strategies (which implies incomplete information) which settle "game of chicken situations" common in plants and animals?

What is the internal logic in the base of the "Junior-Senior example"? There is an asymmetry in the information. Our model analyses the biological case "as if" the Darwinian logic underlying such behavior is the following:

There is asymmetry in the knowledge of the two role types: the Senior type knows both his type and the type of his opponent. The Junior type knows his type but he is not sure whether his opponent is Junior or Senior. In such circumstances there is a range of payoffs in which it is best for the Junior type to always play Dove and not Hawk. Such an example is demonstrated in Figures 6 and 7.

6. Discussion

Evolutionary game theory studies only symmetrical games (Hammerstein and Selten, 1991). Fig. 1 represents a group of biological games that we call *"monomorphic games"*: These are games with a genetically monomorphic population of potential players. In a monomorhic game equilibrium all the individuals in the population must play the same strategy. Thus, the possible payoff pairs for pure strategies in Fig. 1 are (a, a) and (d, d). Indeed, in the prisoner's dilemma game when $b > a$ and $s < d$, (d, d) is an equilibrium payoff. But if $s > d$, the bimatrix game represents the "game of chicken", which has two pure equilibria that are asymmetric and therefore cannot be realized in intraspecific competition with a monomorhic population. We show that by adding further structure in the form of a lottery device, the two pure equilibria of the normal form game can be combined to produce a Nash equilibrium in a semiextensive game[2]. The chance device in nature turns out to be a situation which assigns to the two opponents with the same genotype different information situations, which are known in biology as "roles". In Aumann's model, the lottery between the two pairs of strategies is part of

the solution concept, and indeed in nature the biological correlation device of the roles is part of the solution of the game. (Both the roles and the strategies are chosen simultaneously and the temporal order in Figures 4 and 6 is only to help in understanding the model). This is in contrast with Selten (1980) and Hammerstein and Selten (1991), who interpreted the chance device as part of the model. Overall, this semi-extensive biological game comprises a symmetrical game with asymmetric conflict (Hammerstein and Selten 1991, p. 48). It is worth noting that the chance device generates role types which do not influence the payoff of the game. Both Aumann (1974) in game theory and Maynard-Smith and Parker (1976) in animal behavior have independently dealt with this case.

The competitive interaction among individuals in a natural population can drive the opponents into situations where both will receive small payoff. Such an inefficient situation for both players can result in the following games:

a) In the prisoner's dilemma, at equilibrium, both players get very low payoff (see Fig. 1).
b) In the "game of chicken" (Fig. 1) the only symmetrical equilibrium is that of mixed strategies in which the players receive lower payoff compared with what they could get in the pure equilibria (Fig. 5).

The following question can be raised. Is it possible in nature to achieve more efficient equilibria by natural selections of biological traits? The formal solution to this question was suggested by Aumann with the correlated equilibrium model. To implement Aumann's correlation machine, evolution should develop and invent behavioral or morphological traits with the following characteristics:

a) The pairwise meeting of phenotypes (carriers of traits) are totally or partially correlated.
b) Traits that develop information asymmetry between the opponents.
c) Traits that create and develop a situation of incomplete information for at least one of the opponents.

Indeed, we find in nature a number of behavioral traits that lead to semi-extensive games with correlated equilibrium. Behavioral conventions such as seniority, territoriality, and sexuality in parental care may be viewed as correlation devices which have been selected by evolution because of their highly efficient equilibria. The following characteristics are typical of such traits:

1. Encounters occur mainly among different roles (in totally correlated cases).
2. In cases of encounters between the same roles , evolution will select behavior leading to incomplete information that will result in an efficient equilibrium with the appropriate "correlated equilibrium distribution".
3. The roles do not have an influence on the payoffs.

We suggest that natural situations in which different individuals in the population can behave differently (i.e., choose complementary roles) and thereby render the genotype payoff more efficient, can evolve only if nature builds semi-extensive games in which strict symmetric correlated equilibria (which correspond to ESS's) can be materialized [8]. Thus, asymmetric pairwise interactions are the natural lottery device that maintain correlated strategies.

Thus, the evolution of conventions such as seniority, territoriality and unisogamy (in conflicts of parental care) in nature as well as in human culture, does not have to be interpreted by "altruistic means" or by "species good" (Parker, 1978; Parker and Hammerstein, 1985), but by individualistic behavior (i.e., maximization of fitness) which selects those natural correlation devices which produce the greatest fitness. The pure strategies we observed in nature in such examples are the result of the existence of strict symmetric correlated equilibrias. Several human cultural ceremonies can be interpreted along this line (Selten, 1991).

We now try to explain the adaptive value of sex in animals with two sex types, by means of our model. Many explanations have been suggested to the evolution of sex (Michod and Levin (1988)). Our model is complementary and explains how the specialization of the parents in different roles, when raising the offspring, leads to a biological equilibrium, that is, to an ESS.

Aumann's notion of correlated equilibrium explains the equilibrium between male and female in raising their offspring, although they play different roles and, hence, receive different payoffs. Indeed, the male - female game, for animals, has two asymmetric roles, namely, the two sexes. We may assume that each player has two strategies: D:= feed the offspring; and H:= forage in order to get food for the family. In this way we obtain the situation depicted in Fig 3 when we replace "owner" by "male", and "intruder" by "female". Notice that the equilibrium in Figure 3 is totally correlated and payoff dominates the symmetric Nash equilibrium. Also, the roles of the players are chosen by Nature.

Independently of our work, Cripps (1991) has proved a somewhat more general result than our Theorem 4.1. Whereas our theorem applies to bimatrix games of the special form (A, A^t), Cripps proves an equivalence theorem for general bimatrix games. We do not know of any biological model which is solved by Cripps's result and for which our result does not apply. Furthermore, Cripps does not present any biological applications in his work. Finally, our Theorem A in Appendix A is more general than the Sufficiency Part of Cripps's Theorem (Shmida and Peleg, 1991).

From the theoretical point of view, this study makes a connection between Selten's model and Aumann's "correlated equilibrium distribution". The main theorem shows that the distribution of every ESS in Selten's model is the distribution of a strict symmetric correlated equilibrium, and vice versa. From the biological point of view, our main result may help the biologist in

the following way: Suppose he knows the fitness matrix of a biological conflict. Then, he may ask: Which payoffs are obtainable with the help of some payoff irrelevant asymmetric conflict based on this matrix? The answer to this question is given directly by our theorem using Aumann's definition of correlated equilibrium, without the need to consider all possible payoff irrelevant asymmetric conflicts in Selten's extensive form. In particular, it supplies a computational procedure to obtain all attainable payoffs in "payoff irrelevant asymmetric conflicts".

Compared with Aumann's paper (1974), Selten's theorem (1980) represents a different approach to modelling nature. While Selten identifies the roles and the "basic probabilities" as part of the game model, Aumann represents the correlated equilibrium distributions as part of the solution concept. We fully agree with Professor Selten (per. comm.) that natural situations should be explored in game theory by explicit models and one should explain explicitly Aumann's "correlation device".

But, on the other hand, as we are concerned with biological evolutionary games we prefer to approach the correlation device (i.e., the behavioral games which control the conditional phenotype behavior) and the probabilities of encounters of different roles as part of the solution concept. Because, in biology, there is no temporal order in the semi-extensive "Hawk-Dove game" and evolution has selected both the strategies and the roles "as if" it were a simultaneous rational decision.

Footnotes

[1] The Hawk-Dove conflict corresponds to the "game of chicken" situation (see Aumann 1974, 1987).

[2] A semi-extensive game in our paper is an extensive game in which a chance move chooses different roles (types) which play the same matrix game. In usual extensive games, the payoffs may be arbitrary and are attached to the endpoints of the game tree. The term 'semi-extensive' was suggested to us by R. Selten.

[3] By asymmetric animal conflicts the ethologists (people who study animal behavior) mean that the contestants in the game may have different "roles", e.g., female vs. male; owner vs. intruder; first vs. second (see Maynard-Smith and Parker, 1976 and Hammerstein, 1981).

[4] Anecdotally, what makes the correlation in the sense of Aumann possible is the phenomenon of "uncorrelated asymmetry" (i.e., "payoff irrelevance"—see Sec. 3) in animal conflict. Indeed, we wanted to call our paper "Uncorrelated asymmetry as a mechanism to obtain correlated equilibrium in biological conflicts".

[5] What was considered in Selten (1980) as "information asymmetry" (no contests between opponents with the same information) is named by Hammerstein and Selten (1991) "role asymmetry". We will use only the latter term in the present paper.

[6] A correlated strategy for the "game of chicken" is totally correlated if the pure strategies that it dictates for the players are fully correlated, that is, the pure strategy used by Player I determines the pure strategy used by Player II, and vice versa. A correlated strategy is partially correlated if it is not totally correlated. In biological contests based on the "game of chicken" the totally correlated strategies are:

p	0
0	$1-p$

$0 \le p \le 1$ and

0	1/2
1/2	0

[7] For the formal model to be complete and consistent with the example in Fig. 4, we must add the following assumptions:

1. The probability of finding an empty clearing (being first) is 1/2, and the probability of being second is 1/2.
2. The probability of the arrival of a second male in the interval $[0, T]$ is 1/3. The probability of the arrival of the second in $[T, 3T]$ is 2/3.
3. After time $3T$ the first butterfly leaves.
4. Three or more butterflies never share the same clearance.
5. The probability of simultaneous arrival of two males is 0.
6. If a "senior" butterfly has expelled a "junior", then no new male will enter the clearing before the "senior" leaves.

[8] In interspecific competition there is no need to require symmetric equilibria between the opponents. And, indeed, it is common in such situations in nature to obtain equilibria with payoff pairs (b, s) and (s, b) (see Fig. 1). Such equilibrium points correspond to an evolutionary phenomenon of specialization and differentiation of two different species.

A. Some remarks on Theorem 4.1

The Sufficiency Part of Theorem 4.1 is a special case of the following general result. Let $\Gamma = (K, P, U, C, p, r)$ be a finite n-person game in extensive form (see van Damme (1987, p.100)), let Z be the set of endpoints of Γ, and let $G = (A^1, \ldots, A^n; u^1, \ldots, u^n)$ be a finite n-person game in strategic form. Denote by $\Gamma(G) = (K, P, U, C, p, G)$ the game obtained from Γ by replacing $r(z)$ by G at every $z \in Z$ (i.e., if a play of Γ arrives at $z \in Z$ then the players have to play G in $\Gamma(G)$). A pure strategy of player i, $1 \le i \le n$, in $\Gamma(G)$ is a pair (d^i, δ^i), where d^i is a pure strategy (for i) in Γ and $\delta^i : Z \to A^i$. Clearly, if $\sigma = (\sigma^1, \ldots, \sigma^n)$ is an n-tuple of mixed strategies in $\Gamma(G)$, then σ induces a correlated strategy μ_σ in G.

THEOREM A: If $\sigma = (\sigma^1, \ldots, \sigma^n)$ is a Nash equilibrium in mixed strategies in $\Gamma(G)$, then μ_σ is a correlated equilibrium in G.

PROOF: $\Gamma(G)$ is equivalent to the following game G_C (in extensive form): **Stage 1:** Each player $1 \le i \le n$ reports a pure strategy d^i in Γ to a mediator. We denote by $v(\cdot|d^1, \ldots, d^n)$ the probability distribution on Z which is induced by $d^1, \ldots, d^n$; **Stage 2:** The mediator chooses $z \in Z$ according to the distribution $v(\cdot|d^1, \ldots, d^n)$ and informs each player on z; **Stage 3:** Each i, $1 \le i \le n$, chooses $\delta^i(z) \in A^i$.

Clearly, $\Gamma(G)$ and G_C have the same strategic form. Hence, σ is a Nash equilibrium in G_C. Therefore, by the revelation principle (see Myerson (1991), Chapter 6), μ_σ is a correlated equilibrium of G.

B. The computation of c in Figure 8

We consider the symmetric bimatrix game

a, a	s, b
b, s	d, d

where $b > a > d$, $b > s > d$ and $a > (b+s)/2$.

We are interested in payoffs of symmetric and strict correlated equilibria that Pareto dominate the vector $((b+s)/2, (b+s)/2)$.

Let $\mu =$

α	p
p	β

be a symmetric correlated strategy, that is:

$$\alpha + \beta + 2p = 1 \text{ and } \alpha, \beta, p \ge 0. \quad (1)$$

μ is a strict correlated equilibrium iff

$$\alpha a + ps > \alpha b + pd; \quad (2)$$

and

$$pb + \beta d > pa + \beta s. \quad (3)$$

Clearly, (1), (2), and (3) define a convex set.

The payoff to μ is $(h^1(\mu), h^2(\mu))$ where

$$h^1(\mu) = h^2(\mu) = \alpha a + p(b+s) + \beta d. \quad (4)$$

Obviously,

$$c = \sup\{h^1(\mu) | \mu \text{ satisfies } (1), (2), \text{ and } (3)\}. \quad (5)$$

Because $d < (b+s)/2$ we otain from (1)–(5).

$$\begin{aligned} c &= \sup\{\alpha a + p(b+s) | \alpha a + ps > \alpha b + pd \text{ and } \alpha + 2p = 1\} \\ &= \sup\{a + 2p\frac{(b+s-2a)}{2} | p > \frac{b-a}{2(b-a)+s-d}\} \\ &= \frac{2(b-a)(b+s)/2}{2(b-a)+s-d} + \frac{(s-d)a}{2(b-a)+s-d} \end{aligned}$$

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Recurring Bullies, Trembling and Learning[1]

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Abstract. In a recurring game, a stage game is played consecutively by different groups of players, with each group receiving information about the play of earlier groups. Starting with uncertainty about the distribution of types in the population, late groups may learn to play a correct Bayesian equilibrium, as if they know the type distribution.

This paper concentrates on Selten's Chain Store game and the Kreps, Milgrom, Roberts, Wilson phenomenon, where a small perceived inaccuracy about the type distribution can drastically alter the equilibrium behavior. It presents sufficient conditions that prevent this phenomenon from persisting in a recurring setting.

Keywords. Recurring Game, Social Learning, Chain Store Paradox

1 Introduction

In a recurring game, a stage game is sequentially played by different groups of players. Each group, before its turn, receives information about the social history consisting of past plays of earlier groups. Game theorists have studied such recurring situations using various dynamics like fictitious play, last period best response, and random matching, since Nash in his dissertation (1950).

Of interest here is a Bayesian version of a recurring game, where each stage game player is uncertain about the types of his or her opponents. Observed social histories are used by players to update beliefs about the unknown distribution of types in the population. With time, players' behavior converges to that of the Bayesian equilibrium of the stage game, as if the true distribution of types in the population were known.

In a recent paper (Jackson and Kalai, 1995a) we present a general model of recurring games and sufficient conditions that yield such social convergence. Our purpose in this chapter is to apply this approach to Selten's (1978) chain-store games and study the implications on the phenomenon illustrated by Kreps and Wilson (1982) and Milgrom and Roberts (1982) (KMRW for

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short, also sometimes called the "gang of four"). A specific question is whether this phenomenon (described in more detail below) can persist in a recurring setting. To better fit the recurring setting we replace Selten's chain-store story with a strategically equivalent story about a bully terrorizing a finite group of individuals, the challengers. The rest of the introduction presents a verbal description of our model and conclusions. A broader survey of the related literature can be found in Jackson and Kalai (1995a).

The "challenge the bully" stage game (the bully game, for short) is played by a bully and L challengers. It consists of L preordered episodes each played by the bully and one challenger. In each episode the designated challenger chooses whether to challenge the bully or not, and if challenged, the bully chooses whether to fight or not. The bully prefers not to be challenged, but if challenged he would rather not fight. The challenger prefers challenging a nonfighting bully, but if the bully fights he would rather not challenge. (The single episode payoffs are represented in Figure 1 of the next section.) Both bully and challenger know the outcome of all previous episodes before making their decisions.

Recall that Selten's paradox is that backward induction applied to this game dictates that all challengers challenge and the bully never fights. Yet, when L is large, common sense suggests that the bully may choose to fight in early episodes in order to build a reputation so that challengers will not challenge.[2] KMRW point out that the backward induction result depends on the complete information assumption. The resolution offered by KMRW is to replace the above game by a Harsanyi (1967–68) Bayesian game in which there is a small commonly known prior probability that the bully is "irrational," e.g., he prefers fighting to any other episode outcome. In such a modified game the more reasonable behavior is obtained as the unique sequential equilibrium outcome, even when the realized bully is rational. Thus the phenomenon illustrated by KMRW is that in a world with a rational bully, a small uncertainty about the rationality of the bully can drastically change the equilibrium outcomes, and in particular, induce the rational bully to fight.

It is not obvious whether this phenomenon will persist if this situation recurs. In other words, if new bullies are always born rational, but challengers are uncertain about this fact, would bullying behavior persist? On the one hand, it seems that statistical updating will lead observers (i.e., players in later rounds) through backward induction to the recognition that the bullies are rational and with it will come challenging and not fighting. On the other hand, a violation of a rational act by a bully that wishes to appear irrational, is more convincing when done in view of such learning. Thus, one is left with the question of whether such learning will lead the eventual discovery of the rationality of the population and thus the unraveling of the KMRW

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[2] There are, of course, Nash equilibria of this type; however, they do not survive backward induction.

phenomenon, or whether the actions of the players will slow the learning so that the rationality of the population is never learned.

To meaningfully address this question, we extend the KMRW game to a "doubly Bayesian" recurring game with two "priors." One prior, τ, is a fixed probablity distribution according to which a bully–type is drawn before each stage game. We assume, however, that this type–generating distribution is not known to the players. They assume that it was selected before the start of the entire recurring game according to a known probability distribution Γ over a set of possible type generating distributions.[3]

This gives rise to what we call an *uncertain recurring game* whose extensive form game can be described as follows. In period 0 nature randomly selects a type generating probability distribution τ from a set of possible ones, according to the prior probabilities $\Gamma(\tau)$. Players know that a selection was made according to Γ, but no information is given about the realized τ. For period 1, one bully and L challengers are selected. The bully is randomly assigned a type according to the fixed (unknown) type–generating distribution τ. Only the bully is informed of its realized type and the $L + 1$ players proceed to play the L episodes of the bully game. Their realized play path becomes publicly known. The game recurs, following the same procedure. In each period $t > 1$ a new set of challengers and a bully are selected. Again, the bully is privately assigned its type according to the same unknown τ and the players proceed to play the L episodes of the bully game, with the play path publicly revealed.

To study the recurring version of the KMRW phenomenon we will focus the above described game, with a realiztion of τ that selects rational bullies with probability one, indicating a world in which bullies are really rational.

Note that the above separation to two distributions is necessary since a restriction to a single prior τ, representing both actual type-generation and social beliefs, will force one of the following misrepresentations of the phenomenon. If the single τ assigns probability one to a rational bully, then there would be no social uncertainty about it. On the other hand, if the single τ assigns positive probability to irrational bully types, then in the recurring setting with probability one some irrational types will be realized, leading to the KMRW equilibrium, but in a world which truly has irrational bullies. In either situation, there would be no social learning. That is, observing past

[3] This approach may be viewed as the multi–arm–bandit "payoff" learning model adapted to the Bayesian repeated game "type" learning literature. [See Aumann and Maschler (1967) and followers, and the more closely related Jordan (1991).] The utility maximizing bandit player starts with a prior distribution over his set of possible payoff generating distributions. Ours is a multi–person version, with uncertainty over the distribution generating types, modeled through a prior over a set of possible distributions. A major difference between our model of learning in recurring games and the literature on learning in repeated games is that in our setting players are attempting to learn about the distribution of types that they will face in their stage game, while in a repeated game players learn about the actual opponents they repeatedly face.

stage games would tell a player nothing new about the distribution of types in the population.

Our recuring game definition of a type is also significantly generalized. A type's preferences can depend on the entire social history and not just on his own stage game. So, for example, we can model a "macho bully" whose utility of fighting increases after social histories with many earlier fighting bullies. Or in a multi-generational setting, a bully may prefer to behave like his ancestors.

We refer to the Nash equilibria of the above *uncertain recurring game* as *uncertain Bayesian equilibria.* In playing an uncertain Bayesian equilibrium, expected utility maximizers perform Bayesian updating of the prior Γ to obtain updated posteriors over the set of type generating distributions. As a result, the strategies of each stage game constitute a Bayesian equilibrium of the stage game relative to the perceived distribution over types induced by the updated prior, but not necessarily relative to the true (realized) type generating distribution.

This discrepancy partly disappears however, after long social histories. First, as a consequence of the martingale convergence theorem, the updated prior will converge (almost everywhere). This means that players' learning disappears in the limit. Second, the updated distribution will be empirically correct in the limit. This means that players' learning induced predictions concerning the play path will match the distribution over play paths induced by the true (realized) distribution over types. This second result can be obtained almost directly from a learning result in Kalai and Lehrer (1993). It means that in late games, any discrepancy between the true and the learning induced type generating distributions cannot be detected, even with sophisticated statistical tests.

The fact that players learn to predict the play path with arbitrary precision, does not necessarily imply that they have learned the true type generating distribution τ or the true distribution of strategies (including off the equilibrium path behavior).[4] We present an example of challengers who never challenge, because their initial beliefs assign high probability to fighting bullies, even though the realized bullies would never fight if challenged. No learning occurs because no challenging ever occurs. Moreover, in the example a single tremble by a single challenger will reveal the non fighting nature of the bullies and would cause the entire equilibrium to collapse.

In order to overcome this difficulty, we introduce trembles à la Selten (1975, 1983). The effect of introducing the trembles is to ensure that the

[4] This phenomenon also arises in non-recurring situations, such as those where players play a long extensive form or a repeated game. For detailed analyses of this in such settings, see Battigalli, Gilli and Molinari (1992), Fudenberg and Kreps (1988), Fudenberg and Levine (1993), and Kalai and Lehrer (1993).

social history leads to informative learning.[5] For an exogenously given small positive probability q, we assume that every player chooses strategies that assign probabilities of at least q to each one of his or her actions at every information set.

Assuming that players play an uncertain Bayesian equilibrium with trembling, we obtain the following conclusions for the case where the realized distribution results in only rational bullies: With probability one, in late stage games, the challengers always challenge, and the bullies never fight. These rules are followed with the exception of occasional trembles. This result is obtained regardless of the initial beliefs of the society, provided those initial beliefs assign some positive probability to the distribution which selects only rational bullies.

Thus the introduction of the trembles in the recurring model leads one to eventually play the trembling hand perfect equilibrium (defined with respect to the agent normal form). It is important to remark, however, that the trembles are not working directly (as in the definition of trembling hand perfection), but rather indirectly through the learning that they ensure. This is evident since in early stages, the equilibrium outcomes with trembles can still be that rational bullies fight and challengers not challenge.

We wish to remark that the general message of our results is not simply the eventual decay of the KMRW equilibrium. For instance, if the realized type generating distribution is truly selecting some "irrational" fighting bullies, then this would also eventually be learned and the eventual convergence would be to the KMRW equilibrium – even if initial beliefs placed an arbitrarily high (but not exclusive) weight on rational bullies. Thus the more general understanding of our results is that in a recurring game, given some randomness to induce learning, equilibrium behavior will eventually converge to that as if players were playing an equilibrium knowing the underlying type generating distribution.[6] If players are truly rational, and the game is an extensive form, then this will lead to eventual approximate play of a perfect equilibrium.

2 The Recurring Bully Game

The *stage game* is a *bully game* consisting of one bully player, b, and L challengers, $(c_i)_{i=1,\dots,L}$. In sequential episodes $i = 1, \dots, L$, challenger c_i decides whether to challenge (C) the bully or refrain (R) from doing so. If the challenger does challenge, then the bully has to decide whether to acquiesce (A) or fight (F). The corresponding episode payoffs, to the challenger and bully, are given by the extensive form game pictured in Figure 1.

[5] See the concluding remarks of Fudenberg and Kreps (1995) for the suggestion of a similar approach to ensure learning and convergence to Nash equilibrium in a model with myopic long lived players.

[6] See Jackson and Kalai (1995ab) for general results along these lines.

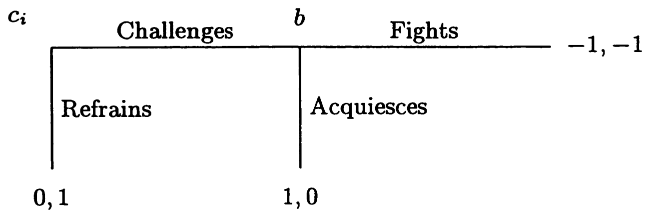

Figure 1

Each player is informed of the outcomes of earlier episodes before making a decision. The payoffs of challengers are determined according to their episode payoffs while the payoff to the bully is the sum of his or her L episode payoffs.

Formally, a *play path* of such a bully game is described by a vector $p = (X_1, \ldots, X_L)$ with each X_i, being in the set $\{(R),\ (C, A),\ (C, F)\}$, describing the outcome of the i^{th} episode. A *partial play path* is such a vector $(X_1, \ldots, X_l)$, but with $0 \leq l < L$. A (behavioral) strategy σ_i of challenger c_i consists of a probability distribution over the actions R and C for every $i - 1$ long partial play path $(X_1, \ldots, X_{i-1})$. A bully strategy, η, chooses a probability distribution over the actions A and F for every partial play path of every length. A vector of strategies $((\sigma_i), \eta)$ induces a probability distribution over the set of play paths and defines expected payoffs for all players in the obvious unique way.

The *recurring bully game* is played in periods $t = 1, 2, \ldots$. In each period t a new group of players (bully and challengers) are selected to play the stage game. Moreover, before choosing their strategies, they are informed of the play paths of all earlier groups.

A *social history* h of length t is a vector of play paths $h = (p^1, \ldots, p^t)$. The empty history $\emptyset$ is the only history of length 0, and H denotes the set of all finite length histories. The players of the recurring bully game are denoted by b^h and c_i^h, where for every fixed h, b^h and c_i^h describe the bully and the challenger to play the i-th episode of the stage bully game after history of h. Thus, as the notation suggests, the players b^h and c_i^h know the social history h that led up to their game. Strategies in the recurring bully game are denoted by η^h and σ_i^h, with the obvious interpretations.

3 Games with Unknown Bully Types

Alternative bully types describe different utility functions that a bully may have in playing a bully game. There is a countable set Θ of possible bully types. For each type $\theta \in \Theta$ there is a function $u_\theta(h, p)$ describing the payoff to bully b^h of the play path p, when he or she is of type θ.

An example of a type is the one already defined whose payoff from any path (regardless of the social history) is the sum of the episode payoffs described earlier (in Figure 1). We refer to these payoffs as the "rational" ones and to this type as the *rational type*, denoted by r.

An example of an "irrational" bully, motivated by KMRW, is one with payoff $k > 0$ for every "strong path" p in which he or she never acquiesces, and $-k$ for weak paths p, exhibiting some acquiescing actions. We keep the terms "rational" and "irrational" in quotes, since "irrational" bullies are supposed to be maximizing their expected payoffs. It is simply that the "irrational" payoffs do not match those of the game in Figure 1. Of course, this is simply a modeling convenience as any behvior can always be made to be expected utility maximizing, if one can arbitrarily manipulate the payoff function.

More complex "irrational" types can condition their payoffs on the observed social history. For example in the type just described we can replace k by k^h, taking on large values after social histories h consisting only of tough play paths and small values for social histories h with weak bullies. This may represent an ego maniac bully who compares himself to earlier ones.

Probability distributions over the countable set of possible bully types, Θ, are referred to as *type-generating distributions*. Every such distribution τ defines a *recurring Bayesian bully game* as follows. After every social history h, nature chooses a bully type θ^h, according to the distribution τ (independently of all earlier choices in the game). The bully b^h is informed of his or her realized type θ^h, before choosing his strategy. All other players are only aware of the process and know the distribution τ by which θ^h was selected.

Strategies in the Bayesian recurring bully game are denoted by (η_θ^h) and (σ_i^h), with η_θ^h denoting the strategy of the bully b^h when he or she is of type θ, and σ_i^h is a challenger strategy as before.

Notice that a Bayesian recurring bully game, with a type generating distribution τ, induces after every history h a *Bayesian bully game* of reputation, similar to KMRW, with bully types payoffs given by $u_\theta(p, h)$ and a prior τ on possible types θ.

We wish to model, however, Bayesian recurring bully games with uncertainty about the type generating distribution. It is such uncertainty that introduces learning from the social history. An *uncertain Bayesian recurring bully game* (uncertain recurring game for short) is played as follows. In a 0-time move, nature selects a type generating distribution τ according to a prior probability distribution Γ. Without being informed of the realized τ, the players proceed to play the τ Bayesian recurring bully game. We assume that the exogenously given distribution Γ has a countable support ($\Gamma(\tau) > 0$ for at most countable many τ's) and that it and the structure of the game are commonly known to all players.[7] Players update Γ based on observed histories and the strategies that they believe to be governing play. This up-

[7] see Jackson and Kalai (1995a) for discussion of extensions to allow for players' prior beliefs to be type dependent.

dated distribution induces a distribution over types, which is the basis for the Bayesian stage game that they play.

Notice that an uncertain recurring bully game may be thought of as a "doubly Bayesian" game since we use a prior to select a distribution that serves as the priors of the stage games to come. The strategies of the uncertain recurring game are the same as the ones of the Bayesian game since the information transmitted and feasible actions are identical in both games. However, as will be seen in the sequel, expected utility computations and Bayesian updating are more substantial in the uncertain recurring game.

An *uncertain Bayesian equilibrium* is a vector of strategies $((\eta_\theta^h)), (\sigma_i^h))$ which are best reply to each other in the extensive form description of the uncertain recurring game defined by the prior Γ.

4 Social Learning

In this section, we consider a given uncertain Bayesian recurring bully game with a prior Γ and fixed strategies $((\eta_\theta^h), (\sigma_i^h))$. We first clarify some probabilistic issues.

A fully described *outcome* of the uncertain recurring game is a sequence of the type $\tau, \theta^1, p^1, \theta^2, p^2, \ldots$. Such an outcome generates progressive socially-observed histories $h^1, h^2 \ldots$ with $h^t = (p^1, \ldots, p^t)$. To describe the probability distribution on the set of all outcomes it suffices to define consistent probabilities for all initial segments of outcomes. We do so inductively by defining $P(\tau) = \Gamma(\tau)$ and

$$P(\tau, \theta^1, p^1, \ldots, \theta^{t+1}, p^{t+1}) = P(\tau, \theta^1, p^1, \ldots, \theta^t, p^t)\tau(\theta^{t+1})\mu(p^{t+1})$$

where $\mu(p^{t+1})$ is the probability of p^{t+1} under the strategies $((\eta_\theta^h), (\sigma_i^h))$ with $h = h^t$ and $\theta = \theta^{t+1}$.

Players, observing only the buildup of histories $h^1, h^2, \ldots$, do not know the chosen type generating distribution τ, but they can generate posterior probability distribution over it using the initial distribution and conditioning on current histories. Thus, their posteriors $\Gamma^0, \Gamma^1, \ldots$, are defined for every type generating distribution $\bar{\tau}$ by $\Gamma^0(\bar{\tau}) = \Gamma(\bar{\tau})$ and $\Gamma^t(\bar{\tau}) = P(\bar{\tau} \mid h^t)$.

Each such distribution over type generating distributions induces a direct distribution over types. Thus we have *updated posterior distributions on the next period type* denoted by $\gamma^0, \gamma^1, \ldots$, with $\gamma^t(\theta) = \sum_{\bar{\tau}} \bar{\tau}(\theta)\Gamma^t(\bar{\tau})$. Although we supress the notation, both Γ^t and γ^t are history dependent.

After every history h^t the bully, b^{h^t}, and the challengers, $(c_i^{h^t})$, play a Bayesian bully game. The bully knows his or her realized type, θ^{t+1}, and it is commonly assumed that he or she was drawn according to the distribution γ^t. If the original strategies $((\eta_\theta^h), (\sigma_i^h))$ constitute an uncertain Bayesian equilibrium of the recurring game, then $((\eta_\theta^h), (\sigma_i^h))$ with $h = h^t$ and $\theta = \theta^{t+1}$ constitute a Bayesian equilibrium of the stage Bayesian game with the prior γ^t.

Of course, the assumed prior, γ^t, against which individual challenger optimal strategies are chosen, is "wrong", since the real prior, by which θ^{t+1} is chosen, is the unknown τ. In Harsanyi's (1967–68) definition of a Bayesian equilibrium this presents no problem. The strategies, with the commonly known assumption that the prior is γ^t, are formally a Bayesian equilibrium. But in a recurring setting, when types are repeatedly drawn, a discrepancy between a real prior and an assumed prior may lead to statistical contradictions. These statistical discrepencies will disappear as players learn from observed histories.

To describe the effects of learning we proceed in two steps. Our first proposition states that players' updated distributions (over distributions) converge almost surely. Effectively, after some random time, players stop learning.

Proposition 1. *For almost every outcome, $\gamma^t(\theta)$ converges to a limit $\gamma(\theta)$ uniformly for all types $\theta \in \Theta$.*

Proposition 1 follows from the martingale convergence theorem. We refer readers to Jackson and Kalai (1995a) for a proof.

Our next proposition states that when players stop learning they have in fact learned all that they could, and so they are arbitrarily correctly predicting the play path after some random time. Proposition 2 can be proven using Proposition 1, or can be seen more directly as a consequence of Theorem 3 in Kalai and Lehrer (1993).[8]

With the fixed stage game strategies, let μ be the probability distribution induced on the stage game play paths by the real prior, τ, and let $\tilde{\mu}$ be the one induced by the assumed prior, γ^t. If for every play path $\mu(p^{t+1}) = \tilde{\mu}(p^{t+1})$ then no observable contradictions, even statistical ones can arise. We refer to the strategies and the assumed prior γ^t as an *empirically correct Bayesian equilibrium* whenever $\mu = \tilde{\mu}$. If for some $\epsilon > 0$ we have $| \mu(p^{t+1}) - \tilde{\mu}(p^{t+1}) | \leq \epsilon$ for all play paths p^{t+1}, we refer to it as an *empirically ϵ-correct Bayesian equilibrium.*

Proposition 2. *In an uncertain Bayesian equilibrium of an uncertain recurring bully game, for almost every outcome and for every $\epsilon > 0$, there exists a time T, such that γ^t, together with the induced $t+1$ period strategies, constitute an empirically ϵ-correct Bayesian equilibrium of the stage game for all $t \geq T$.*

The fact that late period stage-game players play an empirically ϵ-correct Bayesian equilibrium relative to their updated type generating distribution γ^t does not mean that they play a Bayesian equilibrium relative to the correct

[8] Theorem 3 in Kalai and Lehrer (1993) states that Bayesian updating of beliefs containing a "grain of truth" must eventually lead to correct predictions. The restriction in the current model, to a countable set of type generating distributions, implies that for almost every outcome $\Gamma(\tau) > 0$, which implies that the beliefs contain a grain of truth.

distribution τ. It only means that players' predictions concerning the play path are approximately correct. Players may be mistaken concerning off the equilibrium path behavior. This is illustrated in the following example.

Example

Consider three types of a bully, r, f, and a, described by their episode payoffs as follows. The *rational* type, r, has the original extensive game payoffs described earlier. The *fighting* type, f, has a payoff 1 for fighting and 0 for any other outcome (replacing the payoffs in Figure 1), while the *acquiescing* type, a, has a payoff of 1 for acquiescing and 0 for any other outcome. Let ρ, ψ and α, be the type generating distributions that select respectively with probability one the types r, f and a.

Suppose the prior Γ assigns high probability to a world with fighting bullies, i.e., $\Gamma(\psi)$ is high, and only a small probability to α and to ρ. Suppose the real type generating distribution was selected to be α, i.e. only acquiescing bullies are generated. The strategies with all challengers refrain, a and r type bullies always acquiesce, and f type bullies always fight, constitute an uncertain Bayesian equilibrium. This is so because the initial stage game prior γ^0 assigns high probability to a fighting bully and to refrain from challenging is therefore rational. But since there is no challenge in the first period the updated posterior on bully types remains unchanged, i.e. $\gamma^1 = \gamma^0$, the same logic applies to the second period, and so on. Moreover, the induced stage game Bayesian equilibria are empirically accurate. It is clear, however, that these strategies do not induce a Bayesian equilibrium of the stage game relative to the real prior α. In other words, if the challengers knew that their bully is drawn with probability one to be of the a type, then they would challenge.

5 Trembling Rational Players

The uncertain Bayesian equilibrium in the above example is highly unstable. If at any time, in the infinite play of the recurring game, a challenger trembles and challenges, the whole equilibrium will collapse after observing that the bully does not fight. The equilibrium is able to survive only because society never learns what bullies would do if challenged.

If a small amount of noise, in the form of trembles by players, is introduced, then more learning will occur and equilibria such as the one in the example above will eventually be overturned. In short, a small amount of imperfection will lead players to learn behavior at all nodes in the tree. While there are several ways to model such trembles, we follow Selten's (1975) approach by restricting the set of strategies that a player can choose.

Let q be a positive small number describing the probability of minimal trembles. The *uncertain Bayesian recurring bully game* with *q-trembling* is obtained from the usual uncertain recurring bully game by restricting the

players to the choice of behavior strategies that assign probability greater or equal to q to every action available in each information set.

Our next observation is that under q-trembling, if a Bayesian equilibrium is empirically ϵ-correct, then it must also be approximately correct for all conditional probabilities in the game. For example, the probability that the bully fights the i^{th} challenger, conditional on the event that the i^{th} challenger challenges, must be similar when computed by the learning induced distribution γ and by the true (realized) distribution τ.

To see this point, let $\tilde{\mu}$ be the distribution induced on play paths through the learning induced distribution γ, and let μ be the one obtained from the true (realized) distribution τ. The fact that $| \mu(p) - \tilde{\mu}(p) | \leq \epsilon$ for any play path p implies that this can be made (by starting with a smaller ϵ) true for any event (i.e. a set of play paths) in the game. The q-trembling property implies that there is a small positive number s such that the probability of any non empty event in the game is at least s. The conditional probability of event E_1 given E_2 is $P(E_1 \text{ and } E_2)/P(E_2)$. Since we have all denominators (over all possible E_2's) being uniformly bounded from zero, by making ϵ sufficiently small we can make all such ratios when computed by μ or by $\tilde{\mu}$ be within any given δ of each other. Thus we obtain the following.

Define $\tilde{\mu}$ to be *strongly empirically ϵ-correct* if for any two events E_1, and E_2 the conditional probabilities of E_1 given E_2, computed by μ and by $\tilde{\mu}$ are within ϵ of each other.

Lemma 3. *Consider any Bayesian equilibrium $((\eta_\theta),(\sigma_i))$ of a q-trembling Bayesian bully game with an assumed type generating distribution γ and a true type generating distribution τ. For every $\epsilon > 0$ there is a $\delta > 0$ such that if the equilibrium is empirically δ-correct it must be strongly empirically ϵ-correct.*

Notice that by combining the result of Proposition 2 with the above lemma, we can conclude that for almost every outcome of an uncertain Bayesian equilibrium of the recurring game, after a sufficiently long time, stage game players must be playing a strongly empirically ϵ-correct Bayesian equilibrium.

We consider now the special case, of an uncertain Bayesian equilibrium of the recurring game, where the true (realized) type generating distribution, ρ, generates the rational type r with probability one. The prior Γ over type generating distributions can be arbitrary, as long as $\Gamma(\rho) > 0$.

For a game with q-trembling, a bully stage strategy η is *essentially acquiescing* if its probability of fighting at every information set is the minimally possible level q, i.e. is only due to trembling. Similarly, a challenger i strategy is *essentially challenging* if the probability of refraining is q. We can now state the following result.

Proposition 4. *Consider an uncertain Bayesian equilibrium of an uncertain recurring bully game with q-trembles ($q > 0$). If the realized type generating*

distribution places weight one on rational players, then for small enough q ($1/2 > q$) and for almost every outcome there is a finite time T such that the t-period stage game strategies must be essentially acquiescing and essentially challenging for all $t \geq T$.

Given the previous propositions and lemma, the conclusion of this proposition is straightforward. For every $\epsilon > 0$ there is a sufficiently large T so that all the later stage games are played by strongly empirically ϵ-correct Bayesian equilibrium. So all we have to observe is that for sufficiently small ϵ, a strongly empirically ϵ-correct Bayesian equilibrium must consist of essentially acquiescing and essentially challenging strategies. This is done by a (backward) induction argument, outlined as follows.

In the last episode, following any play path, a rational bully must acquiesce with the highest possible conditional probability given the (positive probability) event that the last challenger challenges. Therefore the rational bully must acquiesce with probability $1 - q$ in the last episode. Thus, after time T (as defined in Lemma 3) the last episode challenger's assessed probability of the bully acquiescing in that episode is at least $1 - q - \epsilon$. Therefore, for sufficiently small ϵ and $q < 1/2$ the last challenger's unique best response is to challenge, and so he or she challenges with probability $1 - q$. Since this is true for any play path leading to the final episode, the assessed probability of challenge in the final episode is at least $1 - q - \epsilon$, indepedent of the play path leading to that episode. It follows that a rational bully in episode $L - 1$, will have a unique best response of acquiescing in that episode. The $L - 1$ period challenger, assessing this to be the case with probability at least $1 - q - \epsilon$, will essentially challenge, and so on.

6 Concluding Remarks

The KMRW phenomenon extends to a general folk theorem for finitely repeated games with incomplete information, as shown by Fudenberg and Maskin (1986). The failing of the phenomenon in a recurring setting has parallel implications for this folk theorem. Jackson and Kalai (1995ab) contains general results that have direct implications on this question. Again, we should emphasize that our results imply that under certain conditions players will learn to play as if they knew the realized type generating distribution. To the extent that there exists a true diversity of types in the population, this is learned and players will play accordingly. Thus, the variety of equilibrium outcomes allowed for by the folk theorem can still be realized, but will require a true diversity of types, rather than just a perceived one, in order to survive in a recurring setting.

Stronger, and even more striking, versions of the KMRW phenomenon might be possible in the manner described by Aumann (1992). This would involve higher order misconceptions on the part of players. For example,

replace the KMRW situation, where challengers are uncertain about whether the bully is rational or fight loving, by a situation where all challengers know that the bully is rational, but are uncertain about whether other challengers also know it. If, for instance, challengers believe that other challengers believe that there may be a fighting bully, then the KMRW results might be extended. This situation is incorporated into our model by using the type space to allow an explicit description of the beliefs a player holds about the beliefs of other players (and choosing τ's and Γ to reflect this uncertainty). Proposition 4 covers these cases and thus, if bullies are born rational, then this extended version of the KMRW equilibrium would also unravel in a recurring setting.

Getting back to Selten's paradox, it seems to become more severe in the recurring setting. A bully in later stages of the game who fights the first challenger can be explained as having trembled, and thus is not perceived as being irrational. Moreover, in the equilibrium play in late enough stages, even a bully who fights the first few challengers will be explained as having trembled several times, as this likelihood is larger than the alternative explanation of the bully being irrational. Although in late enough play it is relatively more likely that this behavior is due to trembles rather than irrationality, both of these events were very unlikely to start with. Thus, it may be that the challenger would prefer to doubt the model altogether (or believe that the bully has done so), rather than to ascribe probabilities according to it. Thus we are back at Selten's paradox.

Let us close with two comments relating to "technical" assumptions that we have maintained in our analysis. The restriction in this chapter to countably many types is convenient for mathematical exposition. The generalization to an uncountably infinite set (and an uncoutably infinite set of possible type generating distributions) requires additional conditions, in particular to assure that Proposition 2 extends. Lehrer and Smorodinsky (1994) offer general conditions which are useful in this direction.

One strong assumption we have made is that players start from a common prior over priors. This is not necessary for the results. The content of Propositions 1 and 2 can be applied to situations where each type of player has their own beliefs. The equilibrium convergence result then needs to be modified, since players' beliefs may converge at different rates. When these convergence rates are not uniform, then the conclusions are stated relative to a set of types which receives a probability arbitrarily close to one (under the realized distribution).

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Dumb Bugs vs. Bright Noncooperative Players: A Comparison

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Abstract. Consider a repeated bimatrix game. We define "bugs" as players whose "strategy" is to react myopically to whatever the opponent did on the previous iteration. We believe that in some contexts this is a more realistic model of behavior than the standard "supremely rational" noncooperative game player. We compare how bugs fare over a suitable "universe of games," as compared with standard "Nash," "maximin," and other types of players.

1. Background

In this paper we consider bimatrix games in which the two players are denoted as I and II. Both players have a finite number of (pure) strategies. Let $P_k(i,j)$ be the payoff to player k if Player I uses pure strategy i and Player II uses j $(k = 1,2)$. A pair (i^*, j^*) form a *pure strategy Nash equilibrium* (PSNE) if $\max_i P_1(i, j^*)$ is achieved when $i = i^*$ and also $\max_j\ P_2(i^*, j)$ is attained when $j = j^*$.

Suppose Player I has m pure strategies, and Player II has n. A *mixed strategy* for Player I is a probability vector $(p_1, ..., p_m)$ in which p_i represents the probability that he plays pure strategy i $(i = 1, ..., m)$. Similarly, a generic mixed strategy for Player II is given by an n-vector q. A *mixed strategy Nash equilibrium* (MSNE) is a pair (p^*, q^*) for which $p^* \in \arg\max_p \sum_{i,j} p_i q_j^* P_1(i,j)$, $q^* \in \arg\max_q \sum_{i,j} p_i^* q_j P_2(i,j)$, and either p^* or q^* is nonintegral. A *Nash Equilibrium* (NE) refers to a PSNE or MSNE.

Another way to play a game is to use *maximin strategies.* Mixed strategy vector p^* is a maximin strategy for Player I if it maximizes $\min_q \sum_{i,j} p_i q_j P_1(i,j)$ over all p. Similarly, a maximin strategy for II is any maximizer of $\min_p \sum_{i,j} p_i q_j P_2(i,j)$ over all q. A player's *maximin payoff* is his expected payoff when both players play a maximin strategy.[1]

[1] Note that under this definition, a player's maximin payoff may not be uniquely defined (if one or both players have multiple maximin strategies). However, using the randomly generated cell entries described in Section 4, we find that almost always it will be, and this will serve our purposes in that section.

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$$\begin{pmatrix} (4,1) & (0,0) & (2,2) \\ (0,0) & (9,9) & (0,0) \\ (1,3) & (0,0) & (3,1) \end{pmatrix}$$

Figure 1: A Bimatrix Game

Referring to the game in Figure 1, we may also illustrate the idea of optimal response. Suppose that the players were to select moves (strategies) in sequence. Given some move for one player, the other can calculate his optimal response. For example, if Player II assumes that Player I has chosen his first move, Player II's optimal response is to choose her third move. If Player I believes that Player II has chosen her second move then Player I's optimal response is to choose his second move. The optimal response calls for the simple minded "greedy" rule of selecting the largest item in a row or column.[2] Any PSNE in a bimatrix game has the "optimal response property," i.e., at the equilibrium the action of each is the optimal response to the action of the other.

Rather than view optimal response as a computational device, we may regard it as a description of the behavior of a primitive decisionmaker (or "bug") who learns nothing, but is able to perceive the individual worth of all outcomes in a row or column. Suppose initial strategies for the two players are given. The bug's first move will be to play an optimal response against the other player's initial strategy. In general, its move in the nth repetition of the game will be an optimal response against the opponent's move in the $n-1$st repetition. Hence the bug player is reacting myopically to the *last* move of his opponent, *without taking into account what that opponent might be doing currently.*

We may wish to distinguish between sequential and parallel (simultaneous) motion for our bugs. The spirit of the Nash equilibrium calls for alternate one-player adjustments (see Brown 1951); this gives rise to the notion of *sequential bugs.* When sequential bugs play a game, (after the initial strategies are determined) the choice to move is determined randomly, and then passed back and forth until possibly both players choose not to move. If this happens, they have arrived at a PSNE, and no further motion occurs. If this never happens, then the bugs will end up cycling (among four or more outcomes).

Another possibility is for the players to be *simultaneous bugs*, i.e., they move blindly and simultaneously. This has the feature that the cell to which

[2] There are several alternatives to the assumption that a player looks down a entire row or column of outcomes and picks out the best. For instance, we might restrict the individual to only choosing among immediate neighbors. [This implies that there is some form of metric which defines neighbors. In particular, this would mean that permuting rows or columns would change the game.] Alternatively we might regard any choice that leads to an improvement as equiprobable, etc.

they move may be different from what either expect (but neither is assumed to learn).

To clarify matters, consider the 3 by 3 example of Figure 1, and suppose the "initial strategies" are Player I's first strategy and Player II's second. If sequential bugs are playing this game, and Player I is determined to move first, then he changes to his second strategy, the PSNE payoff (9,9) is reached, and neither player has incentive to move thereafter. If Player II is determined to move first, she changes to her third strategy, obtaining payoff (2,2). Now Player I has incentive to change to his third strategy, obtaining (3,1), and then Player II changes to her first strategy, etc. Continuing this process, we see that ultimately the bugs will cycle through the four "corner" payoffs. We call this cycle a ***sequential bug cycle.***

On the other hand, if simultaneous bugs are playing the game, then Player I's first move (his second strategy) and Player II's first move (her third strategy) are played ***simultaneously,*** resulting in the (0,0) payoff on the right of the second row. From this point, Player I switches to his third strategy, and Player II to her second, yielding the (0,0) payoff in the bottom row. Again the result is a cycle, called a ***simultaneous bug cycle,*** this time among the four (0,0) payoffs in the bimatrix.

In this paper we consider several types of player. The first type is the standard *homo ludens*, or "Nash player," and is wedded to the need for consistent expectations. He is capable of computing equilibrium strategies, knows his and his competitor's payoff functions, and always plays "his part" of a Nash equilibrium. The second type is the somewhat primitive and unintelligent bug described above. The third type is intelligent but highly pessimistic. It believes that the optimal policy is to play maximin on the basis that it does not trust the other player to act rationally or nonmaliciously. The fourth type is a random player, or "Nature," which selects among its moves with equal probability. Finally, we also consider malicious "anti-players," each of whom try to minimize the payoff to the other.

2. Landscapes and Almost All Games

The 2×2 bimatrix game has been studied in some detail. There is a small, but useful literature on counting and characterizing matrix/bimatrix games. References include Rapoport, Guyer and Gordon (1972), O'Neill (1988) and Barany, Lee and Shubik (1992). The attraction of the 2×2 bimatrix game is basically that its major variants can be enumerated and studied exhaustively. Unfortunately the simplicity can also be misleading for the study of many questions we might wish to ask about general bimatrix games. In particular, because each row or column contains only two entries, the shape of the payoff surface is limited in complexity.

For the rest of this paper, we limit our analysis to the class of 2×2 games, but with some observations and qualifications for larger bimatrices.

If we regard the payoffs as being given in von Neumann Morgenstern utilities, then there are infinitely many games of any size. But these games may be classified into a finite set of classes by the ordinal aspects of the payoffs.[3]

We wish to obtain a reasonable measure for all bimatrix games of a given size and then study different forms of behavior over the various classes. For an $n \times n$ bimatrix we generate its payoffs by drawing $2n^2$ times from a rectangular distribution on the interval [0,1]. Topologically speaking, this gives us "almost all" $n \times n$ games.[4]

Limiting our concern at this point to the 2×2 bimatrix game, we draw from 8 i.i.d. uniform random variables to obtain the games and place them in the 78 ordinal categories, ignoring ties.[5]

If we consider large $n \times n$ bimatrices, we may observe that if we regard the n^2 payoffs to each player as describing a payoff surface, this surface may be arbitrarily rough. In biological applications and in military operations research involving duels and search problems, one may wish to consider a game between competitors played on an actual two dimensional surface. It is reasonably straightforward to show the relationship between matrix games and games played over a map.[6]

In many applications of game theory there is a special structure to the game and its payoffs. Sometimes (such as in the functioning of markets) moves may have specific physical meaning, and payoffs vary smoothly with small

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[3] We know that of the 576 ordinal 2×2 games, there are 78 classes of games, each strategically different. Of these 78 classes, 66 contain 8 members, and the other 12 contain only 4 members (due to an extra symmetry). Also, 58 of the classes are of games having a single NE that is pure, 9 having a single NE that is mixed, and 11 having three NEs, two pure and one mixed.

Finally, we note that there are several other ways one might classify all games, for example by their optimal response diagrams (i.e., their labelling functions), or by the structure of their payoff sets.

[4] "All" games would be generated from the open set $(-\infty, \infty)$, while we have generated the games from the closed set $[0, 1]$.

[5] If we permitted ties, there would be over 700 strategically different games to consider. But the games with ties will form a set of measure zero. In applications, it is dangerous to ignore ties, especially if they appear to be a fact of life and there is some feature (such as crudeness of perception) which is generating them. But if we are considering behavior over all possible games, the ignoring of ties can be justified as a close approximation.

[6] In particular, in some applications one may want the two dimensional surface to be the surface of a sphere. In wargaming, if a flat surface is used, the best regular paving is hexagonal. Limiting ourselves to bimatrix games, in order to avoid boundary problems it may be useful to regard the matrix as a torus where the top row is identified as a neighbor of the bottom row, and the left column is identified as a neighbor of the right column. A game with two players and k units of space where the payoff to each is a function of the land occupied by the player and whether it is occupied alone or by both players gives rise to a $k \times k$ bimatrix game.

changes in strategy. For example, the amount of profit a firm may make is usually a smooth function of its sales.

Thus, although it may be desirable to consider all games in some general enquiry, applications tend to be constrained to special subclasses.

3. Two Useful Benchmarks

Prior to considering the dynamics of the various players, we establish lower and upper bounds on the expected payoffs of players in the special case of the 2×2 bimatrix game where each of the eight payoffs is an iid random variable, uniformly distributed on the interval $[0, 1]$. We first consider two agents who play randomly, and then two who play completely cooperatively.

RANDOM PLAYERS: Recall that a "random" player is one who plays each of his pure strategies with equal probability. Hence, if both players play randomly every outcome is equiprobable. Since the expected value to each in a randomly selected cell is .5, the payoff from random play by all is (.5, .5).

JOINT MAXIMIZERS: To evaluate the yield from complete cooperation, we assume the players move so as to maximize the sum of their payoffs.[7] Consider a 2 by 2 matrix where the entry in cell i is a random variable X_i, (i = 1, 2, 3, 4), defined as the sum of two i.i.d. uniform random variables on [0,1]. The payoff for either player (assuming they cooperate so as to achieve the maximum X_i and then split the payoff) is one half the random variable Y, where $Y = \max(X_1, X_2, X_3, X_4)$. The expected value of Y is calculated as follows:

1. It is easy to see that $Pr(X_i \leq x)$ is equal to 0 if $x < 0$, $\frac{x^2}{2}$ if $x \in [0, 1]$, $1 - \frac{(2-x)^2}{2}$ if $x \in [1, 2]$, and 1 if $x > 2$.
2. Hence we have

$$Pr(Y \leq x) = \begin{cases} 0 \, , & \text{if} \quad x < 0 \, , \\ (\frac{x^2}{2})^4 = \frac{1}{16}x^8 \, , & \text{if} \quad x \in [0, 1] \, , \\ (1 - \frac{(2-x)^2}{2})^4 = (-\frac{1}{2}x^2 + 2x - 1)^4 \, , & \text{if} \quad x \in [1, 2] \, , \\ 1 \, , & \text{if} \quad x > 2 \, . \end{cases}$$

3. Since $Y \geq 0$, its expectation is given by $\int_0^\infty (1 - Pr(Y \leq x))dx$ (Hoel–Port–Stone 1971, p. 193). This is equal to $\int_0^1 (1 - \frac{1}{16}x^8)dx + \int_1^2 (1 - (-\frac{1}{2}x^2 + 2x - 1)^4)dx$, which is approximately 1.424.

[7] Hence, we are assuming *transferable utility*, i.e., that the players can transfer payoff from one to the other, and do so in a manner so as to equalize their payoffs. If we wish to consider the "nontransferable utility" case, we have a more complicated problem because there is not necessarily a unique optimal payoff in a game; instead there is in general a Pareto-optimal surface from which any point may be chosen.

4. Each player expects to gain half of this, or .712.

Our results from the "random case" and the "joint maximizer" case imply that nonmalicious players should expect to receive a payoff of somewhere between .5 and .712. However, we should note that it is possible for the players to lower their expected payoffs below .5. In fact, it is feasible (but rather unlikely) that they could collaborate to achieve their *minimal* expected joint payoff. A calculation symmetric to the one presented above gives minimum expected payoffs of (.288, .288).

4. A Comparison of Players

In this section we are concerned with comparing the expected performance of different players, playing a universe of games against a player of the same type. In all instances, we assume players are either not concerned with the type of "opponent" they face, or else they assume that opponent is like themselves. This assumption, in essence, indicates that the players do not learn from history.

In each instance it is important to be specific about the nature of the optimal response: does Player I or Player II move first, with information concerning the behavior of the other, or do they move simultaneously? When players of the same type are playing each other one can argue that if reaction is sequential, order does not matter as it will average out over all games.

4.1 Problems in Comparison

The noncooperative equilibrium solution offers no dynamics, whereas the bugs' optimal response procedure does. It is necessary to take this into account when trying to compare the solutions. One way to do this is to average over any cycle into which bugs may settle. If they eventually "fall into" a PSNE, they are credited with its value.

In nondegenerate cases, an $n \times n$ bimatrix game can have as many as n PSNEs, and we conjecture at most $2^n - 1$ NEs in total (see Quint–Shubik, 1994a). Hence, as the theory of Nash equilibrium provides for existence, but no selection,[8] we have several choices as to how to assign payoffs to "Nash players." If PSNEs exist, we can rule out all MSNEs and treat all PSNEs as equiprobable, or we can treat all equilibria as equiprobable. If (as some do) we have some reservations about MSNEs, we could give them a lesser weight than PSNEs. For our comparisons we treat all PSNEs as equiprobable and all

[8] Unless one wishes to follow the tracing procedure given by Harsanyi and Selten (1988). We do not do so here, but it might be of interest to evaluate the expected payoff from the equilibrium selected by tracing.

MSNEs as equiprobable, but the probability of a PSNE is not necessarily the same as the probability of a MSNE. We then consider the weighted average of the associated payoffs.

In the next section, we propose a dynamic process which produces the weighted average of NE payoffs outlined above.

4.2 Two Players of One Type vs. Two Players of Another Type

Consider a bimatrix game, in which player I is the row player and player II the column player, and where each player has two strategies. Hence the game has the following form:

$$\begin{pmatrix} (a,b) & (c,d) \\ (e,f) & (g,h) \end{pmatrix}$$

The game is to be played repeatedly. We consider five types of player:

1. A NASH PLAYER — this player always plays a (possibly mixed) strategy which is "his part" of a NE. If there are three NEs (= 2 pure and 1 mixed) in the game, then this player plays each pure with probability $\frac{\alpha}{2}$ and the mix with probability $1-\alpha$ ($\alpha \in [0,1]$).[9] If in this case he plays a particular NE and his opponent matches his expectations and plays the "other half" of it, the NE player responds to this by playing according to that NE forever. Otherwise, he re-randomizes his NE strategies on the next iteration.
2. A SEQUENTIAL BUG PLAYER — as described in Section 1.
3. A SIMULTANEOUS BUG PLAYER — as described in Section 1.
   We assume that the "initial strategies" (see Section 1) place the bugs into one of the four cells, each with probability $\frac{1}{4}$. In addition, for the sequential bugs, we assume either player "goes first" with probability $\frac{1}{2}$.
4. A MAXIMIN PLAYER — this player always plays his maximin strategy.
5. A RANDOM PLAYER — this player always plays the mixed strategy $(\frac{1}{2}, \frac{1}{2})$.

EXAMPLE 4.2.1: Consider the bimatrix game

$$\begin{pmatrix} (6,2) & (1,1) \\ (5,3) & (2,6) \end{pmatrix},$$

and let us assume the parameter α is equal to $\frac{1}{2}$.

Now, if two random players were to play this game, all of the outcomes would occur with probability $\frac{1}{4}$, and so the expected payoffs would be $(\frac{7}{2}, 3)$. In addition, we note that the maximin strategy for I is to play his second pure

[9] Hence, $\alpha = \frac{2}{3}$ represents the case where all three NE strategies are played with equal probability; $\alpha = 1$ represents the case where the PSNE strategies are each played with probability $\frac{1}{2}$ and the MSNE strategies are disregarded.

for Player I is again $\frac{1}{2}$. For games in N_1, one can see that the only labelling cycle (see Section 6 of Quint–Shubik–Yan, 1995a) will contain precisely one row and one column, and the row and column will be the ones defining the PSNE. Hence, we see that the bugs must end up at the PSNE.

The analysis of the previous section then gives expected value $\frac{2}{3}$ for this case.

Finally, for games in N_2, we see that with probability .25 the bugs end up in one NE, with probability .25 they end up in the other, and with probability .5 they end up cycling between the two other cells. Hence, if we assume (WLOG) that $a > e$, $g > c$, $b > d$, and $h > f$, the expectation in this case is $E[.25a + .25g + .5(\frac{c+e}{2})|a > e, g > c] = E[\frac{1}{4}(a + c + e + g)|a > e,\ g > c] = \frac{1}{2}$.

Putting this all together, we have $E(h^1) = \frac{1}{8}\frac{1}{2} + \frac{3}{4}\frac{2}{3} + \frac{1}{8}\frac{1}{2} = \frac{5}{8} = .625$.

4.2.5 Nash Players. Let us again use the same technique. For the analysis of games in N_0, let us assume WLOG that $a > e$, $d > b$, $g > c$, and $f > h$. The unique MSNE in this case is for Player I to play $p = (\frac{f-h}{f-h+d-b}, \frac{d-b}{f-h+d-b})$ and for Player II to play $q = (\frac{g-c}{a-e+g-c}, \frac{a-e}{a-e+g-c})$. Given a, b, c, d, e, f, g, and h, the expected payoff for Player I is then $pqa+p(1-q)c+(1-p)qe+(1-p)(1-q)g$, which turns out to be $\frac{ag-ce}{a-e+g-c}$. To find $E(h^1)$ over all games in N_0, we evaluate $E[\frac{ag-ce}{a-e+g-c}|a > e,\ g > c]$, which turns out to be $\frac{1}{2}$.[11]

For games in N_1, it is clear that the payoffs for the players are merely those of the PSNE. Since either type of bug attains the same result, we can use the results from the previous sections to state that the expected payoff in this case is $\frac{2}{3}$.

For games in N_2, assume WLOG that $a > e$, $b > d$, $g > c$, and $h > f$. We note that the probability is $\frac{\beta}{2}$ that either of the two PSNEs will occur, and $1-\beta$ for the MSNE, where $\beta = \frac{\alpha^2}{2-4\alpha+3\alpha^2}$.[12] Hence, Player I receives payoff a with probability $\frac{\beta}{2}$, g with probability $\frac{\beta}{2}$, and $\frac{ag-ce}{a-e+g-c}$ (same analysis as in the N_0 case) with probability $1-\beta$. In expectation this gives him $\frac{\beta}{2}\frac{2}{3} + \frac{\beta}{2}\frac{2}{3} + (1-\beta)\frac{1}{2}$, which is equal to $\frac{1}{2} + \frac{\beta}{6}$.

Putting this all together, we get $E(h^1) = \frac{1}{8}\frac{1}{2} + \frac{3}{4}\frac{2}{3} + \frac{1}{8}(\frac{1}{2} + \frac{\beta}{6})$, which is equal to $\frac{5}{8} + \frac{\beta}{48}$. Hence the Nash players' expected payoffs range between those for the simultaneous bugs ($\frac{5}{8}$ when $\beta = 0$) and those for the sequential bugs ($\frac{31}{48}$ when $\beta = 1$).

4.2.6 Maximin Players. To analyze the case of maximin players, we will use the same general procedure as above, except, instead of "partitioning"[13]

[11] This is a rather complicated quadruple integral, which we evaluated both by longhand and by using Mathematica.

[12] In Example 4.2.1, where $\alpha = \frac{1}{2}$, β was equal to $\frac{1}{3}$.

[13] We did not formally partition Ω, because there is a set of measure zero consisting of games with an infinite number of NEs. The same comment applies to the maximin analysis, where we disregard cases where players have more than one maximin strategy.

$\mu(N_0) = \frac{1}{8}$, $\mu(N_1) = \frac{3}{4}$, *and* $\mu(N_2) = \frac{1}{8}$.

LEMMA 4.2: *Let Ω and μ be defined as above. Let MM_S^1 be the subset of Ω in which Player I has a saddlepoint at a, c, e, or g (i.e., $e < a < c$, $g < c < a$, $a < e < g$, or $c < g < e$ respectively). Similarly, let MM_S^2 be the subset of Ω in which Player II has a saddlepoint at b, d, f, or h (i.e., $d < b < f$, $b < d < h$, $h < f < b$, or $f < h < d$ respectively).*[10] *Then $\mu(MM_S^1) = \mu(MM_S^2) = \frac{2}{3}$.*

4.2.1 Joint Maximizers. If two "joint maximizers" are playing the game, the calculation presented in Section 3 says that the expected payoff for each player is approximately .712.

4.2.2 Random Players. If two "random players" are playing the game, the calculation presented in Section 3 says that the expected payoff for each player is exactly .5.

4.2.3 Sequential Bugs. To find the expected payoff to a pair of sequential bugs playing the game, let us consider the (expected) payoffs h^1 for Player I. [By symmetry, the expected payoff for II will be the same.] We consider in turn games in N_0, N_1, and N_2 respectively.

Given a game is in N_0, WLOG we may assume $a > e$, $d > b$, $g > c$, and $d > h$. In this case the bugs will cycle around the four outcomes, giving an average payoff of $\frac{a+c+e+g}{4}$ to Player I. Hence the expected payoff, given the game is an element of N_0, is $E[\frac{a+c+e+g}{4}|a > e, g > c]$. By symmetry, this is .5.

Given a game is in N_1, Corollary 6.4 of Quint–Shubik–Yan (1995a) implies that the sequential bugs attain the PSNE payoffs. WLOG assume (a, b) is the PSNE payoff; hence $a > e$ and $b > d$. In addition, either $c > g$ or $f > h$. Either way, the expected payoff to I is $E[a|a > e]$, which turns out to be $\frac{2}{3}$.

Finally, consider the case where the game is in N_2. WLOG, $a > e$, $b > d$, $g > c$, and $h > f$. Since we assume a) the bugs "start" in any of the four cells with equal probability, and b) either player moves first with probability .5, symmetry dictates that the bugs are equally likely to end up in either PSNE. Hence the expected payoff for Player I is $E[\frac{a+g}{2}|a > e, g > c] = \frac{2}{3}$.

Putting all of this together gives that $E(h^1) = \sum_{k=0}^{2} E(h^1|$ game in $N_k)\mu(N_k) = \frac{1}{2}\frac{1}{8} + \frac{2}{3}\frac{3}{4} + \frac{2}{3}\frac{1}{8} = \frac{31}{48} = .646$.

4.2.4 Simultaneous Bugs. For simultaneous bugs, we use a similar procedure.

For games in N_0, it is easy to see that a pair of simultaneous bugs will act exactly as do a pair of sequential bugs, and hence the expected payoff

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[10] The significance here is that if a player has a saddlepoint, his maximin strategy will be to play the pure strategy corresponding to that saddlepoint.

strategy, while that for II is to play his first, so that the payoff for maximin players is $(5,3)$.

Next, let us do the analysis for the case of two sequential bugs playing the game. In this case, all is dependent on the initial conditions. If the game "starts" in the upper left cell or the bottom right cell, neither bug will ever move, so the payoffs are in fact those in the cells. If the game begins in the upper right cell with bug player I to move, he will play his second pure strategy, giving payoff $(2,6)$, and then neither bug has incentive to move. Similarly, if the game were to start in the lower left cell, I would play his first pure strategy, and no further moves would be made. A similar analysis would hold if player II were to make the first move. All in all, there is a 50% chance that payoffs $(6,2)$ would be obtained forever, and a 50% chance for $(2,6)$. Expected payoff: $(4,4)$.

The simultaneous bug analysis is the same in the cases where the game begins in the upper left or bottom right cell. However, if the game were to begin in either of the other two cells, *both* players would then move, and the result would be endless cycling between the outcomes $(1,1)$ and $(5,3)$. Hence, the expected payoffs for the simultaneous bugs are $\frac{1}{4}(6,2) + \frac{1}{4}(2,6) + \frac{1}{2}\frac{(1,1)+(5,3)}{2}$, which is equal to $(\frac{7}{2}, 3)$.

Finally, for the NE player-NE player analysis, we note that there are two PSNEs in the game, paying off $(6,2)$ and $(2,6)$. In addition, there is a MSNE which pays off $(\frac{7}{2}, \frac{9}{4})$. Since $\alpha = \frac{1}{2}$, this means that when two NE players play this game, the chances are $\frac{1}{16}$ that they both will play their first pure strategy, $\frac{1}{16}$ that they will both play their second pure strategy, and $\frac{1}{4}$ that they will both play their MSNE strategy. [The other $\frac{5}{8}$ of the time, they will be playing NE strategies that don't "match."] This 1:1:4 ratio means that in the long run, they will attain either PSNE payoff with probability $\frac{1}{6}$, and the MSNE payoff with probability $\frac{2}{3}$, giving expected longrun payoff $(\frac{22}{6}, \frac{17}{6})$.

Comparing the payoffs for Player I, we see that the maximin game is best for him, followed by the sequential bugs game, the NE–NE game, and finally the simultaneous bugs and random players game are tied. For II, we see that the sequential bugs game is best, the NE–NE game is worst, and the other three games are all tied.

To compare how the various types of players fare *over a suitable universe of games*, let us assume that the eight payoffs $a, \ldots, h$ are each i.i.d. U[0,1] random variables. Define Ω as the set of all joint outcomes $(a, \ldots, h)$. Let μ be the probability measure defined on the set of subsets of Ω, defined in the natural way with $\mu(\Omega) = 1$.

Our aim is to calculate the expected payoff for each type of player over the above universe (Ω). To this end, we will need the following simple lemmas:

LEMMA 4.1: *Let Ω and μ be defined as above. Let N_0, N_1, N_2 be the subsets of Ω consisting of those games with 0, 1, and 2* PSNEs *respectively. Then*

Ω into the three sets N_0, N_1, and N_2, we will break it up into the following four sets:

1. Games where both players have a saddlepoint, and Player II's saddlepoint is in the same column as Player I's. Using Lemma 4.2, the measure of this set is $\frac{2}{9}$.
2. Games where both players have a saddlepoint, but Player II's saddlepoint is not in the same column as Player I's ($\mu = \frac{2}{9}$).
3. Games where Player I has a saddlepoint but Player II does not ($\mu = \frac{2}{9}$).
4. Games where Player I has no saddlepoint ($\mu = \frac{1}{3}$).

The Analysis.

1. Suppose in this case WLOG that Player I's saddlepoint is a (so $e < a < c$) and so Player II's is in the first column. Then I's maximin payoff is a, and his expected payoff in this case is $E[a|e < a < c] = \frac{1}{2}$.
2. Suppose in this case WLOG that Players I's saddlepoint is a (so $e < a < c$) and so Player II's is in the second column. Then I's maximin payoff is c, and his expected payoff in this case is $E[c|e < a < c] = 6\int_0^1 \int_e^1 \int_a^1 cdcdade = \frac{3}{4}$.
3. In this case we again assume I's saddlepoint is a, and that $h > b > d > f$. Then I plays $p = (1, 0)$, and II plays $q = (\frac{h-d}{h-d+b-f}, \frac{b-f}{h-d+b-f})$. Hence I's expected payoff is $\frac{h-d}{h-d+b-f}a + \frac{b-f}{h-d+b-f}c$. Since a, c, e, and g are by assumption independent of b, d, f, and h, we get that $E(h^1)$ in this case is equal to $E[a|e < a < c]E[\frac{h-d}{h-d+b-f}|h > b > d > f] + E[c|e < a < c]E[\frac{b-f}{h-d+b-f}|h > b > d > f]$. This is equal to $\frac{1}{2}\frac{1}{2} + \frac{3}{4}\frac{1}{2} = \frac{5}{8}$.
4. For this case, WLOG assume $a > g > c > e$. Then I's maximin strategy is $p = (\frac{g-e}{g-e+a-c}, \frac{a-c}{g-e+a-c})$. Suppose II's maximin strategy is $(x, 1-x)$. Then I's expected payoff is $\frac{g-e}{g-e+a-c}xa + \frac{g-e}{g-e+a-c}(1-x)c + \frac{a-c}{g-e+a-c}xe + \frac{a-c}{g-e+a-c}(1-x)g$, which is equal to $\frac{ag-ce}{g-e+a-c}$. Hence, I's expected payoff in this case is $E[\frac{ag-ce}{g-e+a-c}|a > g > c > e]$, which is equal to $\frac{1}{2}$.[14]

Finally putting this all together, we find that Player I's expected payoff for the entire game is $\frac{2}{9}\frac{1}{2} + \frac{2}{9}\frac{3}{4} + \frac{2}{9}\frac{5}{8} + \frac{1}{3}\frac{1}{2} = \frac{7}{12}$, or approximately .583.

4.3 "Anti-Players"

Although we might consider random behavior as providing a lower bound on payoffs, we could consider malicious, "looking-glass" versions of our players. For instance, we might consider a (sequential or simultaneous) "anti-bug," which always moves to a row (or column) which minimizes his opponent's payoff (instead of maximizing his own). Or, we might consider an "anti-maximin

[14] Again we calculated this integral using both longhand and Mathematica.

player," whose *modus operandi* is to minimize his opponent's maximum payoff. Finally, we can define a "anti-Nash player" similarly. Given the generation of payoffs described in Section 4.2, the following Theorem is obvious:

THEOREM 4.3: *The payoffs to two Anti-Bug players (resp., two anti-maximin players, two anti-Nash players) evaluated over all environments are one minus the payoffs to two Bug players (resp., two maximin players, two Nash players).*

4.4 A Summary of Expected Payoffs for Different Types of Players

Putting together our results from subsections 4.2 and 4.3, we may state a "rank ordering" on all types. The ordering[15,16] is as follows:

1. Cooperative joint maximization (.712)
2. Sequential bugs (.646)
3. Noncooperative players (between .625 and .646)
4. Simultaneous bugs (.625)
5. Maximin players (.583)
6. Random players (.5)
7. Anti-maximin players (.417)
8. Simultaneous Anti-bugs (.375)
9. Anti-Nash players (between .354 and .375)
10. Sequential Anti-bugs (.354)
11. Cooperative joint minimization (.288).

5. A Final Word

In a separate paper, we explore the types of movement that can occur by the two types of bugs in a bimatrix game. Specifically we ask, "What is the longest possible cycle on which the bugs can embark?", "How many cycles of what length can there be?", and "How long can the bugs move before entering a cycle or PSNE?" See Quint–Shubik–Yan (1995a) or Quint–Shubik–Yan (1995b).

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[15] In Quint–Shubik (1994b), the authors ranked the player types according to the criterion: "type x" ranks higher than "type y" if the measure of the subset of Ω in which the "type x" player I does better than the "type y" Player I is larger than vice versa. Not only did this produce a complete ordering of the player types, but the ordering is exactly the same as we obtain here.

[16] In section 4.2 our concern was with the performance of players facing an opponent of the same type. We have not covered the "mixed cases," where a player of one type confronts a player of a different type.

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Communication Effort in Teams and in Games

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Abstract. Quick and cheap communication networks may strengthen the case for the dispersal of organizations from one central site to many local ones. The case may be weakened if the dispersed persons behaved like members of a team before the dispersal but become self- interested players of a game after dispersal. To obtain some insight into these matters, the paper explores one very simple class of models. Each of N persons observes a signal that partly identifies the "week's" (finite) game and then sends a chosen action (strategy) to a Center. Following the receipt of all N chosen actions, the Center puts them into force. Each person's net payoff for the weak is the gross payoff determined for the week's game minus a communication cost. Person i's cost depends on the probabilities with which i sends alternative messages to the Center. We use the "information content" measure of classic Information Theory, multiplied by θ, the time required to transmit one bit. We study the "rule game" in which i chooses a rule that assigns a probability mixture over actions to each of i's signals; in that game i's payoff is i's average weekly gross payoff minus i's communication cost. We find that (i) if an equilibrium N-tuple of rules exists, then it has to be an N- tuple in which true mixtures are not used; (ii) the existence of an equilibrium N-tuple is not guaranteed and the usual appeal to mixing over the possible N-tuples does not help, since such mixing has no meaning in our setting; (iii) even under the strong assumption that the gross payoff is identical for every person, there may be equilibria in which the dispersed persons communicate "too little" (from the team point of view) as well as equilibria in which they communicate "too much"; (iv) for very small θ and very large θ, equilibria exist in the identical-gross-payoff case. A great deal remains to be learned.

1. Introduction

It is widely claimed that the revolution in high-speed communication networks will lead organizations to disperse. An organization which previously operated at a single site, where its members were sufficiently controlled to behave essentially as a team, will now find it advantageous to disperse its members among local sites and to let them communicate via the network. If local sites are cheap compared to a large central site, then abandoning the central site and dispersing to local sites may be good for the organization — even if the tasks the organization wishes each member to perform remain unchanged.

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But whether dispersal is desirable depends on the way the members use the communication network. If the dispersed members continue to behave like a team, then they will impose the "correct" demands on the network. Deciding what to communicate, and then sending it, uses up members' time. The more time the members spend observing their private environments and deciding what to communicate, the better the actions which the organization takes when they are finished. But during the time they spend generating new actions, obsolete actions — or no actions — are in force. A balance has to be struck between the improved payoff due to better actions and the low payoff that is earned while the better actions are being found.

Even if the delay due to communication is not costly, the act of communication (deciding what to send and then sending it) still requires effort, and that effort has to be paid for. A balance then has to be struck between costly communication effort and the improved payoff due to the better actions that more effort produces.

If the dispersed players continue to behave like a team, then they will find the balance that is correct from the team point of view, whether it is delay or effort that has to be weighed against payoff. The difficulty is that once members disperse and leave the controls of the central site behind them, they may start to pursue their self-interest instead of sharing a common goal. They may start to behave, in other words, like players in a game and not members of a team. Will they then choose the "correct" balance between communication effort and payoff improvement — or will they make some other choice? If the equilibrium choice is the wrong one (from the organization's viewpoint) then the benefits of dispersal may be an illusion: while the network is cheap and the abandonment of the central site indeed saves money, the organization may have to undertake serious control expenses if it wants the dispersed members to make good choices. Consequently the case for dispersal may be a weak one. When is there an equilibrium in which the dispersed members communicate too little? When is there an equilibrium in which they communicate too much? If delay is costly, or if effort depends on how fast messages can be sent, then when are the equilibria sensitive to the speed of communication?

One might study a variety of models to obtain some insight into these questions. In this paper we explore, in Section 2, just one class of models. Each person communicates a choice of action to a Center, and the resulting communication effort is simply subtracted from the sender's gross payoff. Special as they are, the models present a number of challenges. Section 3 describes, but does not study, a class of models in which the cost incurred by communication effort depends on the delay that communication causes. Concluding remarks are made in Section 4.

2. A Class of Models in which Communication Effort is Costly

Consider an N-person organization. Every week each person chooses an action. The N choices determine, for each person, a payoff that will be earned that week. To start with, the organization is not dispersed and behaves like a team. It wants the *sum* of the N payoffs to be large, given the communication effort that it makes, and it seeks a good balance between the sum of the N payoffs and the organization's total communication effort. After dispersing, the organization becomes a collection of selfish individuals. Each wants his individual payoff to be large and seeks a good balance between his individual payoff and his individual communication effort. In either case, we begin our description of the typical week by specifying a game g belonging to a finite set G of possible games, which we shall call the *gross-payoff games.*

In the game g each person i has a finite set[1] A^i of possible actions. Let A^* denote $A^1 \times \cdots \times A^N$. If the action N-tuple $(a^1, \ldots, a^N) \in A^*$ is chosen by the N persons, then, in the game g, Person $i, i = 1, \ldots, N$, receives the (gross) payoff $B_g^i(a^1, \ldots, a^N) \in I\!R$.

We shall suppose that as soon as each person has decided on his action for the week, he communicates that choice to a Center. That happens whether or not the organization is dispersed. The Center is not a player but rather a coordinator. He does not know what the week's game is. His function is to insure that the chosen actions are carried out.

We assume that each week's game is drawn from the same population of possible games; this week's drawing is independent of previous drawings. At the start of each week, Person i instantly observes a *signal* w^i that tells i something about the new game g. Each person's signal is independent of every other person's signal. For each person i there is a finite set W^i of possible signals. For generality we shall let Person i respond to the week's new signal by choosing a probability mixture over his $|A^i|$ possible actions.

Specifically, Person i uses a *rule* $p^i : W^i \to P^{|A^i|}$, where $P^{|A^i|}$ denotes the probability simplex in $|A^i|$-space, i.e., the set $\{(p_1, \ldots, p_{|A^i|}) \mid p_a \geq 0, a = 1, \ldots, |A|; \sum_{a=1}^{|A^i|} p_a = 1\}$. Observing the signal $w^i \in W^i$, and using the rule p^i, Person i chooses the action $a \in A^i$ with probability $p_a^i(w^i)$. Person i then tells the Center what his chosen action is.

Given the rule N-tuple $(p^1, \ldots, p^N)$, Person i obtains the *average weekly gross payoff*

$$\pi^i(p^1, \ldots, p^N) = \sum_{\substack{g \in G \\ w^i \in W^i \\ (a^1, \ldots, a^N) \in A^*}} \left(\prod_{1=1}^{N} z_{gw^i}^i p_a^i(w^i) \right) \cdot B_g^i(a^1, \ldots, a^N),$$

[1] In what follows, persons will always be identified by superscripts .

where $z^i_{gw^i}$ is the joint probability that the game is $g \in G$ and i's signal is $w^i \in W^i$. All the joint probabilities are common knowledge.

Now suppose that we measure i's *average weekly communication effort* by some real-valued function J^i defined on the possible rules p^i. (We defer for a moment a discussion of the particular function we shall be using). Then i's *average weekly net payoff* for the rule N-tuple $(p^1, \ldots, p^N)$ is

$$V^i(p^1, \ldots, p^N) = \pi^i(p^1, \ldots, p^N) - J^i(p^i).$$

The game in which Person i's strategy space is the set of rules and the payoffs are given by $V^1, \ldots, V^N$ will be called the *rule game.* It will be useful to define

$$V(p^1, \ldots, p^N) \equiv \sum_{i=1}^{N} V^i(p^1, \ldots, p^N).$$

If the organization is a team, then it uses an N-tuple $(p^1, \ldots, p^N)$ that maximizes V. Our central questions will be these: *When is there a Nash equilibrium of the rule game in which there is "too little" communication — in which the total communication $\sum_i J^i$ is less than it is in the team-optimal solution? When is there a Nash equilibrium in which there is "too much" communication?* The answers will depend on the gross-payoff games and on the communication cost measures J^i.

Consider the special case in which π^i can be written as $\tilde{\pi}^i(J^1(p^1), \ldots, J^N(p^N))$. Here we may view the rule game as a *communication-effort game*, wherein each Person i chooses a communication effort η^i and collects the net payoff $\tilde{\pi}^i(\eta^1, \ldots, \eta^N) - \eta^i$. We are then in familiar territory. As first noted by Holmstrom (Holstrom 1983), there is no Nash equilibrium in which the chosen efforts are team-optimal. Informally speaking, in a Nash equilibrium of the effort game, Person i equates the marginal change in $\tilde{\pi}^i$ (as i varies his effort) to one, whereas Person i "should" equate the marginal change in $\sum \tilde{\pi}^i$ to one. Moreover, if, for every (i, j), $\tilde{\pi}^i$ is increasing in η^j, then in a Nash equilibrium Person i undervalues the marginal contribution of his communication effort to the gross team payoff $\sum \tilde{\pi}^i$ and *too little* communication effort occurs.

It is possible, however (as Holmstrom noted), to shift the incentives towards team-optimal effort by modifying the effort game. If person i receives $\tilde{\pi}^i - \eta^i$ when the entire effort vector chosen by the N persons is team-optimal, but receives $-\eta^i$ otherwise, then a team-optimal vector is a Nash equilibrium. But that version of the effort game would seem to require a drastic reshaping of our organization. After the organization disperses, we imagine individual payoff to be determined following each person's action choice in exactly the same way as it was before dispersal. In the modified game, by contrast, some sort of policing agency would have to determine whether the chosen effort N-tuple is in fact the "correct" one, and would then have to deny everyone any part of the gross payoff if the N-tuple is wrong.

For the communication-effort measure we shall use, it will not be possible to write π^i in the form $\tilde{\pi}^i$. We are, in fact, in quite new territory.

We take our measure of individual communication effort from classic Information Theory. We view Person i, at the start of a week, as a source of $|A^i|$ possible messages to the Center. Given that i uses the rule $p^i(\ )$, our measure is

$$\theta \cdot (\text{ the information content of } p^i),$$

where $\theta \geq 0$ and the information content of p^i is

$$I^i(p^i) \equiv - \sum_{a^i \in A^i} (\text{ probability that } i \text{ sends } a^i)\cdot \log_2(\text{ probability that } i \text{ sends } a^i).$$

I^i reaches its maximum when all the probabilities are equal and its minimum when one of them is one while the others are zero. The probability that Person i sends the action a^i to the Center is

$$\sum_{g \in G} \sum_{w^i \in W^i} z^i_{gw^i} p^i_{a^i}(w^i).$$

We shall refer to the game in which each person i's payoff is $\pi^i(p^1, \ldots, p^N) - \theta I^i(p^i)$ as *the rule game for θ*.

The term "information content", as just defined, is widely used in Information Theory. To obtain one interpretation of $I^i(p^i)$ and of $\theta I^i(p^i)$, first note that some of i's action messages will be sent more frequently than others. Imagine i to code each of the $|A^i|$ possible messages as a binary string, in such a way that the Center can decode each string without ambiguity; the Center can distinguish any string from its predecessor. Suppose that the transmission of one bit requires time θ and that all other elements of the act of communication are instantaneous. Then, using an appropriate coding, the average weekly length of the binary string that i sends is between $I^i(p^i)$ and $I^i(p^i) + 1$. So the average weekly time that Person i, or someone in i's employ, has to spend in transmitting the week's binary string is between $\theta I^i(p^i)$ and $\theta I^i(p^i) + \theta$.[2] We suppose that the transmitter has to be paid for his time, and that the payment comes out of i's "pocket"and has to be subtracted from π^i to obtain i's net payoff. The case $\theta = 0$ (instantaneous transmission) will be an important benchmark. In the rule game for $\theta = 0$, i's payoff is just $\pi^i(p^1, \ldots, p^N)$.

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[2] See, for example, Abramson (1963), Chapter 4. To obtain another interpretation of $I^i(p^i)$, imagine that each week the organization draws not once but K times from the population of possible games and their accompanying signals. The K games then take place simultaneously during the week. Person i sees one signal for each of the K games and chooses an action for each game in response to that game's signal. Since all K games and their accompanying signals are drawn independently from the same population of games and signals, Person i will use a common rule p^i for each of the K games. Then as K increases, the average length per game of the weekly message i sends to the Center approaches $I^i(p^i)$.

Note that I^i is continuous and strictly concave in p^i. Those will be the two key properties of I^i that we shall use. The results we obtain and the questions we raise will be the same for any communication-effort measure that depends on i's message probabilities and has those two properties.

We would argue that probability-sensitive communication-effort measures have to be serious candidates in modelling organizations. One could certainly consider a measure that does not depend on message probabilities — for example, the total number of possible messages that i can send. Such a measure ignores the saving in average communication effort that coding permits. Once we choose to recognize that saving — and once we note that equal message probabilities are the most demanding case, while having just one possible message is the least demanding — we are driven to measures that depend on the probabilities. Among possible measures of that sort the classic I^i seems the strongest candidate.[3]

We now establish an important fact about the rule game for $\theta > 0$. First define

$$\Gamma^i \equiv \underbrace{P^{A^i} \times \cdots \times P^{A^i}}_{|W^i| \text{ times}}.$$

We have

Lemma 1 *For any $\theta \geq 0$: (i) The rule game has a finite number (possibly zero) of Nash equilibria; if $(\tilde{p}^1, \ldots, \tilde{p}^N)$ is a Nash equilibrium of the rule game, then for every i and every $w^i \in W^i$, $\tilde{p}^i(w^i)$ is not a true mixture (all components of $\tilde{p}^i(w^i)$ are zero except one). (ii) There exists a team-optimal rule N-tuple (a maximizer of $V = \sum_{i=1}^{N} V^i$); the number of optimal rule N-tuples is finite; if $(\bar{p}^1, \ldots, \bar{p}^N)$ is team-optimal, then for every i and every $w^i \in W^i$, $\bar{p}^i(w^i)$ is not a true mixture.*

Proof: For both parts of the Lemma, we note that a strategy for Person i in the rule game is a function from the finite set W^i to the compact convex set $P^{|A|^i}$, and is therefore a point in Γ^i, which is also a compact convex set and has a finite number of extreme points. Thus a rule N-tuple is a point in the set $\Gamma^1 \times \cdots \times \Gamma^N$; that set is again compact and has a finite number of extreme points.

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[3] When i scans his current signal, chooses an action in response, encodes it as a binary string, and then transmits it, he is, of course, engaging in time-consuming efforts other than transmission itself. But one can argue that I^i says something about those efforts as well. Suppose, for example, that i uses just two possible messages in response to his signal. Compare the case where the first message is used far more frequently than the second with the case where they are used equally often. In the first case, it may be easier for i to scan the current signal, since the signals leading to the second message are rare and stand out prominently. The difference in the signal-scanning effort between the two cases is then properly reflected in I^i. Modellers of information-processing efforts are very far indeed from a consensus about a winning approach. We argue that the classic measure fits our setting quite well.

(i) Consider Person 1. The rule $\tilde{p}^1$ maximizes $V^1(p^1, \tilde{p}^2, \ldots, \tilde{p}^N)$ on Γ^1. But $V^1(p^1, \tilde{p}^2, \ldots, \tilde{p}^N)$ is strictly convex in p^1, since $\pi^1(p^1, \tilde{p}^2, \ldots, \tilde{p}^N)$ is linear in p^1 and $I^1(p^1)$ is strictly concave in p^1. That means[4] that $\tilde{p}^1$ is an extreme point of Γ^1, so that for any $w^1 \in W^1$, $\tilde{p}^1(w^1)$ is not a true mixture. Similarly for Persons $2, \ldots, N$. Thus a Nash equilibrium is an extreme point of $\Gamma^1 \times \cdots \times \Gamma^N$. Since that set has a finite number of extreme points, there is a finite number (possibly zero) of Nash equilibria.

(ii) The function $V = \sum_i V^i$ is defined on $\Gamma^1 \times \cdots \times \Gamma^N$ and is strictly convex. Hence some extreme point of $\Gamma^1 \times \cdots \times \Gamma^N$, say $\bar{p}^1, \ldots, \bar{p}^N$, maximizes $\sum_i V^i$ and is a team-optimal rule N-tuple. Moreover every team-optimal rule N-tuple is an extreme point, so there is a finite number of team-optimal N-tuples. Now consider the game in which each person i chooses a p^i and i collects the payoff $\frac{1}{N} V(p^1, \ldots, p^N)$. A team-optimal N-tuple of the rule game must be an equilibrium of the new game. Using the argument in Part (i), we conclude that for any i and any $w^i \in W^i, p^i(w^i)$ is not a true mixture. ∎

Now define

$$E \equiv \{(p^1, \ldots, p^N) \mid p^i \text{ is an extreme point of } \Gamma^i, i = 1, \ldots, N\}.$$

The set E is also the set of extreme points of $\Gamma^1 \times \cdots \times \Gamma^N$. It is finite. The Lemma establishes, for any $\theta > 0$, that

- If there is a rule N-tuple that is a Nash equilibrium of the rule game for θ, then it belongs to E.
- Some member of E is team-optimal in the rule game for θ and every team optimum belongs to E.

Without imposing further conditions, we cannot guarantee that a rule N-tuple which is a Nash equilibrium exists. Theorems on the existence of equilibria when strategy spaces are continua do guarantee the existence of *mixtures of rule N-tuples* with the equilibrium property.[5] But such a mixture, if it is not degenerate, ***has no meaning in our setting.*** If it is not degenerate, such a mixture will require some Person i to mix over two or more rules — for example, to use p^i with probability $\frac{1}{2}$ and p^{*^i} with probability $\frac{1}{2}$. But the resulting expected value of V^i does not properly express i's net payoff after allowing for communication effort. In the expected value of V^i the term $\frac{1}{2} I^i(p^1) + \frac{1}{2} I^i(p^{*^i})$ appears, and that term does *not* equal $-\sum_{a^i \in A^i} [(\text{ probability of sending } a^i) \cdot (\log \text{ probability of sending } a^i)]$.

2.1 Three Examples

We shall consider three two-person examples. In the first two examples, each person has just two possible actions.

[4] See, for example, Luenberger (1973)

[5] See, for example, Fudenberg and Tirole (1992), Theorem 3.1.

In the first example there are four possible gross-payoff games, labelled **TL**, **BR**, **BL**, and **TR**. "T" and "B" stand for Top and Bottom; "L" and "R" stand for Left and Right. Each game occurs with probability $\frac{1}{4}$. Person 1's signal is the first term of the identifying label, and Person 2's signal is the second term. Each of the four gross-payoff games has the strong property that *the two persons receive the same payoff.* We shall call such games *identical-payoff games.* The four identical-payoff games are shown below. Person 1's actions are Top, Bottom and Person 2's actions are Left, Right. If each person chooses the action identified by his signal, then each receives 4. If only one does so, then each receives $4-4\epsilon$. If neither does so, then each receives zero. The common payoff received by each person is shown in each box.

game **TL**

	Left	Right
Top	4	$4-4\epsilon$
Bottom	$4-4\epsilon$	0

game **TR**

	Left	Right
Top	$4-4\epsilon$	4
Bottom	0	$4-4\epsilon$

game **BL**

	Left	Right
Top	$4-4\epsilon$	0
Bottom	4	$4-4\epsilon$

game **BR**

	Left	Right
Top	0	$4-4\epsilon$
Bottom	$4-4\epsilon$	4

We assume $0 < \epsilon < \frac{1}{2}$. From the team's point of view each person ought to take the action identified by his signal.

In view of Lemma 1 we can confine attention to "pure" rule pairs, in which each player chooses one action in response to each signal. In the rule game each person has four possible pure rules. Two of them are constant-action rules, with information content zero. In the other two, a different action is sent to the Center for each signal and the information content is $-(\frac{1}{2} \cdot \log \frac{1}{2} + \frac{1}{2} \cdot \log \frac{1}{2}) = 1$. In the rule game for θ there are two identical rows and two identical columns. Nevertheless, we write out the whole payoff matrix.

	Left always	Right always	Left if R, Right if L	Left if L, Right if R
Top always	$\frac{3}{2}-\epsilon, \frac{3}{2}-\epsilon$	$\frac{3}{2}-\epsilon, \frac{3}{2}-\epsilon$	$\frac{1}{2}-\frac{\epsilon}{2}, \frac{1}{2}-\frac{\epsilon}{2}-\theta$	$2-\epsilon, 2-\epsilon-\theta$
Bottom always	$\frac{3}{2}-\epsilon, \frac{3}{2}-\epsilon$	$\frac{3}{2}-\epsilon, \frac{3}{2}-\epsilon$	$\frac{1}{2}-\frac{\epsilon}{2}, \frac{1}{2}-\frac{\epsilon}{2}-\theta$	$2-\epsilon, 2-\epsilon-\theta$
Top if B, Bottom if T	$\frac{1}{2}-\frac{\epsilon}{2}-\theta, \frac{1}{2}-\frac{\epsilon}{2}$	$\frac{1}{2}-\frac{\epsilon}{2}-\theta, \frac{1}{2}-\frac{\epsilon}{2}$	$-\theta, -\theta$	$2-\epsilon-\theta, 2-\epsilon-\theta$
Bottom if B, Top if T	$2-\epsilon-\theta, 2-\epsilon$	$2-\epsilon-\theta, 2-\epsilon$	$\frac{3}{2}-\frac{3}{2}\epsilon-\theta, \frac{3}{2}-\frac{3}{2}\epsilon-\theta$	$2-\theta, 2-\theta$

The first two rows and the first two columns are identical. If we delete one of the duplicate rows and one of the duplicate columns and if we now suppose that

$$\frac{1}{2} < \theta < \epsilon,$$

then we find that the middle column and the middle row are dominated. We are left with:

	Constant action	Left if L, Right if R
Constant action	$\frac{3}{2}-\epsilon, \frac{3}{2}-\epsilon$	$2-\epsilon, 2-\epsilon-\theta$
Bottom if B, Top if T	$2-\epsilon-\theta, 2-\epsilon$	$2-\theta, 2-\theta$

This is a Battle of the Sexes with two pure equilibria: (*Bottom if B and Top if T, constant action*) and (*constant action, Left if L and Right if R*). Each equilibrium has a total information content of $I^1 + I^2 = 1$. On the other hand, $\frac{1}{2} < \theta < \epsilon$ implies $\theta < \frac{1}{2} + \epsilon$ and $\theta < 2\epsilon$. That implies in turn that the team-optimal rule pair is (*Bottom if B and Top if T, Left if L and Right if R*), with a total information content of *two*. The two persons, as rule-game players in an equilibrium, communicate *less* than they do as a team.

In the second example, there are just two possible gross-payoff games, labelled **0** and **1**. Each occurs with probability $\frac{1}{2}$. Person 1 observes only an uninformative signal and does not know the identity of the week's game. Person 2 knows the identity of the week's game perfectly.

game **1**

	Left	Right
Top	20, 10	−8, 2
Bottom	16, −8	0, 0

game **0**

	Left	Right
Top	40, 2	−2, 8
Bottom	−20, 6	0, 0

The rule game is as follows:

	Left always	Right always	Left if 1, Right if 0	Right if 1, Left if zero
Top always	30,6	-5,5	9, 9-θ	16, 2-θ
Bottom always	-2,-1	0,0	8,-4-θ	-10, 3-θ

We suppose that

$$0 < \theta < 3.$$

Then there are no dominated rows or columns. the only (pure) equilibrium is (*Top always, Left if 1 and Right if 0*), with a total information content of one. But the *team-optimal* rule pair is (*Top always, Left always*), with information content zero. This time, the two persons, as game players in a rule-game equilibrium, communicate *more* than they do as members of a team.

If we now change the payoffs in the lower left box of game 1 to (82, -8), then in the rule game the payoffs in the first box of the bottom row become (31,-1) and the payoffs in the third box of the bottom row become (41, -4-θ). If θ equals 2 (for example), there is then no pure equilibrium in the rule game. There are equilibria which are pairs of mixtures of rules, but — as we argued above — a person's average payoff in such a rule mixture does not incorporate a reasonable measure of that person's communication effort.

If we return to the identical-payoff case, can we again have the game players communicating more than the team members? It is perhaps surprising that even with the strong identical-payoff property this can indeed happen again. That is shown in our third two-person example, which, however, has substantially more actions, gross-payoff games, signals, and possible rules than the preceding two examples.

There are now sixteen possible gross-payoff games, each identified by one of the sixteen possible four-digit binary strings. Person 1's signal is the first pair of digits in the string and Person 2's signal is the second pair of digits. Each person observes the four possible signals 00, 01, 10, 11 with probabilities .3,.3,.2,.2. Each person has seven possible actions which we shall label

$$b_{00}, b_{01}, b_{10}, b_{11}, c_0, c_1, d.$$

Define i's *signal action* to be the "b" whose subscript equals *i's* signal. Define i's *appropriate* c to be that "c" whose subscript equals the first digit in i's signal. Then the typical gross-payoff game may be described as follows:

	signal action	appropriate c	d	other actions
signal action	0	0	1	0
appropriate c	0	1	0	0
d	1	0	0	0
other actions	0	0	0	0

The number in each box of the table is the payoff received by Person 1 and by Person 2. There are many possible rule pairs. But only three pairs guarantee — for $\theta = 0$ — that in every gross-payoff game each person collects 1. In two of those pairs one person always takes the signal action while the other always takes *d*. In the third pair each person always chooses the appropriate *c*. If Person *i* always takes (and sends) the signal action, then I^i equals

$$-2(.3(log.3) + .2(log.2)),$$

which equals 1.97 to two decimal places. If Person *i* always takes (and sends) the appropriate *c*, then I^i equals

$$-(.6(log.6) + .4(log.4)),$$

which equals .97 to two decimal places.

To analyze the rule game it suffices to study the following three-by-three submatrix of the rule-game payoff matrix. The submatrix deals with the rules that belong to the three pairs just described.

	always take signal action	always take appropriate *c*	always take *d*
always take signal action	$-1.97\theta, -1.97\theta$	$-1.97\theta, -.97\theta$	$1 - 1.97\theta, 1$
always take appropriate *c*	$-.97\theta, -1.97\theta$	$1 - .97\theta, 1 - .97\theta$	$-.97\theta, 0$
always take *d*	$1, 1 - 1.97\theta$	$0, -.97\theta$	$0, 0$

Now consider the relation between the three-by-three submatrix and the matrix of the full rule game. First notice that for any $\theta \geq 0$, the rule pair (***take appropriate c-action, take appropriate c-action***) is a team optimum in the game defined by the three-by-three submatrix. For that rule pair, the team earns a total average gross payoff of 2 and a net payoff of $2 - 1.94\ \theta$. Second, note that in any box of the full matrix that is outside the submatrix, average gross payoff is less than two. So for $\theta = 0$, and also for sufficiently small positive θ, that rule pair remains team-optimal in the full rule game. For sufficiently small θ, moreover, no other rule pair of the full game is team-optimal.

Finally, note that in any box that lies in the extension of a row of the submatrix to the full matrix, Person 1's communication cost remains what it was in that row of the submatrix, while 1's average gross payoff is no higher than it was in that row of the submatrix. Similarly for Person 2 and the extension of any column of the submatrix. It follows that for any θ, any equilibrium of the three-by-three game remains an equilibrium of the full rule game.

If θ is less than $\frac{1}{2.94}$ (sufficiently less to overcome rounding inaccuracies in the logarithms), then in the three-by-three game the rule pair whose payoffs

are in the lower left box, and the pair whose payoffs are in the upper right box, are equilibria. If $\theta < \frac{1}{.97}$ then a third equilibrium (for any positive θ) is the team-optimal pair already noted, whose payoffs are in the middle box. None of the three equilibria is Pareto-dominated by any of the others. Moreover, for sufficiently small θ, no equilibrium rule pair of the full rule game that one might find outside the submatrix can Pareto-dominate any of the three equilibria of the three-by-three game.

Among the three equilibria of the rule game it is the team-optimal pair that has the *lowest* communication effort (for positive θ); that total is 1.94 θ. The other two equilibria have a total effort of 1.97 θ.

We conclude: for sufficiently small positive θ there are undominated equilbria of the rule game in which total communication effort is more than in the team optimum.

2.2 A General Proposition about Team-Optimal Total Information Content.

We shall establish that the team has a well-behaved "demand curve" for total information content. As θ — the "cost" of transmission — rises, the information content of the team's messages cannot increase. In fact its "demand curve" — the graph of $\underline{I}$ — is a descending sequence of flat lines.

We shall need some definitions. The rule N-tuple $(p^1, \ldots, p^N)$ will often be denoted p. We let $\pi(p), C(p)$ stand, respectively, for the average total gross payoff $\sum_{i=1}^{N} \pi^i(p)$ and the total information content $\sum_{i=1}^{N} I^i(p^i)$. For any $\theta \geq 0$ we define:

$$L(\theta) \equiv \{p \mid p \text{ is team-optimal in the rule game for } \theta\}$$

$$L^*(\theta) \equiv \{p \mid p \text{ is an admissable equilibrium in the rule game for } \theta\}.$$

"Admissable" means Nash and not Pareto-dominated.

We next define two finite sets whose elements are levels of total information content:

$$I(\theta) \equiv \{r \in I\!R \mid r = C(p); p \in L(\theta)\}$$

$$I^*(\theta) \equiv \{r \in I\!R \mid r = C(p); p \in L^*(\theta)\}.$$

(In a more cumbersome notation, we can write $C(L(\theta))$ and $C(L^*(\theta))$ for these two sets). We let $\bar{I}(\theta), \underline{I}(\theta)$ denote, respectively, the largest and the smallest element of $I(\theta)$; and we let $\bar{I}^*(\theta), \underline{I}^*(\theta)$ denote the largest and smallest element of $I^*(\theta)$.

Theorem 1 *For every $\theta \geq 0$, the set $I(\theta)$ is a singleton at all but a finite number of non-negative values of θ. Where it is not a singleton, it is finite. The functions $\underline{I}$ and $\bar{I}$ are non-increasing.*

Proof: From Lemma 1, we know that the set $\{p \mid p \in L(\theta); \theta \geq 0\}$ is a subset of the finite extreme-point set E. For any fixed rule N-tuple p in E, the total

team payoff in the rule game for θ is a function of θ, namely $\pi(p) - \theta C(p)$. Define

$$T(\theta) \equiv max_{p \in E}(\pi(p) - \theta C(p)).$$

Note that for any $\theta \geq 0$, all rule N-tuples p in $L(\theta)$ have the same value of T, while all other rules have a lower value. The function T is the upper envelope of a finite collection of straight lines with nonpositive slopes. The function T must therefore be convex; the slopes of the straight lines defining T must get smaller in absolute value as we move in the direction of increasing θ.

For all positive θ in a neighborhood of zero, $T = \pi(p^*) - \theta C(p^*)$, which can also be written $\pi(p^*) - \underline{I}(0)\theta$, where p^* is a member of $L(0)$ for which total information content $C(p)$ is minimal.

At some $\tilde{\theta}$, the line $\pi(p^*) - \underline{I}(0)\theta$ intersects a new line of the envelope, namely $\pi(p^{**}) - C(p^{**})\theta$, which can also be written $\pi(p^{**}) - \underline{I}(\tilde{\theta})\theta$, where p^{**} is a member of $L(\tilde{\theta})$ for which the total information content is minimal. At $\tilde{\theta}$, both p^* and p^{**} are team-optimal rule N-tuples. The set $I(\tilde{\theta})$, consisting of total-information-content levels for team-optimal rule N-tuples at $\tilde{\theta}$ is finite. Its smallest member is $C(p^{**}) = \underline{I}(\tilde{\theta})$ and its largest member is $C(p^*) = \underline{I}(\tilde{\theta})$.

The pattern is the same for the remaining corners of the envelope T. For any θ that is at a corner, the set $I(\theta)$ is finite and its smallest member $\underline{I}(\theta)$ is the absolute value of the slope of the new straight line of the envelope. The new straight line is smaller, with regard to the absolute value of its slope, than the preceding straight line.

The number of corners — at which the set $I(\ )$ is not a singleton — is finite since each successive corner is defined by two members of the finite set E and one of those members is unique to the corner.

All parts of the Theorem are therefore established. ■

Figure 1 and Figure 2 illustrate the pattern for a very simple case, in which there are just three rule N-tuples, called p^*, p^{**}, and $\tilde{p}$; their total information content is, respectively, $1, \frac{1}{2}$, and zero. The envelope has just two corners. The function $\underline{I}$ consists of three flat pieces, for the values of θ that are not at a corner of the envelope, plus $\{\frac{1}{2}, 1\}$ for the corner value $\theta = 2$, and $\{0, \frac{1}{2}\}$ for the corner value $\theta = 4$.

2.3 The Case of Identical Payoffs in the Gross-Payoff Game

The third of our three examples showed that even if we impose the strong identical-payoff condition, there may be equilibria with "too much" communication as well as equilibria with "too little". What general statements about equilibria *are* implied if we add the identical-payoff condition? The following Theorem shows that if we add the condition, then for very small θ and for very large θ the anomalies that appear in the examples disappear. For such θ, existence of equilibrium is guaranteed. Moreover for very small θ, the rule N-tuples used in a rule game, and those used in a team optimum, are both

FIGURE 1

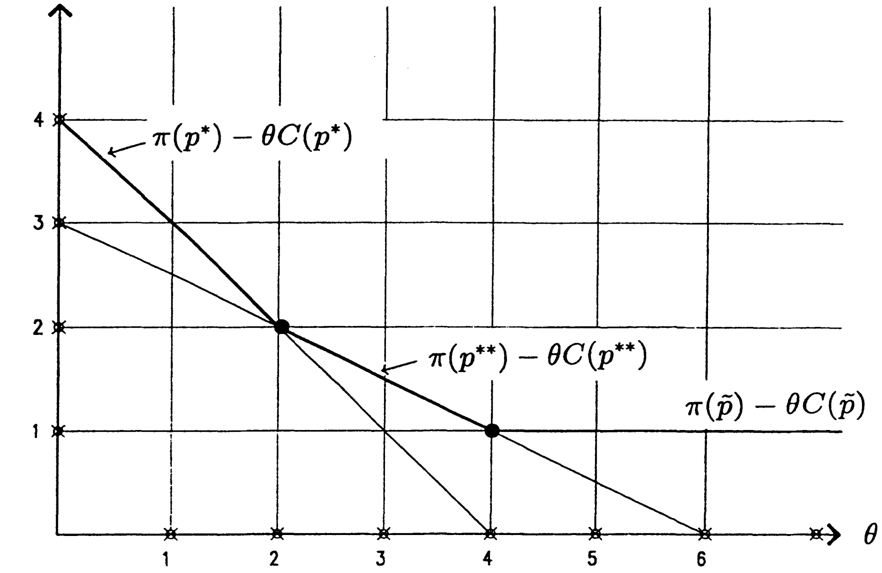
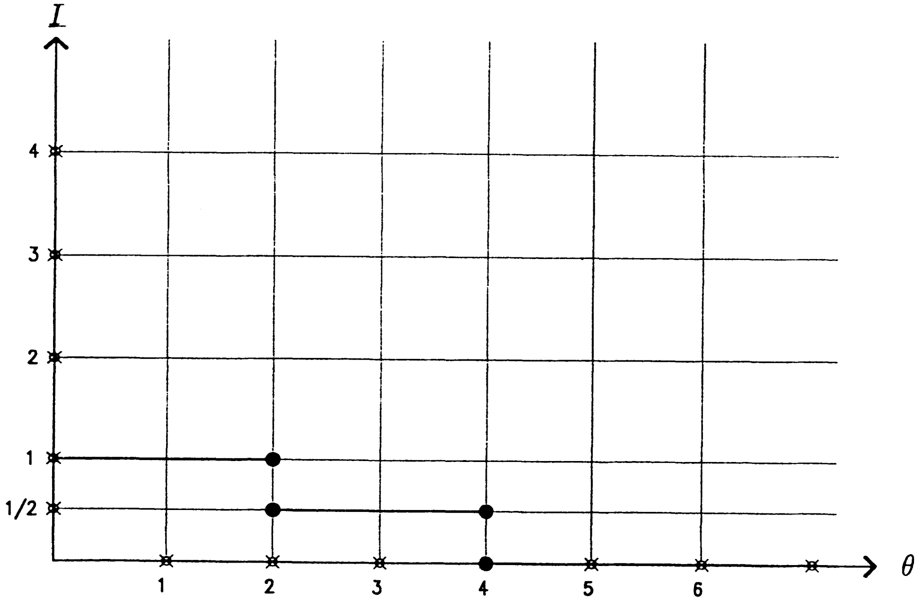


FIGURE 2



subsets of the rules used when $\theta = 0$. For very large θ, the set of equilibrium N-tuples is the same as the set of team optima.

Theorem 2 *If all of the gross-payoff games have identical payoffs, then:*

(1) For θ sufficiently small: (a) an admissable equilibrium of the rule game exists; (b) $L^*(\theta) \subseteq L^*(0) = L(0)$; and (c) $L(\theta) \subseteq L(0)$.

(2) For θ sufficiently large, an admissable equilibrium of the rule game exists; $L(\theta) = L^*(\theta)$; and $\underline{I}^*(\theta) = \underline{I}(\theta) = \bar{I}^*(\theta) = \bar{I}(\theta) = 0$.

(3) Suppose the rule game has an equilibrium for every $\theta \geq 0$. Then for any $\theta \geq 0$, $\underline{I}(\theta) = 0 \Rightarrow \underline{I}^*(\theta) = 0$.

Proof:

(1)(a) We consider the finite rule game, in which every Person i chooses from the finite set Γ^i. The sets $L(0)$ and $L^*(0)$ are identical; they consist of the N-tuples in which the common average gross payoff, received by every person, is maximal. We now describe a procedure which leads to a rule N-tuple that is an admissable equilibrium if θ is sufficiently small. Define an N-tuple to be *valid* if it belongs to $L(0)$. We first note that if θ is sufficiently small, then given any valid N- tuple, any person i will only be interested in deviating from that N-tuple if the resulting deviation preserves validity (the new N-tuple remains in $L(0)$) while lowering i's information effort I^i.

We start our procedure with an arbitrary member of $L(0)$, say $(p^1, \ldots, p^N)$. Now we ask Person 1 to replace p^1 by $p^{*^1} \in \Gamma^1$, where p^1 preserves validity and has a lowest information cost I^1 among all the rules in Γ^1 that preserve validity. Next we ask Person 2 to consider the new N-tuple $(p^{*^1}, p^2, p^3, \ldots, p^N)$ and to replace p^2, by p^{*^2}, which preserves validity and has the lowest value of I^2 among all the rules in Γ^2 that preserve validity. Continue in this manner until it is Person N's turn. After Person N's turn we have obtained an N-tuple $p^* = (p^{*^1}, \ldots, p^{*^N})$. We then go through the same N steps for p^*. If no person i replaces p^{*^i} with a different rule when it is his turn, then p^* is an admissable equilibrium. If someone does make such a replacement, then complete the second sequence of N steps and start a new sequence with the N-tuple so obtained. The procedure cannot cycle, since at least one person's information content decreases in each completed sequence. Moreover there must be a terminal sequence, since every Γ^i is finite.

(1)(b) Suppose that $\tilde{p}$ is an equilibrium in the rule game for θ but that (contrary to assertion (1)(b)), $\tilde{p}$ does not belong to $L(0)$. Let $\bar{p}$ denote the rule N-tuple found by the procedure described in the proof of (1)(a). The N- tuple $\bar{p}$ belongs to $L(0)$ and yields the N-tuple of net payoffs $(H - \theta I^1(\bar{p}^1), \ldots, H - \theta I^N(\bar{p}^N))$, where $H = \frac{1}{N} \max_{p \in E} \pi(p)$ is the average gross payoff received by every person for any rule N-tuple belonging to $L(0)$. On the other hand, for $\tilde{p}$, the N-tuple of net

payoffs is $(Q - \theta I^1(\bar{p}^1), \ldots, Q - \theta I^N(\bar{p}^N))$, where $Q = \frac{I}{N}\pi(\tilde{p})$. Since $\tilde{p}$ does not belong to $L(0)$, we have $Q < H$. That means that for θ small enough, $\tilde{p}$ is Pareto-dominated by $\bar{p}$ while at the same time $\bar{p}$ is, as shown in the proof of (1)(a), an equilibrium. So while $\tilde{p}$ may be an equilibrium, it is not admissable.

(1)(c) The proof is straightforward.

(2) If θ is sufficiently large, then for every Person i, every rule $p^i \in \Gamma^i$ with $I^i(p^i) > 0$, is dominated by some no-communication rule (with $I^i(\ ) = 0$), under which i chooses a constant action, regardless of signal. Discarding all dominated strategies, we are left with the gross-payoff game. Since every action N-tuple in that game has the same payoff for every person, an equilibrium exists. Proofs of the remaining statements in (2) are straightforward.

(3) We are given that for some θ, some $p^* \in E$ is a no-communication rule N-tuple which is also team-optimal, i.e., $C(p^*) = 0$ and

$$\pi(p^*) \geq \pi(p) - \theta C(p), \text{ all } p \in E.$$

The identical-payoff property of the gross-payoff games implies that $\pi^1(p^*) = \cdots = \pi^N(p^*) \equiv T$.

But p^* is also an equilibrium. For suppose not. Then for some person — say Person 1 — there is a better reply to $({p^*}^2, \ldots, {p^*}^N)$ than ${p^*}^1$. Let $\tilde{p}^1$ denote that better reply, i.e.,

$$V^1(\tilde{p}^1, {p^*}^2 \ldots, {p^*}^N) = Z^1 - \theta I^1(\tilde{p}^1) > V^1(p^*) = T,$$

where $Z^1 \equiv \pi^1(\tilde{p}^1, {p^*}^2, \ldots, {p^*}^N)$. Hence

$$(\dagger) \qquad Z^1 > T.$$

The identical-payoff property of the gross-payoff game implies that for every person $i > 1$, $V^i(\tilde{p}^1, {p^*}^2, \ldots, {p^*}^N) = Z^1 - I^i(p^i)$. But, in view of (†), that means that $(\tilde{p}^1, {p^*}^2, \ldots, {p^*}^N)$ achieves a higher total team payoff than p^* does, which is a contradiction. Thus zero is the lowest total information content among the equilibria of the rule game. ∎

If we want to obtain similar characterizations of team information content and game-equilibrium information content for values of θ that are not very large or very small, then, it appears, we have to add further conditions on the gross-payoff games. The identical-payoff condition, strong as it is, does not suffice.

3. Models with Delay

In the model thus far, Person i is the only person to suffer directly when i increases his own communication effort, from which all may benefit. Suppose

instead that i's increased effort imposes a cost on everyone because of the delay it causes. Suppose that the week's new actions cannot be taken until everyone has been heard from.

Then in the simplest variation of the preceding model, the Center receives the N messages *in sequence.* It takes the average time $\theta \sum_{i=1}^{N} I^i(p^i)$ until all N mesages have been received. We then define i's net payoff in the rule game for θ, when the rule N-tuple p is used, to be

$$V^i(p) \equiv \pi^i(p) - \frac{1}{N}\theta \sum_{1=1}^{N} I^i(p^i).$$

A penalty equal to the delay is subtracted from the week's gross payoff, and everyone is assessed an equal share of that penalty. (One might imagine the organization to have a client who will pay the organization in a given week, where the game is g and the actions are $(a^1, \ldots, a^N)$, the net amount $\sum_i B_g^i(a^1, \ldots, a^N)$ minus a penalty equal to the time until the week's actions are actually "delivered" to the client).

If, on the other hand, the N messages are received *simultaneously*, then the time $\theta \max_i(I^i(p^i))$ elapses until the new actions are taken, and we have

$$V^i(p) \equiv \pi^i(p) - \frac{1}{N}\theta \max_i(I^i(p^i)).$$

In a more ambitious variation, it remains the case that new actions cannot be taken until everyone has been heard from, but the penalty due to delay is now the payoff that is foregone until the new actions have been found and taken. Take the week to be the unit in which time is measured. For $Q \geq 0$, interpret $Q \sum_i B_g^i(a^1, \ldots, a^N)$ as the gross payoff that accumulates during an interval of length Q in which the game is g and the actions are $(a^1, \ldots, a^N)$. Make the rather drastic simplifying assumption that until the week's new actions have been taken, *zero* payoff accumulates. Suppose (to simplify again) that for all possible rule N-tuples $(p^1, \ldots, p^N)$, $\theta \max_i I^i(p^i) < 1$ and $\theta \sum_{i=1}^{N} I^i(p^i) < 1$. (The week's new actions are always taken before the week is up, whether transmissions are simultaneous or sequential). Then if message transmissions are simultaneous, i's net payoff in the rule game becomes

$$V^i \equiv \pi^i(p^1, \ldots, p^N) \cdot (1 - \theta \max_i I^i(p^i)).$$

If messages are received in sequence, then we have

$$V^i \equiv \pi^i(p^1, \ldots, p^N) \cdot (1 - \theta \sum_{i=1}^{N} I^i(p^i)).$$

All the questions we have uncovered in our previous effort-is-costly model arise again in all of the four variants.[6] In the simultaneous-transmission models, we have now lost the strict convexity of V^i, and have therefore also lost the useful fact that the number of pure equilibria is finite. The equilibrium-existence question remains in all four models. One can again not use rule mixtures to insure existence of meaningful equilibria. In all four models there appears to be even more reason to believe that when pure equilibria exist they may include both equilibria in which there is too much communication and equilibria in which there is too little, even in the identical-payoff case. The behavior of the functions $\underline{I}$ and $\underline{I}^*$ is far from clear *a priori* even for the identical-payoff case.

Both the models of Section 2 and the delay models just sketched could be altered in one respect. Instead of viewing the Center as a passive robot who simply turns the action choices of the other persons into actions that are physically in force, we could let the Center be a robot who works harder. We could let each person's message to the Center be an arbitrary message about the sender's current signal instead of restricting it to be an action choice. The Center translates the messages received into the actions that are to be taken, using a rule that is best from the organization's point of view.[7]

4. Concluding Remarks

The popular view that cheap and quick communication strengthens the case for the dispersal of organizations turns out to rest on rather shaky ground. If dispersal means that team members become game players, then they may

[6] In a still more ambitious model, one would drop the assumption that the successive weeks' games are serially independent and would suppose that they are generated by a stochastic process — a Markov chain, for example. Then i's accumulated payoff during the part of the week in which the new actions are being found and transmitted is not zero, as in the version above. Rather it equals the payoff specified by this week's game for last week's actions multiplied by $\theta \max_i I^i(p^i)$ in the simultaneous-transmission case and by $\theta \sum_{i=1}^{N} I^i(p^i)$ in the sequential-transmission case. V^i becomes i's average total weekly payoff — the old-actions total payoff plus the new-actions total payoff — over all realizations of the process.

[7] That sort of model would be closer in spirit to the Theory of Teams (Marschak and Radner, 1971), which compares alternative "information structures" with respect to average gross team payoff, though not with respect to communication cost. A structure specifies who knows what about the team's current environment (the current game, in our setting). Suppose the Center knows the rule used by every person i to select a message given i's signal. Then when the N persons send messages that are proposed actions, the Center acquires information about the week's environment (game). When they send arbitrary messages instead, the Center acquires different information. Models with arbitrary messages would generally be harder to study than ours. The equilibrium-existence question, in particular, would not be easier.

communicate too little (from the organization's viewpoint) and they may communicate too much. We appear to be very far indeed from the familiar setting of the effort models first explored by Holmstrom. We have done little more here than to open an agenda.

The most striking finding is that if communication effort is going to be measured in a way that depends on message probabilities, then the customary role of mixtures in guaranteeing the existence of equilibrium no longer helps. Even the Harsanyi interpetation of an equilibrium of mixtures (Harsanyi, 1973) fails. In the two-person case, that interpretation asks 1 to think of 2's mixture as portraying 2's alternative types, not observable to 1. For each of 2's types, 2 uses a different rule. But 1 cannot meaningfully view the payoff he earns when he chooses a rule as the average he earns over 2's possible types — since that average again fails to incorporate a meaningful measure of 1's communication effort for his chosen rule.

We appear to be in a new area where one central issue is "too few" equilibria instead of the more familiar "too many". It was the work of Reinhard Selten that went a very long way in meeting the challenge of "too many". Possibly it will take someone with his vision and ingenuity to clear up the present puzzle, using concepts and approaches not yet imagined.

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Endogenous Agendas in Committees

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1. Introduction

When a committee faces the task of making a collective decision on several issues procedural matters seem to play an enormous role. The process of decision making typically involves a preliminary phase of negotiation, which may take the form of either an informal behind-the scene bargaining or a round-table discussion prior to voting. In this paper we will formulate the process of such decision making as a non-cooperative bargaining game, and analyze the effect of the agenda on the efficiency and stability of the bargaining outcomes. In Winter (1993) it was shown that if the underlying preferences of the committee members are such that there exists a consensus over the importance of issues, then, in order to guarantee efficiency, issues should be negotiated in the order of their importance. Our main purpose here is to explore two models, in addition to those in Winter (1993), in which no agenda is exogenously determined prior to the bargaining. In one model the agenda itself is negotiated by the committee members before dealing with issues of substance; in the other, the agenda is determined endogenously, i.e., each proposer can at any time propose an alternative on any unsettled issue. We will show that these models can only partially replace the "important issues first" rule for maintaining efficiency and stability.

Our approach here is one of multilateral bargaining theory. Reinhard Selten and John Harsanyi were the first to introduce extensive games into models of multi-person negotiations and coalition formation. Harsanyi (1974) and Selten (1981) can also be regarded as pioneering contributors to the literature on the non-cooperative foundations of cooperative game theory. Selten's (1981) paper introduces a bargaining model that involves sequential proposals of a coalition and a payoff distribution for its members[2]. An alternative model, introduced by Selten (1992), is the demand commitment bargaining, where players publicly announce their requests from cooperation. Both models are based on an underlying cooperative

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[1] I would like to thank the German-Israeli Foundation for financial support.

[2] Our bargaining models bellow are very much in the spirit of Selten's proposal model. However the underlying problems are different since we are interested in committees and not in characteristic function games.

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game in coalitional form. Selten's interest in the relationship between non-cooperative equilibrium behavior and cooperative solution concepts was motivated by his awareness that understanding this relationship is essential for understanding the discrepancies between game theoretic predictions and actual behavior in experiments. The bargaining models considered here are designed to fit the context of committees, and are based on sequential proposals as well. Each issue is negotiated through a series of sessions and the outcome on an issue is determined once it is supported by a majority. We commence in Section 2, with a formal description of committees. In Section 3 introduces the benchmark bargaining model, which is based on simultaneous proposals on all issues. In Section 4 we review the main result in Winter (1993) regarding issue-by-issue bargaining; roughly, this result asserts that the efficiency and stability of the bargaining outcome can be guaranteed in agendas where important issues are discussed first. Section 5 presents a model in which the agenda is negotiated by the members of the committee prior to discussing issues of substance. We show that such a procedure fails to guarantee the efficiency of the bargaining outcome. Section 6 concludes with a model of endogenous agenda; here, any member can choose an issue to propose as long as this issue is not yet settled. We show that such a model can guarantee the efficiency and stability of the bargaining outcome, but only for committees that deliberate no more than two issues and when each issue involves yes-no voting.

2. Committees

A committee must reach a decision on k issues $\{1,2,\ldots,k\}$. An issue is a finite set of alternatives A_i. The final process of the bargaining should induce a Platform a $= (a_1,a_2,\ldots,a_k)$, which is an element of the product set $A = \Pi A_i$. Thus, on each issue one alternative has to be chosen. The committee consists of a set N of n-members. Each member i has an ordinal strict preference relation on A, which we denote by $\succ_i$. Members' preferences on A are not necessarily separable -there may be interaction between the issues. The distribution of power within the committee is represented by a collection of winning coalitions W. Each coalition in W has the power to impose any decision it favors. For example, if the decision is taken by means of simple majority voting then W consists of all coalitions with at least n/2 members. But the structure of winning coalitions may also allow for decision making that requires unanimity, in which case W consists only of the grand coalition. The standard property which we assume on W is that there are no two disjoint winning coalitions. To summarize, a committee is a represented by a 4-tuple $(N,A,(\succ)_{i\in N},W)$.

The notion of stability that interests us here is based on the concept of core. Given a committee $C = (N,A,(\succ)_{i\in N},W)$, a platform a is said to be stable if it cannot be blocked by any winning coalition, i.e., there exists no S in W and a

platform b such that all members in S prefer b to a ($b \succ_i a$ for all $i \in N$). Note that the notion of stability automatically guarantees Pareto efficiency.

We describe the process of collective decision making to determine a platform as a non-cooperative bargaining game. Our aim is to consider several bargaining models and determine whether equilibrium behavior in these models necessarily leads to an efficient/stable bargaining outcome. We start with a model in which all issues are negotiated simultaneously.

3. The Simultaneous Bargaining Model

When bargaining commences a chair i in N is determined exogenously. Bargaining then takes place in sessions. At the beginning of each session the chair submits the first proposal. A proposal is a pair (S,a) where S is a winning coalition containing i, and a is a platform in A (i.e., a proposal that specifies an outcome on each issue). Once (S,a) is proposed, each member in S either accepts the proposal or rejects it. Responses are carried out sequentially according to some exogenously specified order[3]. If all the members accept (S,a), the bargaining terminates with the outcome a. If some member j in S rejects the proposal he has to propose an alternative proposal, say (T,b), which has to be tabled again. This process continues until each committee member has had the opportunity to propose at least once[4]. When all members have proposed at least once, the chair closes the session and a new session opens with an initial proposal by the chair. The whole process continues until a platform is chosen.

The procedure described above gives rise to an extensive-form game with perfect information and an infinite horizon. The solution concept we use for analyzing this game as well as the subsequent ones is subgame perfect equilibrium. As we have said before we would like to focus on the occurrence of inefficiencies due to the multiplicity of issues. To abstract away all other sources of inefficiencies we would confine ourselves to stationary equilibria. In stationary equilibria players' actions can depend only partly dependent on the history of the game. Specifically, we will be interested in equilibria satisfying the following properties:

1. In responding to a proposal, the player may condition his action only on the proposed platform, i.e., each player has some cut-off point in his ordering, above which he accepts any proposal. A platform bellow this cut-off point will always be rejected.

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[3] The results of the paper hold for every choice of such exogenous order.

[4] Note that each session can potentially be infinite. We however assume that reaching an agreement is the basic common interest (i.e., no agreement is inferior to any other outcome) so that indefinite disagreement cannot be sustainable in equilibrium.

2. In proposing, a player can condition his action on events that occurred within the current session, but not on events in previous sessions, i.e.,

Conditions 1 and 2 should not be taken as behavioral assumptions regarding players' behavior. They are imposed in order avoid the well-known phenomenon of the "Folk Theorem" in multilateral bargaining, by which non-stationary equilibria sustain almost every possible outcome (see, Chatterjee et al (1993), and van Damme et al (1990), where this phenomenon occur even in 2-person bargaining).

We now show that when all issues are negotiated simultaneously, strategic behavior leads to an efficient platform. Furthermore:

Proposition 1: For every committee $C = (N,A,(\succ)_{i \in N},W)$ a platform a is stable if and only if a is sustainable by a stationary subgame perfect equilibrium of the simultaneous bargaining game.

The proof of Proposition 1 is a direct consequence of Winter (1993), Proposition 5.1. The simultaneous consideration of all issues in effect reduces the multi-issue problem to a single issue. On a single issue, inefficiency or instability cannot be sustained because a blocking coalition would necessarily negotiate a dominating outcome, which (with a backward induction argument) will be accepted. This argument is made formal in Winter (1993). In fact this result (as well as the subsequent results) is in some respect robust to the bargaining procedure. In particular, the Proposition holds also in a model without sessions. Namely, a model in which a rejection always triggers a new proposal by the rejector. However, such a model will require a stronger notion of stationarity for the results.

In spite of the advantage, in term of efficiency, in negotiating several issues simultaneously, committees are often reluctant to use this type of bargaining procedure. Typical arguments against such a regime include the fact that decision making is much more complex when all issues are considered simultaneously, and that it allows for more "log rolling", i.e., trade of interest (see Riker, 1969). Thus, most committees prefer to negotiate one issue at a time. We now examine the strategic behavior in this alternative procedure, focusing on the possibility of guaranteeing efficiency and stability as a result of equilibrium behavior.

4. Issue-by-Issue Bargaining

In this alternative model issues are negotiated in sequence. At the beginning of the bargaining an exogenous Agenda, i.e., an order of issues, is fixed. The bargaining starts by discussing the first issue on the agenda using the same procedure as in Section 3, i.e., a proposal on A_1 consists of a pair (S,a_1) where a_1 is in A_1 and S is

in W. If a proposal is rejected, the bargaining continues session after session, according to the same rules[5] as in Section 3, on A_1 alone. If (S,a_1) is accepted, then a_1 is determined to be the outcome on the first issue and the bargaining proceeds to the second issue in the same manner. Bargaining terminates after all issues are discussed and a platform $a = (a_1,a_2,\ldots,a_k)$ is determined. Note that when discussing a certain issue the players can keep track of the decisions made on previous issues, and are allowed to make their decisions dependent on this information.

Does the issue-by-issue game guarantee efficiency as does simultaneous bargaining? Unfortunately not. In Winter (1993) it is shown that the fact that a decision on one issue can be made contingent on previous decisions on other issues may yield serious inefficiencies as a result of stationary[6] subgame perfect equilibrium play. However the situation is different if individual preferences represent a consensus over the importance of issues. Many committee situations are characterized by the fact that all parties share the same view regarding the importance of issues even if they disagree on the preferred outcome on each issue. In times of war, for example, parliaments would often agree that issues of national security are more important than economic issues. We now turn to such committees to extend the Winter (1993) results on the efficiency of bargaining outcomes in such committees. But first, some notations which will enable us to define the notion of "order of importance".

Let A be a set of platforms and $a = (a_1,\ldots,a_k)$ in A. Suppose that $b_j \in A_j$ is an alternative on issue j. We denote by $a|b_j$ the platform obtained from a by replacing a_j with b_j. Furthermore, we write $a|b_j,b_m = (a|b_j)|b_m$.

Let i be a committee member with a preference relation $\succ_i$ on A. We say that issue j is more important for i than issue m (with respect to $\succ_i$) if the following holds:

For each platform a, if $a|b_j,b_m \succ_i a|c_m$ for some $b_j \in A_j$, b_m and c_m in A_m, then $a|b_j,b'_m \succ_i a|c'_m$ for every b'_m,c'_m in A_m. That is, the ranking of any two platforms that differ only on issues j and m should be independent of the choice of issue m.

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[5] Specifically; If a player in S rejects (S,a_1) then he submits a proposal himself on A_1. The session continues until every one had a chance to propose at least once in the current session. Then a new session starts again on A_1.

[6] Stationarity in this context means applying properties 1 and 2 of Section 3 on each issue separately, i.e., when discussing the jth issue, if at some stage a_j was rejected by player i, then i will reject a_j also in subsequent responses to a_1. Recall however, that players are allowed to make their action on one issue dependent on the decisions taken on former issues.

We say that the preference profile $(\succ_i)_{i\in N}$ represents consensus over the importance of issues if the order of importance induced by the individual preferences is complete, transitive, and identical for all players[7].

In Winter (1993) it is shown that even with under consensus over the importance of issues, stationary subgame perfect equilibria may yield inefficient outcomes. However, if issues are discussed in the order of their importance, then no inefficiency or instability can result from an equilibrium play.

Proposition 2 (Winter 1993): Let $C = (N,A,(\succ)_{i\in N},W)$ be a committee in which $(\succ)_{i\in N}$ represents a consensus over the importance of issues. Consider the issue-by-issue bargaining game with an agenda that is consistent with the order of importance of issues, i.e., the most important issue is discussed first, the next most important second, and so on. Then, a platform a is sustainable by a subgame perfect equilibrium outcome if and only if it is stable.

The intuition behind Proposition 2 and example 1 is quite clear. Suppose the bargaining starts with a less important issue. Since the committee members can make their decisions on one issue dependent on previous issues, they will be "overly concerned" with the effect this decision on the more important issue. This may prevent them from making the "right" decision on less important issues, using them as signals or threats. If, however, the bargaining is carried out by order of importance of issues, then when discussing a certain issue it is known that subsequently only less important issues will be considered, and everybody can act as if this issue is the only issue on the agenda.

We have seen that by discussing issues by order of importance one can guarantee the stability and the efficiency of the equilibrium outcome. We now examine two additional mechanisms that allow for more flexibility in the choice of the agenda and check whether they can endogenize the positive effect on efficiency seen in Proposition 2.

5. Bargaining Over the Agenda

The bargaining model discussed in Section 4 is based on the exogenous determination of the agenda. Obviously, since the agenda has a dramatic effect on the bargaining outcome, different members may prefer different agendas so that the agenda itself becomes an issue. We now consider a model in which committee members start by bargaining over the agenda before discussing the issues of substance. Let G be the set of all k! agendas; at the beginning of the bargaining G

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[7] The requirement that individual preferences induce an order of importance is weaker than lexicographic preferences. In lexicographic preferences if two platforms a and b coincide on the first j issues, then their ranking depend on a_{j+1} and b_{j+1} only. In our preferences this is not necessarily the case.

is considered as an issue and the agenda is negotiated according to the same procedure as in Section 4, i.e., the chair starts with a proposal (S,g), where S is a winning coalition and g is an agenda. If the proposal is accepted by all the members in S, then g is fixed as the agenda. If some i in S rejects the proposal, he comes up with an alternative proposal. This continues in the same manner until each player has had a chance to propose an agenda at least once, after which a new session is opened by the chair with a new proposed agenda. Once the agenda is determined, bargaining continues according to the chosen agenda by means of the issue-by-issue bargaining game discussed in Section 4.

Does this process of negotiation over the agenda necessarily imply that the more important issues will be discussed first? Is it at least the case that this mechanism guarantees a stable platform as a result of equilibrium behavior? Unfortunately,

Proposition 3: In the negotiated-agenda bargaining game every stable platform is sustainable by some SSPE, but there may be non-stable bargaining outcomes that are sustainable by an SSPE.

Proof: We start with the second assertion. Consider the following committee with two issues, two alternatives on each issue, and 4-person simple majority voting (three votes win):

$A_1 = \{a_1,a_2\}$, $A_2 = \{b_1,b_2\}$. The preferences are:

1: $(a_1,b_1) \succ (a_1,b_2)^* \succ (a_2,b_2)^* \succ (a_2,b_1)$
2: $(a_2,b_2) \succ (a_2,b_1) \succ (a_1,b_2) \succ (a_1,b_1)$
3: $(a_2,b_1) \succ (a_2,b_2) \succ (a_1,b_2) \succ (a_1,b_1)$
4: $(a_1,b_2) \succ (a_1,b_1) \succ (a_2,b_2) \succ (a_2,b_1)$.

Note that the outcomes (a_1,b_2) and (a_2,b_2) are in the core; the other two are non-core outcomes. Note also that all players agree that the first issue is more important than the second. There are two possible agendas, one in which the most important issue is discussed first (denoted by [1]) and the other in which the second issue is discussed first (denoted by [2]). Consider the subgame starting after agenda [1] was chosen (the most important issue is discussed first). Since (a_2,b_2) is a core platform of C, we know by proposition 2 that there exists some SSPE on this subgame that yields the platform (a_2,b_2). Let s_1 denote this strategy combination, and let s_2 denote the SSPE that yields (a_1,b_1) on the subgame starting after the agenda [2] was chosen (as in example 1). We now specify the SSPE in the negotiated-agenda game that yields the non-stable platform (a_1,b_1).

In the first phase of the bargaining:

1: Proposes ([2],{1,3,4}) and accepts only the agenda [2].
2: Proposes ([2],{1,2,4}) and accepts every agenda.

3: Proposes ([2],{1,3,4}) and accepts every agenda.
4: Proposes ([2],{1,3,4}) and accepts only the agenda [2].

After the agenda is chosen:
If the chosen agenda is [1], then all play s_1.
If the chosen agenda is [2], then all play s_2.

To verify that this is indeed an SSPE note first that this strategy combination specifies an SSPE on each subgame that occurs after the agenda was determined. It is, therefore, enough to check that the behavior prior to the choice of agenda, as specified above, is part of an SSPE. This follows from the fact that given s_1 and s_2 players 1 and 4 prefer agenda [2], while players 2 and 3 prefer agenda [1].

The first assertion follows from the fact that for any agenda and for any stable platform a is supported by some SSPE [see the proof of Proposition 5.1 in Winter (1993)]. This means that any such platform is also supportable by an SSPE in the negotiated-agenda game: simply take any strategy combination that yields some SSPE supporting platform a on every subgame starting after the agenda was chosen.

We have seen that allowing players to bargain over the agenda is not sufficient to induce stable outcomes as a result of equilibrium behavior. We now move to an alternative model, in which the agenda is not fixed at any stage of the bargaining but is determined endogenously in the course of the negotiation. We will show that for two-issue committees with yes-no voting such model yields positive results.

6. Open-Agenda Bargaining

In the open-agenda bargaining game proposers choose the issues on which they wish to propose a decision.

Bargaining starts with a proposal by the chair. A proposal (j,a_j,S) now consists of an issue j, which has not yet been settled, an alternative to this issue, and a coalition S in W. If everybody accepts the proposal the jth issue is settled with the outcome a_j, and the chair submits a new proposal (i,a_i,T), $i \neq j$. If the proposal is rejected by some player m, then m has to submit a new proposal. At every stage the proposed issue must be one that has not yet been settled. Bargaining terminates when all issues are settled and a platform is obtained.

We now show that when the committee engages in "simple" decision making, i.e., when there are only two issues, each with two alternatives, then the open-agenda model yields the same effect as the "important issue first" model. Note that the domain of platforms with two alternatives for each issue is an important domain because it involves all issues on which yes-no decisions made.

Proposition 4: Consider a committee $C = (N,A,(\succ)_{i\in N},W)$, where A consists of two issues, each with two alternatives, and there is a consensus over the importance of issues. In the open-agenda bargaining game, platform a is supportable by an SSPE if and only if it is stable.

Proof: We again start by showing that all SSPE outcomes are stable. Without loss of generality assume that the first issue is more important and that (a_1,b_2) is an equilibrium platform. Since the first issue is more important, we can reduce the preferences on platforms to preferences on this issue (because they are independent of the choice made on the other issue). This means that there is no S in W such that all players in S prefer a_2 to a_1. Otherwise consider the coalition T, which is proposed before a_1 is accepted, and consider i in $T\cap S$. i can guarantee a_2 by proposing (S,a_2), which must be accepted by the members of S (because of the stationarity). Now suppose, by way of contradiction that (a_1,b_2) is not in the core. Then, given the arguments above, there must be some $S\in W$ such that all members of S prefer (a_1,b_1) to (a_1,b_2). Now let T be the coalition proposed at the beginning of the game and take again i in $S\cap T$. Consider a proposal of (S,a_1) by i. If this proposal is accepted, b_1 will obviously be chosen on the second issue. Otherwise, one of the responders to b_2 in S should reject it and propose b_1, which must be accepted. Since all players expect b_1 to be chosen on A_2, all members in S must accept a_1 on A_1. Thus by proposing a_1 before A_2 is discussed player i guarantees the platform (a_1,b_1) which he prefers to the equilibrium outcome. This contradicts the fact that (a_1,b_2) is an equilibrium platform.

To show the converse, assume without loss of generality that (a_1,b_1) is stable. Consider the following SSPE: Each player proposes (N,a_1) as long as A_1 is not yet settled. Each player rejects (S,a_j) if and only if he or some other player who has not yet responded to this proposal prefers a_1 to a_j. Once A_1 is settled, the remaining subgame is identical to single-issue bargaining. Thus we can specify here an SSPE in which b_1 is played after a_1 was chosen and b_j is played after a_2 was chosen, where b_j is such that there exists no $S\in W$ with $(a_2,b_{j \bmod 2}) \succ_k (a_2,b_j)$ for k in S [see, for example, Winter (1993) Theorem 5.1].

It can be shown that open-agenda cannot guarantee stability when the committee has to discuss more than two issues.

We use the simplest example to demonstrate this statement, which is 2-person, 3-issue, 2 alternatives each.

Example 1:

$N = \{1,2\}$, $W = \{N\}$, $A = A_1 \times A_2 \times A_3$, $A_1 = \{a_1,a_2\}$, $A_2 = \{b_1,b_2\}$, $A_3 = \{c_1,c_2\}$.

Preferences:

1: $(a_1,b_1,c_1) \succ (a_1,b_1,c_2) \succ (a_1,b_2,c_2) \succ (a_1,b_2,c_1) \succ$
$(a_2,b_2,c_1) \succ (a_2,b_2,c_2) \succ (a_2,b_1,c_2) \succ (a_2,b_1,c_1)$

2: $(a_2,b_1,c_2) \succ (a_2,b_1,c_1) \succ (a_2,b_2,c_1) \succ (a_2,b_2,c_2) \succ$
$(a_1,b_2,c_2) \succ (a_1,b_2,c_1) \succ (a_1,b_1,c_1) \succ (a_1,b_1,c_2)$

Note that with respect to this preference profile both members agree that A_1 is the most important issue, A_2 is the second most important, and A_3 is the least important. Note also that (a_2,b_1,c_1) is inefficient since both prefer (a_2,b_1,c_2). Nevertheless, this platform is an SSPE outcome in the open-agenda game. We will not specify the complete strategy combination here, but briefly describe the way one can construct such an equilibrium. Along the equilibrium path, the players start with proposing and accepting first c_1 then b_1 and then a_2. Now, if the second issue is settled before c_1, the players play a_1 on the first issue. Similarly, if the first issue is settled with a_2 before the third, the players play a_2 on the second issue afterward.

A similar example can be constructed for the case where only two issues are discussed, but there are more than two alternatives on each issue.

7. Conclusions

The fact that the agenda of multi-issue negotiations has a direct influence on the equilibrium outcome begs the question how one should construct the agenda. Proposition 2 suggests that given consensus over the importance of issues, important issues should be discussed first in order to guarantee stability and efficiency. Propositions 3 and 4 explore the possibility of attaining efficiency through endogenizing the agenda. We have seen that if the agenda is treated as an issue negotiable by members of the committee, then a "wrong" agenda can still be determined in equilibrium, i.e., some equilibria may yield inefficient outcomes. However, if no agenda is fixed and each player can choose the issue on which he wants to propose, then if the underlying problem is simple enough (i.e., two issues, yes-no voting) efficiency and stability can be guaranteed. With more complex platform spaces there would be much more room for contingent behavior. This has a negative effect on the efficiency of the equilibrium outcomes. Our results suggest that under such circumstances, guaranteeing an efficient outcome requires that the agenda be imposed exogenously.

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The Organization of Social Cooperation: A Noncooperative Approach

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Abstract. A noncooperative game model of organizations for social cooperation is presented based on Selten's (1973) model of cartel formation. The possibility of organization with enforcement costs is studied in an n-person prisoners' dilemma situation.

Key words. collective action, cooperation, organization, prisoner's dilemma

1. Introduction

Selten (1973) presents a noncooperative game model of cartel formation among Cournot oligopolistic firms. In the model, the cartel imposes quotas on its members, if such an agreement is reached. Selten shows that five is the critical number of firms for voluntary formation of a cartel. If the number of firms is less than five, then a cartel is formed. Otherwise the probability of the cartel is very small. The purpose of this essay is to show that Selten's pioneering model can be applied to the general problem of organizations for social cooperation.

In recent literature on game theoretic analysis of cooperation, two main approaches have dominated: repeated game approach and evolutionary game approach. The central result, termed the Folk Theorem, in the repeated game shows that cooperation can be attained among self-interested individuals if a game situation is repeated without any limit, and if players do not heavily discount future payoffs. The mechanism of attaining cooperation is mutual punishment among individuals, for defection from cooperation. See Aumann and Shapley (1976) and Taylor (1987) for the repeated game approach.

In evolutionary game theory, it is assumed that a game is played within a population of players from generation to generation. The distribution of strategies over the population is emphasized. The higher the reproductive success one strategy attains, the more likely that strategy is inherited by the next generation. One of the main interests is what kind of strategy can survive in such a selection process, similar to the Darwinian process of natural selection. In his well-known

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computer tournaments of an iterated prisoners' dilemma game, Axelrod (1984) shows us that the TIT FOR TAT strategy (cooperate first, and imitate what the opponent did) is very successful in a certain environment. For surveys on evolutionary game theory, see Maynard Smith (1982) and Hammerstein and Selten (1994).

Although the two approaches have contributed much to our understanding of social cooperation, some aspects seem not to be fully understood. Both approaches assume players to be "passive" to the social environment they face in the sense that they regard the rules of a game as given. In our view, however, one of the important activities of the players as "social beings" is to change and reconstruct the rules of their game, if necessary. The purpose of our research beginning in Okada and Kliemt (1991) is to develop a game theoretic framework for considering the problem of social cooperation from this aspect of institutional arrangements. In particular, we consider the situation where individuals in a society voluntarily form an organization for collective action of cooperation.

We define an organization as a social relationship created by its members for their common benefit. A primary function of the organization is enforcement of its objectives by punishing the opportunistic behavior of members. We explicitly formulate an agency to perform enforcement, and introduce costs associated with establishing it. In our model, the possibility of organization depends upon such enforcement costs in the organization and free-riding behavior of nonmembers.

This essay is organized as follows. Section 2 describes a society as an n-person prisoners' dilemma and presents a noncooperative game model of social organizations. Some implications of the model are given about the formation of organizations. Section 3 extends the model to the case of heterogeneous players. The last section provides two applications of the model: environmental pollution and emergence of the state, and critically discusses our model with regard to future research.

2. A Game Model of Social Organizations

Our society is described as an n-person prisoners' dilemma. Let $N = \{1,...,n\}$ be the set of players. Every player has two actions: C (cooperation) and D (defection). Player i's payoff is given by

$$f_i(a_i, h) \qquad a_i = C, D \quad \text{and} \quad h = 0, ..., n-1,$$

where a_i is player i's action and h is the number of other players who select C. In this section, we assume that all players have identical payoff functions. Player index i is omitted in notations when no ambiguity arises. This assumption of homogeneous players will be relaxed in Section 3.

The payoff function of every player has the following properties:

Assumption 2.1. (1) $f(D, h) > f(C, h)$ for all $h = 0, ..., n-1$, (2) $f(C, n-1) > f(D, 0)$, (3) $f(C, h)$ and $f(D, h)$ are increasing in h.

The implications of the assumption are as follows. Property (1) means that action D dominates action C for every player, i.e., each is better off by taking D than C regardless of what all the other players select. Thus the action combination $(D,...,D)$ is a unique Nash equilibrium point of the game. The equilibrium payoff $f(D, 0)$ is called the noncooperative payoff in the prisoners' dilemma. On the other hand, property (2) means that if all players jointly select C, they are all better off than the Nash equilibrium point. Cooperation is beneficial in the society. Finally, property (3) means that the more other players cooperate, the better every player's life is, regardless of that player's action. In other words, cooperation by one player gives positive externality to other players' welfare. An example of the payoff function is illustrated in Figure 2.1. It is easily seen from Figure 2.1 that there exists a unique integer s_0 $(2 \leq s_0 \leq n)$ such that

$$f(C, s_0-2) < f(D, 0) \leq f(C, s_0-1). \tag{2.1}$$

The integer s_0 shows the minimum number of cooperators that guarantees each of them a payoff higher than or equal to the noncooperative payoff.

Society as the prisoners' dilemma is in an anarchic state of nature in which there exists no social order to enforce collective actions upon self-interested players. Therefore, the natural outcome of that society is the Nash equilibrium point where no players cooperate. Scholars in the theory of Social Contract have presumed that players in society enter, with their free will, into a contract to organize a social institution for enforcing cooperation. We investigate the possibility of voluntary formation of such a social institution, by using game theoretical models. In this paper, we call a social institution created by players for cooperation an *organization*.

An organization in society is formulated by a quadruple $\varphi = (S, p, i^*, C)$. A subset S of N is the set of all members composing the organization. We often refer to the cardinality of the set S as the size of the organization. The second element p, a nonnegative real number, is a punishment imposed on any member in case of defection. When punished, the payoff of a member is reduced by the amount p. The third element i^* is the special agent, called the *enforcement agent*, that does enforcement work for the organization. The final element C is a function assigning to each size s of the organization the costs $C(s)$ of enforcing cooperation on its members. In general, there are many kinds of enforcement cost. One of the

major enforcement costs is the payment, denoted by w, to the enforcement agent by the members of the organization.[1] Others include costs of monitoring members' actions and of punishing deviators. Enforcement costs minus the payment to the enforcement agent is denoted by $M(s)$. Then, the enforcement cost $C(s)$ can be represented by $C(s) = w + M(s)$. The existence of enforcement costs plays an important role in our study of the formation of the organization.

The *constitutional rules* of an organization are regarded as rules for defining the four components above. For instance, there are at least two rules, *free* and *compulsory*, for defining its members. Under the free-participation rule, every player in the society is at a liberty to decide whether to participate in the organization. In contrast, under the compulsory rule, all players must participate in the organization. The punishment level may be determined by a certain mechanism of collective decision (voting and negotiations, etc.) within the organization. There are two types of enforcement agent of the organization: external and internal. In the case of the *external* agent, an agent outside the society is employed for enforcement work in the organization.[2] The payment to the external agent may be determined in a labor market outside the society. For example, if the labor market is competitive, the payment to the enforcement agent will be determined at the opportunity cost of enforcement labor. In the case of the *internal* agent, one member (or members) of the organization is selected as the enforcement agent by political, social or cultural mechanisms. Rules for defining the payment to the internal agent tend to be more complicated than for the external agent. This is due to the emergence of a distributional conflict between the enforcement agent (enforcer) and other members (enforcees) of the organization. In general, as with the punishment, the payment to the internal enforcement agent may be determined by a collective decision mechanism in the organization. Exploitation by either side (enforcer or enforcees) is an extreme case. Finally, the constitutional rules of the organization should include a rule for distributing the enforcement cost among members.

The research agenda in the formation of organizations is divided into two levels:

(1) the constitutional choice problem: how is a constitutional rule of the organization selected?,
(2) the organization formation problem: under a given constitutional rule, is the organization formed? If it is, what components does it have?

Since an answer to the first problem naturally depends upon that to the second problem, it is reasonable to consider first the organization formation problem. In the rest of the section, we consider the constitutional rule of free participation and

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[1] We assume that players have transferable utility. This assumption can be relaxed in a more elaborate model.

[2] An example of the external agent is where a country employs foreign workers for various kinds of public service.

the external enforcement agent as a benchmark of our analysis, and assume that the enforcement cost is exogenously given. Our analysis focuses on what kind of organization can be created under this constitutional rule. We will discuss the constitutional choice problem in the last section.

Our basic model of organization formation is described below as a multistage game.

(1) The participation decision stage

All players i (= $1,\dots,n$) in the society decide independently and simultaneously whether to participate in the organization. Let S be the set of all participants who become the members of the organization.

(2) The bargaining stage for punishment and cost allocation

Enforcement cost $C(s)$ of the external agent i^* is exogenously given. The punishment p and the allocation of enforcement cost $C(s)$ are determined through negotiations among all members. If no agreement is reached, the organization is not formed.

(3) The action decision stage

Every player in the society selects an action, C or D, with or without the organization $\varphi = (S, p, i^*, C)$, depending upon the bargaining outcome in stage (2). The punishment is imposed only on defecting members of the organization.

In each stage, players know perfectly the results of previous stages when they make decisions. Our primary solution concept in analyzing the game is that of a subgame perfect equilibrium point.

To simplify analysis, we assume that punishment p has only two possible values: 0 and p^*.[3] The punishment level p^* is sufficiently high to the extent that cooperation gives a higher payoff to every member of the organization than defection. We assume that the punishment level is determined by unanimity among the members. We also assume that the total enforcement cost $C(s)$ is equally allocated among all members. By these assumptions, the bargaining stage of the model is reduced to the following game. Every member in S selects independently either 0 or p^* as the punishment level. The payoff of each member is given by

$$F(C, s-1) \equiv f(C, s-1) - \frac{C(s)}{s} \tag{2.2}$$

if all members select p^*, and $f(D, 0)$, otherwise. To derive these payoffs, we use the fact that in the final stage of action decision every nonmember of the organization always selects defection and that every member selects cooperation if

[3] See Okada (1993) for a detailed analysis which allows a continuum range of punishment levels.

and only if the punishment level is p^*. We call the payoff $F(C, s-1)$ in (2.2) *the cooperative payoff of the organization.*

We assume that the condition of the prisoners' dilemma (Assumption 2.1) holds true if one replaces $f(C, h)$ with $F(C, h)$ as the payoff function in case of cooperation.

Assumption 2.2. The cooperative payoff $F(C, s-1)$ of the organization is increasing in size s, and $F(C, n-1) > f(D, 0)$.

Figure 2.1 illustrates the properties of the payoff functions $f(C, h)$, $f(D, h)$ and the enforcement cost function $C(s)$ satisfying Assumptions 2.1 and 2.2. Similarly to (2.1), we can see that there exists a unique integer s^* $(2 \le s^* \le n)$ such that

$$F(C, s^*-2) < f(D, 0) < F(C, s^*-1).\text{[4]} \qquad (2.3)$$

The integer s^* shows the size of the organization such that (i) cooperation in the organization is more beneficial to each member than in the noncooperative equilibrium point in the prisoners' dilemma, and (ii) this property never holds if one member deviates from the organization. We call the integer s* the *minimum size of the organization.*

We first analyze the Nash equilibrium point of the reduced bargaining stage with the payoffs given in (2.2). Since the effective punishment level p^* is determined by unanimity, there are many "trivial" Nash equilibrium points leading to disagreement (or zero punishment level) regardless of the organization size. For example, all situations where two groups each consisting of more than one member make different choices, are Nash equilibrium points leading to disagreement. We want to exclude these trivial equilibrium points peculiar to the unanimous voting rule. A simple way to do this is to employ a strict equilibrium point (a Nash equilibrium point in which every player is worse off in case of unilateral deviation from the equilibrium) as the (local) solution in this stage.[5] In what follows, we mean a strict equilibrium point whenever we refer to an equilibrium point in pure strategy. We are mainly concerned with when agreement on effective punishment can be supported in equilibrium. The following theorem is easily proved.

[4] It may be possible that equality $F(C, s^*\text{-}1) = f(D, 0)$ holds in (2.3). In this case, all members of the organization with size s^* are indifferent about establishing the organization or not, but their decision matters much to nonmembers. In order to avoid the problem of which indifferent action is selected, we simply assume as a regularity condition that the strict inequality holds in (2.3).

[5] In general, we can employ the "trembling-hand" perfect equilibrium point of Selten (1975).

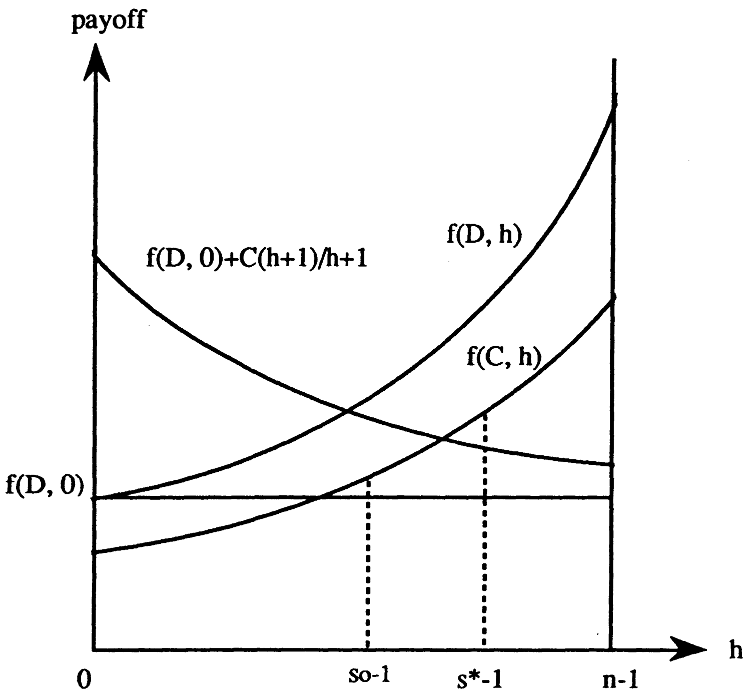

Figure 2.1. An n-person prisoners' dilemma

Theorem 2.1. *Agreement on effective punishment p^* is reached in a unique equilibrium point of the reduced bargaining stage if and only if the organization size is greater than or equal to the minimum size s^*.*

The theorem implies that the success of an organization depends upon a simple criterion, size, in our framework. The minimum size s^* of the organization is the critical size for its success. If the size of the organization is smaller than s^*, then it fails to achieve cooperation among members.

In view of Theorem 2.1, the participation stage is reduced to the following game. Every player i (= $1,...,n$) has two actions: participate (1) or not (0). For an action combination $b = (b_1,...,b_n)$, let $s(b)$ be the number of all participants. Then, every player i obtains the payoff $E_i(b)$ below.

Case (i) $0 \leq s(b) \leq s^*-1$: $E_i(b) = f(D, 0)$,

Case (ii) $s^* \leq s(b)$:

$$E_i(b) = \begin{cases} F(C, s(b)-1) & \text{if } b_i = 1 \\ f(D, s(b)) & \text{if } b_i = 0. \end{cases}$$

The payoff structure of the participation stage is summarized from the point of view of each player as follows. There are three possible outcomes: (i) an organization is formed without the player's participation, (ii) an organization is formed with participation, (iii) no organization is formed. The following theorem characterizes pure equilibrium points of the participation stage.

Theorem 2.2. *An action combination $b = (b_1,...,b_n)$ in the participation stage is a (strict) equilibrium point if and only if exactly s^* players participate in the organization.*

Proof. The *if* part is proved from inequalities $f(D, 0) < F(C, s^*-1)$ and $F(C, s^*) < f(D, s^*)$. Any action combination with s ($> s^*$) participants is not a Nash equilibrium point because every member is better off under unilateral deviation from the organization. If $s < s^*$, then the action combination is not a strict equilibrium point.[6] Q.E.D.

The theorem first implies that if $s^* = n$, the full organization can be attained in an equilibrium point of the participation stage. In this case, moreover, we can see that from the point of view of every player the action of participation weakly dominates that of nonparticipation. Therefore, there is no Nash equilibrium point in mixed strategy. We can derive from these results the conclusion that all players in the society participate in the organization for cooperation in this case. Second, the theorem implies that in general cases only a minimal organization with size s^* can be supported in a pure equilibrium point due to every player's incentive to free-ride the organization. Then, a further question arises: "Who participates in the minimal organization?" Various kinds of formal and informal institutions such as moral attitudes, seniority-rule, leadership and sequential moving may be needed to answer the question. In an institution-free society, a symmetric equilibrium point seems to be more appropriate as our solution than an asymmetric one.[7] A trivial symmetric (non-strict) equilibrium point is the situation that no players participate. The next theorem shows the other symmetric equilibrium point.

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[6] If $s = s^*-1$, it is not a Nash equilibrium point, and if $s < s^*-1$, it is a Nash equilibrium point, but not a strict one. See Okada (1993).

[7] Roughly speaking, a symmetric equilibrium point is a Nash equilibrium point in which identical players employ identical strategies.

Theorem 2.3. *There exists a unique symmetric equilibrium point in mixed strategy in the participation stage. Participation probability t of every player is the solution of the equation*

$$\sum_{k=s^*}^{n-1} \binom{n-1}{k} \Big/ \binom{n-1}{s^*-1} \cdot \left(\frac{t}{1-t}\right)^{k-(s^*-1)} \frac{f(D,k)-F(C,k)}{F(C,s^*-1)-f(D,0)} = 1. \qquad (2.4)$$

Proof. For a mixed strategy combination $q = (q_1,...,q_n)$, let $E_i(q)$ be the expected payoff of player i. A symmetric mixed strategy combination with participation probability t (> 0) is a Nash equilibrium point if and only if $E_i(q/1) = E_i(q/0)$ for every i, where $q/1$ and $q/0$ mean the strategy combinations obtained from q if only player i's strategy q_i is replaced with the pure strategy of participation and of nonparticipation, respectively. We can see that this condition is equivalently reduced to (2.4). The left-hand side of (2.4) is a continuous and increasing function of t over [0, 1), and moreover it takes zero if $t = 0$, and diverges to infinity as t goes to 1. Therefore, (2.4) has a unique solution. Q.E.D.

The theorem implies that participation probability of every player depends upon three factors: population n, the minimum size s^* of the organization and the ratio of payoff differences, $\{f(D, k) - F(C, k)\}/\{F(C, s^*-1) - f(D, 0)\}$ for each $k = s^*,...,n-1$. The last factor, called the *incentive ratio*, means the ratio of player incentive (payoff gains) to deviate from the organization of size k + 1 over incentive to join the minimal organization.

Since the pioneering work of Olson (1965), the effect of group size on the likelihood of group success has been one of the central questions in the theory of collective action. The next theorem gives one implication of our model to the group size problem in terms of player participation probability. For the proof, see Okada (1993).

Theorem 2.4. *Assume that the minimum size s^* of the organization and the incentive ratios are constant with respect to population n. Then, participation probability t(n) decreases and converges to zero as n goes to infinity.*

When a society is so small that the population is equal to the minimum size s^* of the organization, all players join the organization for cooperation. If the population grows, the probability of every player participating in the organization decreases, becoming close to zero in a very large society. In this sense, social cooperation is more likely in a small society than in a large society.

3. Heterogeneous Players

In the previous section, all players are assumed to be identical: they have the same payoff functions. We here consider how heterogeneity among players affects the formation of an organization.

One simple way to introduce heterogeneity among players into our model is to assume that the minimum size s^* of cooperation defined in (2.3) is not necessarily common to all players. Taking this idea, we assume that for every player i (= 1, ..., n) there exists a unique integer s_i ($2 \leq s_i \leq n$) such that

$$F_i(C, s_i - 2) < f_i(D, 0) < F_i(C, s_i - 1) \tag{3.1}$$

where the functions f_i and F_i correspond to player i's payoff functions f and F, respectively, in the previous section.

The game model of organization formation in the previous section can be easily extended to the heterogeneous case. Keeping the same assumptions about the enforcement costs and their allocation rule, our analysis of the last two stages, the bargaining stage for punishment and cost allocation and the action decision stage, can be applied with minor modifications. Therefore, it suffices to analyze only the participation decision stage.

The participation decision stage of the organization game with heterogeneous players is described as follows. Each of n players has two actions: participate in the organization (1) or not (0). Player i (= 1,...,n) is characterized by the minimum size s_i ($2 \leq s_i \leq n$) of cooperation defined by (3.1). For each subset S of N and each $t = 2,...,n$, let $g_S(t)$ be the number of members of S whose minimum sizes of cooperation are t. The function $g_S(t)$ represents the distribution of members' characteristics in organization S in terms of their minimum sizes of cooperation, and we call it the *characteristic distribution* of organization S.

Unanimous agreement on effective punishment is reached in the bargaining stage for an organization S if and only if its size satisfies $s_i \leq |S|$ for every member i. Such an organization is called *successful*. From the fact above, we can derive the payoff of every player in the participation decision stage as follows. For an action combination $b = (b_1,...,b_n)$, let $S(b)$ denote the set of all participants. Then, the payoff $E_i(b)$ of player i is given by:

(i) if $S(b)$ is successful,

$$E_i(b) = \begin{cases} F_i(C, |S(b)| - 1) & \text{if } b_i = 1 \\ f_i(D, |S(b)|) & \text{if } b_i = 0, \end{cases}$$

(ii) otherwise, $E_i(b) = f_i(D, 0)$.

Theorem 3.1. *An action combination $b = (b_1,...,b_n)$ in the participation decision stage with heterogeneous players is a (strict) equilibrium point if and only if the set $S(b)$ of participants satisfies one of two properties below.*

(1) $S(b)$ is a minimal successful organization,
(2) $S(b)$ satisfies $g_S(2) + ... + g_S(|S|) = |S|$ and $g_S(|S|) \geq 2$.

Proof. (1) Let $b = (b_1,...,b_n)$ be a (strict) equilibrium point. Suppose, on the contrary, that property (1) does not hold. Consider first the case that $S(b)$ is not successful. Then, every player i's payoff is $f_i(D, 0)$. If one participant deviates from $S(b)$, the payoff is either $f_i(D, 0)$ or $f_i(D, |S(b)|-1)$, depending upon whether the remaining organization is successful. Since $f_i(D, |S(b)|-1) > f_i(D, 0)$, this contradicts that the action combination b is a strict equilibrium point. Consider next the case that $S(b)$ is a successful organization but not a minimal one. Then, there exists some member i of $S(b)$ such that the organization remains successful despite individual deviation. In this case, player i obtains the payoff $f_i(D, |S(b)|-1)$ higher than the payoff $F_i(C, |S(b)|-1)$. A contradiction. It can be easily proved that if property (1) holds, the action combination b is a strict equilibrium point.
(2) We prove that two properties in the theorem are equivalent. First, we can see without much difficulty that an organization S of N is successful if and only if $g_S(2) + ... + g_S(|S|) = |S|$. Let S be a successful organization. Suppose that $g_S(|S|) \geq 2$. If any one member deviates from S, the resulting organization is no longer successful. Thus, S is a minimal successful organization. Suppose that $g_S(|S|) = 1$. Then, if player i with $s_i = |S|$ deviates from S, the resulting organization is still successful. Finally, suppose that $g_S(|S|) = 0$. Then, the minimum size s_i of cooperation for every member of S is less than or equal to $|S| - 1$. This implies that if any member deviates from S, the resulting organization is still successful. Q.E.D.

Theorem 3.1 shows that a (strict) equilibrium point of the participation decision stage necessarily leads to a minimal successful organization. The theorem also characterizes a minimal successful organization by using the characteristic distribution function of organizations. An organization is successful if and only if its size can meet the minimum size of cooperation for every member. The minimality of a successful organization is given by the very simple condition of $g_S(|S|) \geq 2$. That is, the organization must have more than one member to whom its size is at the minimum level for beneficial cooperation. Figure 3.1 illustrates an example of the characteristic distribution of players and a minimal successful organization. In the figure, each block corresponds to one player and the shaded area shows a set of players forming a minimal successful organization.

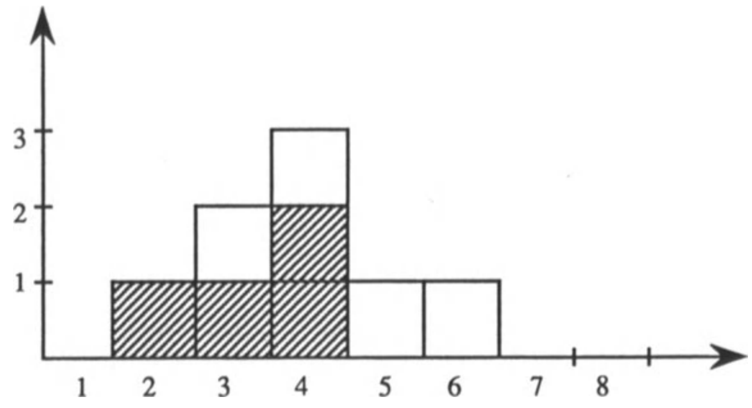

Figure 3.1. The characteristic distribution of players

The following corollary about the possibility of full cooperation is easily obtained from Theorem 3.1.

Corollary. *The largest organization N can be formed in an equilibrium point of the participation decision stage if and only if* $g_N(|N|) \geq 2$.

The corollary says that if there are at least two players whose minimum sizes of cooperation are n, it is possible in equilibrium that all players cooperate in the society with heterogeneity. In contrast, such full cooperation can be attained in the society with homogeneous players only when *all* players' minimum sizes of cooperation are n. These results seem to suggest that full cooperation may be more likely in a heterogeneous society than in a homogeneous society.

4. Discussion

We provide two applications of our model of organizations for social cooperation.

(1) Environmental Pollution

Environmental pollution is a typical and serious example of social dilemma in which cooperative actions by players are needed in order to escape tragedy. In

Okada (1993), we considered voluntary organization for environmental regulation by applying our model to the "Lake" game (Shapley and Shubik 1969), in which factories decide whether to treat wastes before discharging them into a lake. A simulation of the model shows that probability of organization converges to some positive value (around 0.69) when the number of factories becomes large, despite the asymptotic behavior of each factory's participation probability converging to zero (Theorem 2.4). It may be possible that (partial) cooperation can be attained even in a large society.

(2) The emergence of the state

The state is a complex political organization. Although there have been diverse points of view on it in the literature, one of the traditional explanations for the state is the idea of the Social Contract. People in a society make a contract to create the state in order to avoid "Warre of every one against every one" (Hobbes, 1651) in the state of nature. The primary function of the state is to defend its constituents from invasion by foreigners and injuries inflicted on one another, so that they may nourish themselves and live contentedly. To achieve such a common benefit, the state has enforcement power to control the constituents' behavior. If one takes this point of view to the state based on the Social Contract, a theoretical issue arises: how is formation of the state consistent with the free-rider problem?

In Okada and Sakakibara (1991), we investigated the emergence of the state as a tax enforcement institution in a simple dynamic model of a production economy. In the economy, individuals of nonoverlapping generations produce private goods by utilizing social overhead capital as public goods. Due to the free-rider problem, there is no accumulation of capital stock without the state. We considered a democratic state of which constitutional rule is characterized by free participation and an internal enforcement agent. The enforcement agent is elected from among participants. All participants decide by unanimous voting the payment to the enforcement agent before the election.

It is shown that there exists the critical level of social productivity, measured in terms of the capital stock gifted by the previous generation. If the society is so "rich" that social productivity is beyond the critical level, the state is never established. If not, the formation of the state is stochastically determined (see also Theorem 2.3). Under the assumption that population and marginal productivity of the public good are constant over generations, we can show that social productivity stochastically converges to (and beyond) the critical level over a sufficiently large number of generations. In this sense, the end of the state necessarily comes in the long run. The main reason for this result is that the payment to the enforcement agent increases as the capital stock accumulates. If this enforcement cost exceeds the critical level, no enforcees find it beneficial to be burdened with such cost.

To conclude this essay, we present some critical discussion of our analysis. First, our model assumes that any nonmember of an organization can perfectly free-ride cooperative actions by its members. Of course, the possibility of free-riding depends upon specific social contexts in which our game situation is

embedded. For example, in the case of the state, it may be practically difficult for nonmembers of the state to free-ride a common benefit provided by the state. Once established, the state tends to be a "Leviathan" possessing enough physical power to prevent free-riding. On the other hand, in the environmental pollution problem, many empirical studies show that it is quite difficult for voluntary associations for regulation to prevent nonmembers' free-riding. We think that our model allowing free-riding by nonmembers can serve as a benchmark of our analysis of organizations.

Second, our model does not treat the enforcement agent as a real player. We assume that the enforcement agent does its work sincerely. A serious problem in our framework of institutional arrangements is: "Who will take care of the care-takers?" (Arrow 1974, p.72).[8] To investigate this problem of controlling the enforcement agent, we will have to elaborate our model of organizations with a richer internal structure. The division of authority is an important issue.

Third, our model considers the formation of only one organization in a society. It will be interesting to extend our model so that we can consider the formation of multiple organizations, and to analyze conflict and cooperation among them.

Finally, the most criticized aspect of our analysis is probably its static nature. The central questions in the theory of organizations are: how does the institutional structure of the organization evolve in the process of social development; and how do organizations affect the development of a society? In a recent work (Okada, Sakakibara and Suga 1995), we extend our analysis of the emergence of the state in this direction. We consider the centralized political system (monarchy) of the state as the alternative system to democracy and investigate how the political system of the state evolves in the process of economic development. It is shown that monarchy may be selected as the result of a social contract by individuals when social productivity is low, and democracy is selected when social productivity becomes sufficiently high and close to the critical level. The political system tends to evolve from monarchy to democracy in the long run of economic development. An interesting theoretical issue in our future research is to advance our analysis of organizations toward a game theory of social development.

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[8] Kliemt (1990) discusses some issues arising when the enforcer is a strategic player.

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Reinhard Selten Meets the Classics

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Abstract: Most of the more empiristically minded among the modern classics of political philosophy tried to give a rational choice account of the emergence and maintenance of social order. Subsequently we shall relate some aspects of Reinhard Selten's work to this tradition. His insights can further our understanding of fundamental classical queries: Jean Bodin's quest for understanding the implications of the sovereign's lack of commitment power, Benedict de Spinoza's scrutiny of opportunism in trust relationships, John Locke's problem of founding all legal institutions on individually rational consensus in anarchy, and Thomas Hobbes' efforts to explain all aspects of social order in terms of individually rational choice. Relating simple game models to excerpts from classical texts sheds modern game theoretic light on the works of the classics and some classical light on game theory.

Introduction and Overview

All modern classics of social theory were struggling in one way or other with the question whether, and if so, how the emergence and maintenance of social order can be explained in terms of individually rational choice. Recent non-co-operative game theory offers new and fruitful tools for addressing this fundamental problem of all social theory. At the same time going back to the modern classics and their most fundamental problem may put into perspective the game theoretic enterprise as well.

We hope to demonstrate subsequently that some of the basic insights of Reinhard Selten and the classics are quite intimately related to each other. We shall start by putting Jean Bodin's argument that sovereign power of necessity must be absolute into Reinhard Selten's game theoretic perspective (I.). It is then illustrated that this perspective also sheds some new light on Benedict de Spinoza's theory of contracting (II.), the Lockean idea of social contract (III.) and finally the so called "Hobbesian problem of social order" (IV.).

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I. Reinhard Selten Meets Jean Bodin

Jean Bodin is generally regarded as a propagandist of absolutism, as a kind of ideologue favoring the absolute and unrestricted power of the sovereign. Adherents of limited government therefore tend to reject Bodin's arguments for normative reasons. But rather than dismissing the unwelcome arguments out of hand they deserve to be taken seriously. Modern game theory suggests that Bodin's basic insight was a much deeper one than presumed by most political theorists. Bodin neither assumed that sovereigns were power seeking individuals -- at least not to a greater extent than ordinary individuals -- nor did he suggest that they should act as such. He rather doubted that a rational sovereign *could* in fact restrict his own sovereignty even if he wanted to:

> "If then the soueraigne prince be exempted from the lawes of his predecessors, much lesse should he be bound vnto the lawes and ordinances he maketh himselfe: for a man may well receiue a law from another man, but impossible it is in nature for to giue a law vnto himselfe, no more than it is to command a mans selfe in a matter depending on his owne will: For as the law saith, *Nulla obligatio consistere potest, quae voluntate promittentis statum capit,* There can be no obligation, which taketh state from the meere will of him that promiseth the same: which is a necessarie reason to proue euidently that a king or soueraigne cannot be subiect to his owne lawes. And as the Pope can never bind his owne hands (as the Canonists say;) so neither can a soueraigne prince bind his owne hands, albeit that he would." (Bodin 1576/1606, 92).

Since the sovereign prince *cannot* bind his own hands he and his subjects suffer a utility loss. The prince as well as his subjects could be better off if only he could bind himself by his own word. But since the sovereign reigns supreme, he is beyond bounds (or absolutus). As the highest (or sovereign) normative authority in a state he cannot bind himself by (self-enacted) norms. He cannot give "binding" promises and therefore is "bound" to be non-trustworthy. Subjects know this (as does the sovereign himself). Therefore the sovereign's subjects know that their wealth and property are unsecure.

As long as the supreme power in the state cannot restrict its own future exploitative actions and interests, the threat of future exploitation cannot be controlled -- not even by the sovereign himself. Economic history illustrates that this may lead to severe problems for all. In particular, economic growth may be reduced considerably whenever individuals are expecting that they might fall prey to their governments' exploitative acts. Citizens will under-invest in long run projects. The accumulation of capital and wealth will be sub-optimal. Resources will be misallocated. Capital will be used in ways that cannot be taxed with ease rather than in the most productive ways.

On the other hand, if people could trust their prince distortions in the allocation of resources would be reduced. Citizens could then rationally incur the risk of investing into activities that in principle would make them vulnerable to

exploitation. If only they could trust that their prince -- or, for that matter, their parliament -- will not act opportunistically this might make all better off -- including prince or parliament (cf. Jones 1981 and Rosenberg and Birdzell, 1986 for historical evidence that restrained governments do better).

Still, who would trust an unconstrained sovereign? The prince may well understand that all including himself could conceivably be better off could his subjects rationally trust him. However, since he cannot commit to a future course of action merely by his own prior acts of will, he and his subjects are facing a dilemma: Being rational they understand that trustful cooperation among rational individuals would be better for all individuals concerned and at the same time they understand that, being rational, this course of action is closed off.

In a more formal vein the underlying dilemma can be illustrated by the *elementary game of trust* of figure 1:

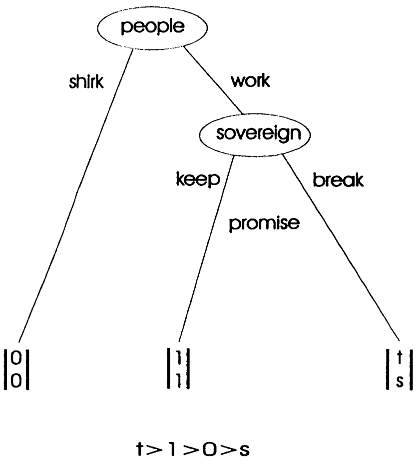


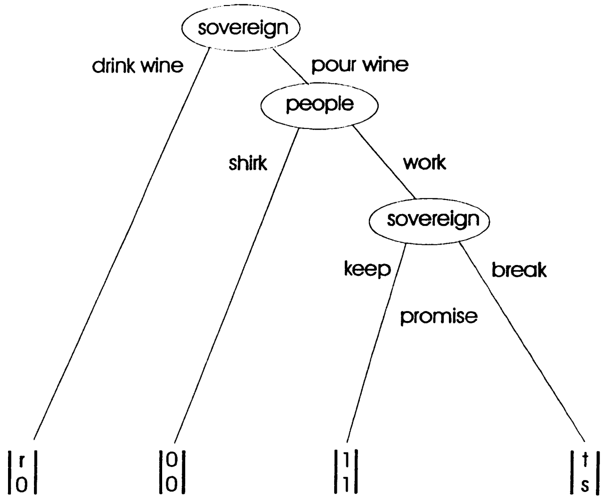
Figure 1

In this tree it is assumed that all individuals of the general population are identical twins who receive the same payoff. The first component of each pay off vector refers to the sovereign while the second indicates the preferences of the people. Since t>1 the sovereign will exploit his people if they choose to work. Anticipating the sovereign's opportunistic behavior the people will shirk or more generally shy away from "promising" investments (cf. for an alternative approach the „peasant-dictator game" Van Huyck et al. 1995).

The proper subgame of figure 1 has exactly one equilibrium. In the terminology of Reinhard Selten (1965): The strategy combination "shirk, break promise" is the only subgame perfect equilibrium of the game of trust. A rational sovereign will exploit his people if they offer him a chance. Should he declare in

so many words that he has no ill intentions this will not change the situation. Enacting some law will not work either, since the sovereign being endowed with supreme legal authority can renounce the law whenever this suits his purposes. Thus, the sovereign cannot alter the expectations that his subjects rationally entertain with respect to his behavior in the proper subgame. The strategy combination ("work", "keep promise") is not an equilibrium if the rational sovereign is unbound by his own orders.

The preceding argument seems quite compelling from the point of view of modern non-cooperative game theory. Nevertheless, modern philosophers tried to cast some doubts on it. Again their argument can be illustrated by an elementary game tree (cf. figure 2):



t>1>r>0>s

Figure 2

Now the sovereign first decides whether or not he should drink or pour out his best wine. This decision seems quite unrelated with his later decisions as a ruler. Nevertheless, some modern theorists suggest that the sovereign can overcome his lack of commitment power, by pouring out the wine and thus signaling his "good" intentions (cf. for instance Reny 1992 and Bicchieri 1989): Everybody -- including the prince and his subjects -- anticipates that a rational prince is going to behave opportunistically rational in the final subgame should people decide on working rather than shirking. Anticipating his people's anticipation of his breach of promise the rational prince will drink his wine. The prince also anticipates that his people expect this. But the prince can falsify the latter anticipation by pouring

out the wine. Doing this he signals that he is willing to behave in non-subgame perfect ways. He signals that he is going to violate Jean Bodin's precepts of "princely" behavior, or so the argument goes.

Put that bluntly the modern philosophers' argument does not seem very appealing, to say the least. But if any further defense of Bodin against his disrespectful colleagues is needed one can trustfully turn to Reinhard Selten again, and bring into play the concept of a perfect equilibrium (cf. 1975).

Princes may be quite old before they can get on the throne, even the youngest after seizing power may once in a while loosen their grip and some might drink more than their due. In any case, their hands like any other person's may tend to tremble. Thus, though trying to drink his wine, many a prince will once in a while pour, rather than drink it. If this is so, the citizens, after observing that the wine has been spoilt rather than being drunk, cannot infer that a prince intended to behave contrary to his preferences. Maybe he was drunk and after the sobering effect of tragically loosing his best wine will be even more eager to tax his citizens because he wants to buy new wine.

More seriously, rational individuals in analyzing a game will always discriminate between "plan and play". From their point of view, introducing some "trembles" such that all moves are "kept in play" so to say, is motivated by their analytical aims rather than by empirical assumptions. To put it slightly otherwise, among individuals who ascribe full rationality to each other, the assumption of small trembles -- in particular uniform trembles -- is not justified by potential deficiencies of rationality, but quite to the contrary rather expresses that fully rational individuals will always analyze and keep in mind the whole game tree when planning their play.

More realistically, the sovereign body of a modern parliamentary democracy is constituted anew every election period. Every freshly elected parliament is a new agent that is only loosely linked with the old one. Therefore any sovereign body faces seemingly insurmountable difficulties in signaling intentions for a subsequent one. Citizens who are aware of this could not rationally assume that "declarations of intent" of a former parliament signal any intentions of a later one, unless there would exist some commitment mechanism transcending the powers of the sovereign parliament. With shifting majorities this would hold true even if the people themselves formed the sovereign. As Bodin had it, if the ultimate power to make the law is vested with the sovereign and if the sovereign is assumed to make choices according to the preferences prevailing in each instance of choice, then it can hardly be imagined how a sovereign (person, parliament, or people) could *choose* to become committed.

This again may be captured well by a game theoretic notion frequently employed by Reinhard Selten, namely that of an agent normal form (cf. Harsanyi and Selten 1988, 34). In the agent normal form of the game of figure 2 the sovereign is split into two players. One player chooses between drinking and pouring the wine and one chooses between keeping or breaking his promise. If we apply the "present motives" interpretation of utility, the game tree and, for that

matter, the agent normal form show the preferences governing decisions of decision makers at every instance of choice. Therefore the notion of the agent normal form is almost implied by that of a utility index representing all relevant motives of the players at all their decision nodes.

Clearly, if Bodin is taken by his word, the agent normal form captures the essential feature of his concept of sovereignty. We might therefore feel that his concept of sovereignty takes us to extremes that conceivably may have some relationship with princely behavior but certainly not with our more pedestrian day to day problems. However, if we strictly stick to the assumptions of forward looking rational choice, Bodin's notion of a sovereign prince directly applies to anybody who is entitled to make some decision as a "sovereign" decision maker. Making that "democratic" move we automatically move on to the general problems of contracting and promising faced by all individuals in their several rational capacities. Benedict de Spinoza addressed these problems in most perceptive and in any case "classical" ways.

II. Reinhard Selten Meets Benedict de Spinoza

In his discussion "of the foundations of the state" in the "theologico-political treatise" Benedict de Spinoza says:

> "Now it is a universal law of human nature that no one ever neglects anything which he judges to be good, except with the hope of gaining a greater good, or from the fear of a greater evil; nor does anyone endure an evil except for the sake of avoiding a greater evil, or gaining a greater good. That is, everyone will, of two goods, choose that which he thinks the greatest; and of two evils, that which he thinks the least. I say advisedly that which he thinks the greatest or the least, for it does not necessarily follow that he judges right. This law is so deeply implanted in the human mind that it ought to be counted among the eternal truths and axioms.
> As a necessary consequence of the principle just enunciated, no one can honestly forego the right which he has over all things, and in general no one will abide by his promises, unless under the fear of a greater evil, or the hope of a greater good... Hence though men make promises with all the appearances of good faith, and agree that they will keep to their engagement, no one can absolutely rely on another man's promise unless there is something behind it. Everyone has by nature a right to act deceitfully, and to break his compacts, unless he be restrained by the hope of some greater good, or the fear of some greater evil." (Spinoza, 203-204)

This is a remarkably clear statement of basic assumptions of a homo oeconomicus model of human behavior. It, quite strikingly, alludes to subjectivism and at the same time spells out some of the implications of the assumptions of individually rational choice behavior for the institutions of promising and contract: Without sanctions promises and contracts will be void.

Traditionally, at this juncture, some external enforcement mechanism based on the sovereign power of the state is wheeled in. Since, however, the sovereign -- reigning absolute -- suffers from the same deficiencies and problems as his rational subjects, such an exogenous explanation will eventually not suffice. An account of how promise keeping may be explained endogenously in social interaction should be given.

Under Spinoza's behavioral assumptions a social institution of promising can emerge and be maintained only if under most circumstances no individual of the relevant reference group (family, tribe, nation, human society in general) can gain from breaking promises. In each instance it must pay off in terms of expected future rewards or punishments to keep one's promises. Evidently, without an external enforcement mechanism, this can be achieved only if individuals are engaged in ongoing interactions or repeated games.

To have something specific in mind, imagine that two individuals 1 and 2 have to play the game of trust of figure 1 repeatedly. On each round of play with equal probability both adopt the roles of the first and second mover respectively. Clearly, if the interaction is repeated indefinitely, it follows from the Folk Theorem that on each round of play choosing to work in the role of the first and keeping the promise in the role of the second mover can be justified as a subgame perfect equilibrium outcome.

Those who argue that subgame perfectness captures the most fundamental aspect of future directedness of individually rational choice are at least to a certain extent right. However, they better listen to what else the inventor of the concept has been saying.

First of all Reinhard Selten would in all likelihood reject that the assumption of an infinite number of interactions is a reasonable approximation of any real world process. Although some experimentalists (e.g. Weg, Rapoport and Felsenthal 1990) claim to study such situations, they in fact cannot, as long as any round of play requires a positive minimum time.

Secondly, the objection against the assumption of indefinite repetition can be strengthened by drawing attention to Reinhard Selten's seminal paper "Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit" (Selten 1965) where he considered a dynamic game with both an infinite and a finite horizon. Selten insisted that the solution for the infinite horizon game has to be asymptotically convergent, i.e. must be approximated by the solutions of the finite horizon games as the number of repetitions goes to infinity. However, as is well known, arguments following the logic of the Folk Theorem do not comply with this reasonable requirement of approximation.

Thirdly, Reinhard Selten could also object to the use -- or abuse -- of the Folk Theorem in recent social theory by drawing attention to his concept of subgame consistency (cf. Selten and Güth 1982, as well as Harsanyi and Selten 1988). In an infinitely repeated game of trust all subgames, where the second mover has to choose initially between keeping and breaking the promise, are strategically equivalent. Subgame consistency requires that therefore their solutions should be

the same (up to isomorphisms). But, then, the second mover's choice must either be always to keep or always to break the promise. Since the previous behavior of a second mover does not influence later decisions of a first mover, always to keep one's promise is suboptimal in these subgames. Thus Folk Theorem arguments cannot provide a normatively convincing potential explanation of how the institution of promising could emerge and be maintained among rational individuals.

It seems that any institution similar to our real world institutions of promise and contract will violate some plausible rationality requirement or other (cf. Lahno 1994 for a proof that the requirement of subgame perfectness necessarily conflicts with our real world practices of promising). But if the premise that all rational actions are chosen in view of the subjectively expected causal consequences of the actions (each taken separately) must be given up in any case, we may quite naturally wonder whether a more fundamental change of perspective or approach may not be desirable.

It may be worth recalling first that classical empiristic modeling of social and in particular political interaction was followed by evolutionary modes of thought (cf. on the to a large extent undeservedly ill-reputed history of social Darwinism for instance Hofstadter 1969). Analogously, perhaps, "classical" game theoretic modeling will be followed by a switch towards evolutionary game theoretic modeling.

Again we find Reinhard Selten on the forefront of research (cf. Selten 1983 and Hammerstein and Selten 1984). But rather than considering his views in the abstract let us return to the example of the game of trust, though now in an evolutionary setting. If in such a setting both individuals can adopt both roles -- that of a second and that of a first mover -- with equal probability, the disposition to work rather than to shirk should be driven out by the evolutionary process. As long as there is a positive probability that any individual will work, exploitation will be superior. If it is superior to exploit others the proclivity to exploit will spread. This in turn will make it even less rewarding to work. There seems to be but one stable state in which this process comes to an end: a state in which nobody works. However, if work is avoided completely all strategies prescribing shirking in the role of the first and arbitrary behavior in the role of the second mover fare equally well against each other; i.e. no such strategy is evolutionarily stable.

Thus the concept of an evolutionarily stable strategy evidently has its own limitations. These can be overcome if we take recourse to Reinhard Selten's concept of a limit evolutionarily stable strategy (Selten 1988). Applying that concept to the elementary trust game of figure 1 we assume that "work" is chosen with some (arbitrarily small) positive minimum probability. Since in a complex environment phenotypes will tend to make mistakes, this assumption will be empirically sound in general. Under this assumption always exploiting others will be evolutionarily stable.

More generally, considerations like the preceding ones may give rise to a more indirect evolutionary approach (cf. on applying this to the evolutionary game of trust Güth and Kliemt 1995). Since it is possible in this approach to treat as endogenous parameters like preferences or behavioral dispositions it offers promising avenues for future research; but rather than speculating about the future, let us stick to the past and turn to how the classics in fact did explain the emergence of social institutions, namely by a social contract to that end.

III. Reinhard Selten Meets John Locke

Modern political theorists like Buchanan, Nozick and Rawls have revived the basic ideas of the so-called old contractarians Hobbes, Locke and Kant. In particular they have scrutinized to what extent a social order could be justified by a conceivable consensus of rational individuals. Much of the discussion has, of course, focused on what kind of consensus is conceivable under what kinds of circumstances. Less interest was devoted to the problem whether, and if so, under which circumstances, a contract is itself conceivable, if contracting must take place in a state of nature. But how could individuals conceivably contract with each other in a state of nature in which the institution of contracting is lacking? Moreover, if this is not conceivable, how could the standard contractarian logic that a conceivable contract provides normative criteria for right and wrong in real world matters be valid?

None of the contractarians has yet managed to solve these problems convincingly. Still, since all normative political theories seem to be deficient in one way or other, and contractarianism is at least a well developed one, it is worthwhile to explore how far it goes. In doing so we can turn to John Locke and conceive of social contracting basically as a process of club formation. In his discussion "of the beginning of political society" John Locke says

> "(m)en being, as has been said, by Nature, all free, equal and independent, no one can be put out of this Estate, and subjected to the Political Power of another, without his own *Consent.* The only way whereby any one devests himself of his Natural Liberty, and *puts on the bonds of Civil Society* is by agreeing with other Men to joyn and unite into a Community, for their ... greater Security against any that are not of it. This any number of Men may do, because it injures not the Freedom of the rest; they are left as they were in the Liberty of the State of Nature. When any number of Men have so *consented to make one Community* or Government, they are thereby presently incorporated, and make *one Body Politick* ... And thus that, which begins and actually *constitutes any Political Society,* is nothing but the consent of any number of Freemen..." (Locke 1690/1963, 374-377).

One should note that according to this passage it is *any* number that may consent to form a body politic. Those who do not want to join are free to stay out. At the same time, they do not have a veto against any number of other individuals going

ahead in forming such a club. This differs for instance from the Buchanan-Tullock notion of unanimous consent granting a veto to each individual of a predefined collective (cf. 1962) as well as from Kantian contract notions starting from all rational beings as an exogenously determined collective of normatively relevant individuals.

Having a veto implicitly presupposes a collective monopoly over actions to which the veto applies. But from the point of view of individualist values the assumption of a normatively justified collective monopoly seems a non-starter. The monopoly is exactly what should be justified as a result of the contractarian argument. Likewise, the Kantian assumption that the collective of normatively relevant individuals is exogenously determined, is stronger than necessary within a contractarian approach. At least in a way it seems conceivable that moral and legal communities form as self-selected groups or clubs.

A most austere process of forming associations in anarchy is again suggested by one of Reinhard Selten's models. In his study of cartel formation (cf. on this 1973) he suggested how such a club may form -- not necessarily an all inclusive one. First of all, individuals decide whether they are in principle interested in joining the club. Those who are will bargain then about the terms of the club's bylaws or "constitution". Finally the interaction itself takes place. When interacting with each other, insiders of the club are bound by the rules of the club, while outsiders interact unrestrictedly.

The game of trust under random matching of partners as previously discussed may serve as an illustration here. Assume that the game is played by n (≥ 2) individuals in the following way: As before, each individual expects with probability 1/2 to adopt the role of a first mover and with probability 1/2 that of a second mover. However, now, after formation of the club, members are certain to be matched with a club member, whereas non-members will be excluded from interaction with members and therefore interact with non-members exclusively.

The sole purpose of forming the club is controlling individual behavior such that "work and keeping promises" will emerge as the general behavioral pattern of interaction between insiders. Control is costly, though. Whenever an individual performs an act of control it incurs a positive monitoring cost c. Since every member must monitor every other one the monitoring costs incurred by each member of a club with m, $2 \leq m \leq n$, individuals are (m-1)c. A non-member receives a pay off of 0 while members get 1 on each round of play. Therefore individuals will want to join such a club if $1-(m-1)c \geq 0$ or $c \leq 1/(m-1)$. Thus only in case of $c \leq 1/(n-1)$, the all inclusive club will form (cf. for some further details of applying Reinhard Selten's ideas on club formation to Lockean contractarianism, Okada and Kliemt 1991).

This simple model sheds some light on social contracting as a process of club formation like the one envisioned by Locke and Nozick. Unless c is extremely small relative to "1" the size of clubs with decentralized monitoring is severely restricted. Thus the standard contractarian assumption that eventually an all-inclusive club will form spontaneously in anarchy, seems quite doubtful. For, the

formation of larger groups in which individuals can trustfully interact with each other in situations of the structure of the game of trust seems to depend on some central monitoring and control (cf. for a model in which "extrinsic" is substituted by "intrinsic control", cf. Güth and Kliemt 1995). If, however, central monitoring and control is necessary from the outset the aforementioned latent circularity of contractarian reasoning immediately re-emerges. Since this problem of circularity is nothing but a variant of the so-called Hobbesian problem of social order, let us finally turn to Hobbes.

IV. Reinhard Selten and the Hobbesian Problem of Social Order

Alfred North Whitehead once remarked that after Plato all philosophy became a commentary on Plato's work. Though it would be somewhat exaggerated too, it would not be grossly misleading either to say that all modern political philosophy after Hobbes in one way or other turned into commentary on Hobbes. In any case, Hobbesian views are deeply entrenched in economics and game theory. Above all, like Spinoza, modern theorists in these fields still tend to accept the Hobbesian view that

> "(i)t is also manifest, that all voluntary actions have their beginning from, and necessarily depend on the will, and that the will of doing, or omitting aught, depends on the opinion of the good and evil of the reward or punishment, which a man conceives he shall receive by the act or omission; so as the actions of all men are ruled by the opinions of each" (De Cive 1651/1982, 75) Therefore, "insomuch as when they shall see a greater good, or less evil, likely to happen to them by the breach, than observation of the laws, they will wittingly violate them." (De Cive 1651/1982, 63) "We must therefore provide for our security, not by compacts, but by punishments." (De Cive 1651/1982 72/73)

However, taking into account what has been said before, how could we, as rational Hobbesian men, ever hope to "provide for our security"? As rational citizens (cive) we are aware of Bodin's problem. We know that ,unless we can answer the perennial question of "who will guard the guardians" ("quis custodiet ipsos custodes?"), there will be no security against an enforcement agency vested with the ultimate power of imposing punishments. We also know that unless we can solve Spinoza's contract problem we cannot as rational individuals deliberately create that supreme power in the first place. So, are we as rational men due to fall prey to our own rational capacities?

We certainly would if we were merely rational beings (the most forceful critique of the economic model applied to law still being Herbert Hart's 1961 analysis of "the concept of law"). But we are also endowed with some extra rational faculties which we can exploit in rational ways. In assessing the scope and limits of these faculties and their several relationships to the rational choice perspective of pure game theory, it is illuminating again to turn to Reinhard

Selten's work. Very markedly he endorses the view that we should not try to patch up pure game theory such that it better conforms with social reality. We should rather acknowledge that the assumption of strictly rational forward looking individual choice -- except perhaps for extreme cases -- is not suitable for explanatory purposes (for experimental evidence on this cf. Selten 1979). If we aim at explaining real world processes we better stay clear of counterfactual assumptions of ideally rational behavior.

Not only cognitive limitations of human decision makers, who can at best be boundedly rational, cast into doubt the assumption that pure or strict rational choice approaches can provide true explanations; it is also obvious that human subjects often are in one way or other intrinsically motivated by their values rather than by the consequences of their acts. Rational choice analysis may model all human individuals as sovereign decision makers who are normatively unbound in their rational decision making. But even Bodin's sovereign prince is human after all. As a human being he is not only cognitively limited but also operates under some more or less moral restraints. He like his subjects will tend to keep his promises if "t-1" is small and thus the temptation to act "opportunistically rational" is not too strong. As experienced social beings his subordinates will anticipate this. Likewise, if they encounter other authorities facing low opportunity costs, as for instance judges or bureaucrats, they may expect norm guided rather than opportunistic behavior from those and so will they themselves be guided primarily by norms and rules of thumb if opportunity costs are low (cf. on the relationship between costs and utilitarian ethics Selten 1986).

Working with idealized rational choice models may serve several non-explanatory purposes, though. Thinking through their implications may be of philosophical interest in its own right. As (boundedly) rational beings we quite naturally wonder what a world of fully rational beings would look like and in particular whether there can be some theory of rational behavior in such a world that could become common knowledge without becoming self-defeating (Harsanyi and Selten 1988 being the best answer to such queries yet). Moreover, rational choice analyses can serve as a bench mark telling us where the crucial problems of social order presumably are located. After all, without rational choice analyses, those critics of the approach who point to the unsolvability of the Hobbesian problem of social order -- and quite rightly so -- would not even know what the problem is. Finally, acknowledging that there are certain limits to rational choice approaches does in no way preclude using them within their limits wherever we can. Reinhard Selten contributed much to shifting out those limits and at the same time to our understanding of their precise nature. But he also taught us some lessons about the merits of engaging in truly "experiental" and in the last resort experimental science of human behavior.

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Equilibrium Selection in Linguistic Games: Kial Ni (Ne) Parolas Esperanton?

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Resumo. Kun elirpunkto en artikolo de Selten kaj Pool uzante la teorion de ekvilibroselektado evoluigitan de Harsanyi kaj Selten, ni analizas la risksuperajn ekvilibrojn de lingva ludo. En modelon kun du grupoj da homoj kun malsamaj gepatraj etnaj lingvoj ni enkondukas kroman interetnan planlingvon, esperanton, kies lernado estas multe pli facila ol tiu de etnolingvo. El la kvar striktaj kaj simetriaj ekvilibroj de la ludo – rezignado pri internacia komunikado, uzo de esperanto aŭ unu de la etnaj lingvoj – du aŭ tri, depende de la valoroj de la parametroj de la modelo, povas kunekzisti. Uzante la koncepton de risksuperado por selekti inter tiuj kunekzistantaj ekvilibroj, ni montras, ke la enkonduko de esperanto por kelkaj realismaj parametrovaloroj estas la ununura solvo. Ĝi ankaŭ estas efika solvo, sed ĉiuj efikaj solvoj pere de esperanto ne realigeblas pere de tia risksupera ekvilibro. Fine ni spekulative pritraktas la eblon analizi la lingvouzon pere de evolua ludo.

Abstract. Taking our point of departure in an article by Selten and Pool and using the equilibrium selection theory developed by Harsanyi and Selten, we analyze the risk dominant equilibria of a language game. Into a model with two groups of people with different native ethnic languages, we introduce a further interethnic planned language, Esperanto, the learning of which is much easier than that of an ethnic language. Of the four strict and symmetric equilibria of the game – absence of international communication, use of Esperanto or one of the ethnic languages – two or three might coexist, depending on the values of the parameters of the model. Using the concept of risk dominance in order to select from those coexisting equilibria, we show that the introduction of Esperanto for some realistic parameter values is the unique solution. It is also an efficient solution, but all efficient solutions with Esperanto are not realised by such a risk dominant equilibrium. Finally, we explore in a speculative manner the possibility of analyzing the use of language with an evolutionary game.

1. Introduction

If strangers with different native languages want to communicate, at least one has to invest in learning a new language. Traditionally this would have meant that at least one party would learn another ethnic language.

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Since an ethnic language has evolved over a long time span, it is usually rather unsystematic due to imported words, phrases and grammatical rules. Famous examples - terrifying millions of pupils - are irregular verbs and famous tongue breakers (like chaiselongue or portemonnaie which were imported from the French into the German language in spite of the phonetic differences between the two languages). To cut a long story short: learning another ethnic language is tedious and often fails (our use of English in this essay may be a good example).

Although the community of Esperantists also favors a more cosmopolitan view of the world, one of the essential reasons for developing Esperanto has been to allow for communication without the enormous investment of learning another ethnic language. Esperanto as a planned language relies on considerably fewer rules and allows for nearly no exceptions. Through its clear and systematic agglutinate structure, it achieves with simple means a great precision and flexibility of expression. This economy of communication is unmatched by just about any ethnic language. It is estimated that to learn Esperanto one needs less than one third of the efforts required for learning English by continental Europeans in spite of the close relationships between nearly all European languages. See for instance Frank and Szerdahelyi (1976) or Formaggio (1990).

But, of course, if two strangers want to communicate in Esperanto, both have to learn it. Although one can imagine that children are brought up learning Esperanto, which indeed is the case in many families where parents from different countries have met through the Esperanto movement, we will here neglect such a possibility. Esperanto was not developed to be a substitute for ethnic languages.

There are many obvious reasons why Esperanto till now has failed to become a frequently used means of communication, e.g. the success of American English as the international language of science, business, and pop(ular) music. We do not want to discuss such reasons in general, but rather develop a simple linguistic game by which some of these reasons can be exemplified and explored more thoroughly. Since most linguistic games are usually special coordination games (every generally coordinated behavior is a strict equilibrium), our main methodology is the theory of equilibrium selection as developed by Harsanyi and Selten (1988).

To the best of our knowledge there exist only two studies trying to explore the strategic (dis)advantages of relying on Esperanto as a communication device, namely the ones by Selten and Pool (1991 and 1995) who both are Esperanto activists. The latter of these studies reviews the related literature which we do not want to repeat here. In section 2 we will discuss these contributions before introducing our own, simpler model which will be analyzed in sections 3, 4, 5, and 6 before concluding in section 7.

2. The Study by Selten and Pool

The study by Selten and Pool (1995) was originally written in Esperanto. The present edition, however, has a German translation and there are extended abstracts written in English and other languages. Players in their linguistic game all speak an ethnic language. To communicate with foreigners they can learn other ethnic languages or a planned language whose learning costs are far below those of any ethnic language.

One can describe the linguistic game of Selten and Pool as one with n countries and different ethnic languages $l = 1, ..., n$ plus the planned language $l = 0$. Denote by $\mathcal{L} = \{0, 1, ..., n\}$ the set of all languages. An individual $i\,(l)$ whose native ethnic language is $l \in \mathcal{L} \setminus \{0\}$ thus has the strategy set

$$S_{i(l)} = \{L \cup \{l\} : L \subset \mathcal{L} \setminus \{l\}\}.$$

Two individuals i and j using $s_i \in S_{i(l)}$ and $s_j \in S_{j(l')}$ can communicate if $s_i \cap s_j \neq \emptyset$, i.e. if they have one language in common. If we denote by $\#\mathcal{X}$ the cardinality of the set $\mathcal{X}$, the total communication gain of individual i is measured by

$$\#\{j : s_j \cap s_i \neq \emptyset\}.$$

Selten and Pool (1991) allow that communication benefits depend on the home country of those whom one can communicate with. For the sake of simplicity we neglect the possibility of weighing communication benefits in this way. Individual $i\,(l)$'s language learning costs are the product of the general learning costs $g\left(s_{i(l)}\right)$ of the language set $s_{i(l)} \setminus \{l\}$, which it wants to acquire in addition to its native ethnic language l, and its inverse individual language learning ability or, reciprocally, its learning cost $c_{i(l)}$. Since inhabitants of the same country can differ in language learning skills, they may choose different strategies even in a symmetric equilibrium of the linguistic game in normal form whose strategy sets are $S_{i(l)}$ and whose payoff functions for all individuals $i\,(l)$ are

$$\#\left\{j : s_j \cap s_{i(l)} \neq \emptyset\right\} - c_{i(l)} g\left(s_{i(l)}\right).$$

For this game model it is shown, in general, that in any equilibrium $s^* = \left(s^*_{i(l)}\right)_{i(l)}$ quite naturally the payoff of an inhabitant $i\,(l)$ of country $l \in L$ decreases if his learning cost parameter $c_{i(l)}$ is higher whereas this is neither true for its general learning cost $g(s^*_{i(l)})$ nor for its communication benefit $\#\{j : s^*_j \cap s^*_{i(l)} \neq \emptyset\}$.

For the special case of $n = 2$ and symmetric cost conditions for both countries (inhabitants of both countries have the same general learning costs for the other ethnic language as well as for the planned one and, furthermore, individual learning costs $c_{i(l)}$ are equally distributed in both countries $l = 1, 2$) an interesting further result confirms the "Esperanto Solution": In any symmetric equilibrium s^* requiring each individual $i\,(l)$ to learn a language,

i.e. $s^*_{i(l)} \neq \{l\}$ for all $i(l)$, $l = 1, 2$, everybody learns the planned language $l = 0$, i.e. $s^*_{i(l)} = \{0, l\}$ for all $i(l)$ and $l = 1, 2$. To rule out learning of the planned language at all, its cost advantage has to be low enough and the general costs $g(\{l, 1\}) = g(\{l, 2\})$ of learning ethnic languages have to be limited.

Unlike Selten and Pool (1991) we will rely on a simpler game model with a finite number of players who, furthermore, will not differ in their language learning skills. Whereas the first assumption needs no justification, the second one is highly restrictive. It has been imposed to simplify our analysis when selecting among equilibria. This assumption is, however, no restriction imposed by our general approach.

It should be noted that the assumption of individual learning skills which apply to all possible languages an individual may want to acquire, as imposed by Selten and Pool (1991), is also rather restrictive. Especially, when including a planned language which tries to avoid all the complications of ethnic languages and therefore does not require as many skills like pronunciation and grammatical background as ethnic languages, individual differences may not matter in the same way as when learning an ethnic language.

In summary, it can be readily accepted that Selten and Pool (1991) analyze a richer game model. The main excuse for relying on a simpler game structure is that we want to investigate our game model more thoroughly, by discussing efficiency aspects and, more importantly, by trying to select a unique solution for all possible parameter constellations.

3. Game Model

As in the main part of the analysis of Selten and Pool (1995) we focus on the special case of two countries with n_1, respectively n_2 inhabitants. Denote by $n_i^j (> 0)$ the number of inhabitants in country i learning language j where $j = 0$ means to learn the planned language and $j \neq 0$, $j \neq i$ to learn the other ethnic language. Communication benefits are measured as in the model of Selten and Pool. The cost of an inhabitant of country i to learn language j is denoted by c_i^j for all $i = 1, 2$ and all $j = 0, 1, 2$ and $j \neq i$ with $0 < c_i^0 < c_i^j$ for $i = 1, 2$ and $j \neq 0$, $j \neq i$.

To compare the situation with and without a planned language two game versions will be analyzed, namely game model G^o with strategy sets $S_k = \{0, 1\}$, where $s_k = 1$ ($s_k = 0$) stands for (not) learning the other ethnic language, and the game model G^m with strategy sets $S_k = \{0, 1\}^2$ where $s_k = (\rho_k, \delta_k)$ means (not) to learn the planned language in case of $\rho_k = 1$ ($\rho_k = 0$), respectively (not) to learn the other ethnic language in case of $\delta_k = 1$ ($\delta_k = 0$).

When analysing the various games in normal form we will impose non-degeneracy assumptions ruling out border cases of no practical relevance.

4. The World Without Esperanto (Game G^o)

In the game G^o for every vector $s = (s_k)_k$ the payoff of an individual i of country 1 is

$$u_i(s) = n_1 - 1 + s_i(n_2 - c_1^2) + (1 - s_i)\sum_j s_j \tag{4.1}$$

where summation over j runs over all inhabitants of country 2. Similarly, the payoff of an inhabitant j of country 2 is given by

$$u_j(s) = n_2 - 1 + s_j(n_1 - c_2^1) + (1 - s_j)\sum_i s_i \tag{4.2}$$

where now summation runs over all inhabitants i of country 1.

Let us assume that side payments are possible and that all inhabitants of the same country use the same strategy (the first assumption requires means, e.g. international money, to transfer utility, the second can be justified by a convincing symmetry requirement). Given these restrictions one can show

Lemma 4.1. *The efficient strategy vector* $s = (s_k)_k$ *is*

(i) $s_k = 0$ *generally for* $min\{n_1c_1^2, n_2c_2^1\} > 2n_1n_2$
(ii) $s_i = 1, s_j = 0$ *for* $min\{n_2c_2^1, 2n_1n_2\} > n_1c_1^2$
(iii) $s_i = 0, s_j = 1$ *for* $min\{n_1c_1^2, 2n_1n_2\} > n_2c_2^1$

where $i(j)$ *runs over all inhabitants of country* $1(2)$.

Proof. Since $s_i = 1 = s_j$ is inefficient due to $c_i^j > 0$, one only has to compare the three strategy vectors (i), (ii), and (iii) which imply the following welfare levels:

$$W(i) = n_1(n_1 - 1) + n_2(n_2 - 1) \tag{4.3}$$
$$W(ii) = W(i) + 2n_1n_2 - n_1c_1^2 \tag{4.4}$$
$$W(iii) = W(i) + 2n_1n_2 - n_2c_2^1 \tag{4.5}$$

Thus the lemma is obvious. ∎

What can be expected, however, in strict (symmetric) equilibria is stated by

Lemma 4.2. *The strict and symmetric equilibria of* G^o *are*

(i) $s_k = 0$ *generally for* $c_1^2 > n_2$ *and* $c_2^1 > n_1$
(ii) $s_i = 1, s_j = 0$ *for* $n_2 > c_1^2$
(iii) $s_i = 0, s_j = 1$ *for* $n_1 > c_2^1$.

Proof. According to (i) learning costs exceed communication benefits whereas according to (ii), respectively (iii), the opposite is true if inhabitants of country 1, respectively 2, learn the other language. Since learning costs are positive all three equilibria are strict in the sense that one loses when deviating unilaterally. ∎

According to the theory of equilibrium selection, developed by Harsanyi and Selten (1988), the solution of the game is the strict and symmetric equilibrium which risk dominates the other[1]. Only equilibria (*ii*) and (*iii*) described in Lemma 4.2 can coexist, namely in the case of condition

$$n_2 > c_1^2 \quad \text{and} \quad n_1 > c_2^1. \tag{4.6}$$

Applying the Harsanyi and Selten theory of equilibrium selection to all games satisfying condition 4.6 yields

Theorem 4.1. *If condition 4.6 holds, the strict equilibrium* (*ii*) *risk dominates the strict equilibrium* (*iii*) *if*

$$n_2 c_2^1 > n_1 c_1^2 \tag{4.7}$$

whereas (*iii*) *risk dominates* (*ii*) *in case of*

$$n_1 c_1^2 > n_2 c_2^1. \tag{4.8}$$

The formal proof of Theorem 4.1 can be found in Appendix A. One first derives the bicentric prior strategies by imagining that all other players know already whether the solution is (*ii*) or (*iii*). The vector of best replies is already a strict equilibrium, namely (*ii*) in case of (4.7) and (*iii*) in case of (4.8). Thus it is not necessary to compute the tracing path according to Harsanyi and Selten (1988).

The solution of Theorem 4.1 is also the efficient solution of G^o. It means that the inhabitants of that country with lower total learning costs should learn the other country's language.

5. When Esperanto is Available (Game G^m)

If the planned language $l = 0$ is available, the payoff of an individual i of country 1 for every vector $s = (\rho_k, \delta_k)_k$ is

$$\begin{aligned} u_i(s) &= n_1 - 1 + \delta_i(n_2 - c_1^2) + (1 - \delta_i)\sum_j \delta_j \\ &\quad + \rho_i \left(\left((1 - \delta_i) \sum_j \rho_j (1 - \delta_j) \right) - c_1^0 \right). \end{aligned}$$

$u_j(s)$ can be derived analogously by exchanging indices. The planned language is the only efficient generally spoken language if condition

$$\min\{2n_1 n_2, n_1 c_1^2, n_2 c_2^1\} > n_1 c_1^0 + n_2 c_2^0 \tag{5.1}$$

[1] Although Harsanyi and Selten (1988) gave priority to payoff dominance they now both seem to accept that payoff dominance should be neglected, see Selten (1995) and Harsanyi (1995).

holds. $2n_1n_2 > n_1c_1^0 + n_2c_2^0$ guarantees that general learning of $l = 0$ provides more communication benefits than costs. If $n_1c_1^2$, respectively $n_2c_2^1$ exceed the cost $n_1c_1^0 + n_2c_2^0$ of learning $l = 0$, this is, furthermore, the cheapest way of guaranteeing maximal communication benefits.

The strict and symmetric equilibria in game G^m are described by

Lemma 5.1. *The not necessarily unique strict and symmetric equilibria $s_k = (\rho_k, \delta_k)$ of G^m are*

(o) $s_k = (0,0)$ *generally for* $c_1^2 > n_2$ *and* $c_2^1 > n_1$
(i) $s_k = (1,0)$ *generally for* $n_2 > c_1^0$ *and* $n_1 > c_2^0$
(ii) $s_i = (0,1), s_j = (0,0)$ *for* $n_2 > c_1^2$
(iii) $s_i = (0,0), s_j = (0,1)$ *for* $n_1 > c_2^1$

where $i(j)$ runs over all inhabitants $i(j)$ of country $1(2)$.

Proof. It does not pay to learn another ethnic language if $l = 0$ is universally spoken. General learning of $l = 0$ requires $n_2 > c_1^0$ and $n_1 > c_2^0$. If one of the inequalities is wrong, nobody would learn $l = 0$ in equilibrium since $c_1^0, c_2^0 > 0$. The remaining part of Lemma 5.1 follows from Lemma 4.2. ∎

All possible constellations of equilibria are shown in Table 5.1. We still assume that $c_1^2 > c_1^0$ and $c_2^1 > c_2^0$.

country 1\2	$c_1^2 > c_1^0 > n_2$	$c_1^2 > n_2 > c_1^0$	$n_2 > c_1^2 > c_1^0$
$c_2^1 > c_2^0 > n_1$	(*o*)	(*o*)	(*ii*)
$c_2^1 > n_1 > c_2^0$	(*o*)	(*o*),(*i*)	(*i*),(*ii*)
$n_1 > c_2^1 > c_2^0$	(*iii*)	(*i*),(*iii*)	(*i*),(*ii*),(*iii*)

Table 5.1. Possible equilibria in G^m

6. When is Esperanto (Universally) Risk Dominant?

Assume that at least two equilibria, stated in Lemma 5.1, exist. The equilibrium (*i*) is called (universally) risk dominant if it in pairs risk dominates (*o*), (*ii*) respectively (*iii*). Since risk dominance is, in general, no transitive relation, there may not exist a universally risk dominant strict equilibrium. We first neglect such possibilities and concentrate on the condition for the "Esperanto Solution" (*i*) to be (universally) risk dominant. Afterwards it will be shown that for the game at hand intransitivity of risk dominance is impossible.

The risk comparison between (*i*) and (*ii*) is described by

Lemma 6.1. *Assume that both equilibria (i) and (ii) of game G^m coexist. Then (i) risk dominates (ii) if*

$$n_1 c_1^0 + n_2 c_2^0 < n_1 c_1^2 \tag{6.1}$$

whereas (ii) risk dominates (i) if

$$n_1 c_1^0 + n_2 c_2^0 > n_1 c_1^2. \tag{6.2}$$

A detailed proof can be found in Appendix B. By exchanging indices one can derive from Lemma 6.1 also the results for risk comparison between (*i*) and (*iii*):

Corollary 6.1. *Assume that both equilibria (i) and (iii) of game G^m coexist. Then (i) risk dominates (iii) if*

$$n_1 c_1^0 + n_2 c_2^0 < n_2 c_2^1 \tag{6.3}$$

whereas (iii) risk dominates (i) if

$$n_1 c_1^0 + n_2 c_2^0 > n_2 c_2^1. \tag{6.4}$$

Now the possibilities for learning Esperanto (*i*) to be the unambiguous solution are:

– (*o*) and (*i*) are the only strict equilibria, i.e.

$$c_1^2 > n_2 > c_1^0 \quad \text{and} \quad c_2^1 > n_1 > c_2^0 \tag{6.5}$$

– and either (*ii*) or (*iii*) or both, (*ii*) and (*iii*), coexist.

Since for the latter case the conditions for (*i*) to be (universally) risk dominant are stated by Lemma 6.1 and Corollary 6.1, it only remains to explore the risk dominance relationship between (*o*) and (*i*) for condition (6.5). The result for condition (6.5) is

Lemma 6.2. *Under condition (6.5) the strict equilibrium (i) risk dominates (o) if*

$$n_1 n_2 > n_1 c_1^0 + n_2 c_2^0$$

whereas (o) risk dominates (i) if

$$n_1 n_2 < n_1 c_1^0 + n_2 c_2^0.$$

The detailed proof of Lemma 6.2 can be found in Appendix C. We now can state the conditions which, according to game-theoretic reasoning specified by the concept of (universal) risk dominance, imply the Esperanto Solution (*i*).

Theorem 6.1. *The Esperanto Solution (i) is the unique solution of the game G^m in case of one of the following conditions:*

1. $c_1^2 > n_2 > c_1^0$ *and* $c_2^1 > n_1 > c_2^0$ *and* $n_1 n_2 > n_1 c_1^0 + n_2 c_2^0$

2. $n_2 > c_1^2 > c_1^0$ *and* $c_2^1 > n_1 > c_2^0$ *and* $n_1 c_1^2 > n_1 c_1^0 + n_2 c_2^0$
3. $c_1^2 > n_2 > c_1^0$ *and* $n_1 > c_2^1 > c_2^0$ *and* $n_2 c_2^1 > n_1 c_1^0 + n_2 c_2^0$
4. $n_2 > c_1^2 > c_1^0$ *and* $n_1 > c_2^1 > c_2^0$
and $n_1 c_1^2 > n_1 c_1^0 + n_2 c_2^0$ *and* $n_2 c_2^1 > n_1 c_1^0 + n_2 c_2^0$

Theorem 6.1 is nothing more than a summary of our previous results. For each component of Table 5.1 with (*i*) being an equilibrium we report the conditions of universal risk dominance of (*i*).

For the case of two equilibria it is obvious that the respective condition 1, 2, or 3 is both necessary and sufficient. For the case of three equilibria, namely (*i*), (*ii*), and (*iii*), we have imposed the rather stringent solution requirement that (*i*) has to risk dominate the other two strict equilibria. According to the Harsanyi and Selten theory of equilibrium selection the strict equilibrium (*i*) could also become the solution in case of cyclical risk dominance relationships. Such a possibility has been neglected so far since we wanted to explore the conditions when the Esperanto Solution (*i*) is the unique solution in the sense of that it pairwise risk dominates all other strict equilibria.

Since the minimal formation of G^m (Harsanyi and Selten, 1988) spanned by the two strict equilibria (*ii*) and (*iii*) is G^o, Theorem 4.1 implies that (*ii*) risk dominates (*iii*) if $n_1 c_1^2 < n_2 c_2^1$ and vice versa. From this result it follows that risk dominance in G^m is always transitive if three equilibria coexist. If, for example, (*i*) risk dominates (*ii*) and (*ii*) risk dominates (*iii*) which in turn risk dominates (*i*) this would require

$$n_1 c_1^0 + n_2 c_2^0 < n_1 c_1^2 < n_2 c_2^1 < n_1 c_1^0 + n_2 c_2^0$$

and thus a contradiction. Hence (*i*) (learning Esperanto) is the solution of G^m whenever it risk dominates (*ii*) and (*iii*).

Corollary 6.2. *The conditions of Theorem 6.1 are not only sufficient but also necessary for the solution* (*i*) *of the game* G^m.

7. Discussion

In most cases the risk dominant equilibrium is the more efficient one. The only exception is when the strict equilibria (*o*), i. e. no learning at all, and (*i*), i. e. learning of Esperanto, coexist. In this case (*i*) is more efficient than (*o*). For the parameter region satisfying

$$2n_1 n_2 > n_1 c_1^0 + n_2 c_2^0 > n_1 n_2$$

the more efficient equilibrium (*i*) is, however, risk dominated by (*o*). Thus the solution is not always efficient. In all other comparisons of two strict equilibria, the more efficient equilibrium risk dominates.

In order to explore the implications of Theorem 6.1, we make the simplifying assumptions that

$$c_1^0 = c_2^0 =: c \quad \text{and} \quad c_1^2 = c_2^1 =: \beta c.$$

We further define

$$n := n_1$$

and write

$$n_2 = \alpha n.$$

That is, β measures the learning costs of an ethnic language in terms of the cost of learning Esperanto and α the size of country two in terms of country one. Without loss of generality we can assume that $\alpha \geq 1$.

Under these assumptions, the conditions for the Esperanto Solution in Theorem 6.1 reduce to the following:

$$\beta > 1 + \alpha \quad \text{and} \quad n > c\left(1 + \frac{1}{\alpha}\right) \tag{7.1}$$

and the efficiency condition (5.1) to

$$\beta > 1 + \alpha \quad \text{and} \quad n > \frac{1}{2}c\left(1 + \frac{1}{\alpha}\right). \tag{7.2}$$

As also explained above, (7.1) implies (7.2). In the introduction we noted that the size of β is about 3. Hence, in our two-country world the Esperanto Solution is the unambiguous solution if the second country is not more than twice the size of the first one and if the advantage of communicating is sufficiently large in relation to the language learning costs, indeed, greater than what is necessary in order to make language learning efficient. Hence, an imposed solution would under certain conditions be more efficient than the (universally) risk dominant equilibrium. We conjecture that in the many-country case the condition for the Esperanto solution would be even less strict than condition (7.1) since the largest alternative ethnic language would have a smaller fraction of total population than in the two-language case where it, by necessity, has more than half.

This brings us back to the question in the title. We feel that the answer may have to be sought outside equilibrium selection theory, but not necessarily outside game theory, namely in an evolutionary approach. Language learning decisions are made individually, gradually and in a decentralized manner under the influence of the prevailing status quo, not the final equilibrium. The usage of various languages will hence develop gradually, as an individual, acting according to our payoff function in sections 4 and 5, will only decide to learn a language if a certain minimum critical mass of speakers already exists. The more users a language has, the larger is the incentive for an individual to learn it. This is probably the main reason for the rapid increase in the use of the American idiom in international communication in the world of today and the restriction of Esperanto to a relatively small group of specially motivated users. The move to the risk dominant equilibrium would require a coordinated collective action in the international arena.

A. Proof of Theorem 4.1

To determine the bicentric priors for the equilibria (*ii*) and (*iii*) one has to compute the minimal value of z for which

$$n_1 - 1 + n_2 - c_1^2 \geq n_1 - 1 + (1 - z)n_2$$

holds. The value $z = \frac{c_1^2}{n_2}$ determines the a priori-probability $p_1 = \frac{n_2 - c_1^2}{n_2}$ by which the inhabitants i of country 1 learn country 2's language. By symmetry one derives $p_2 = \frac{n_1 - c_2^1}{n_1}$ as the a priori-probability by which inhabitants j of country 2 learn the language of country 1.

Given that an inhabitant i of country 1 expects all other players to use their bicentric prior strategy his best reply is $s_i = 1$ if

$$n_1 - 1 + n_2 - c_1^2 > n_1 - 1 + \sum_{k=0}^{n_2} k \binom{n_2}{k} \left(\frac{n_1 - c_2^1}{n_1} \right)^k \left(\frac{c_2^1}{n_1} \right)^{n_2 - k}. \quad \text{(A.1)}$$

The sum corresponds to the expected value of the binomial distribution

$$\sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k} = p \cdot n.$$

Thus (A.1) is equivalent to

$$\begin{aligned} n_1 - 1 + n_2 - c_1^2 &> n_1 - 1 + \frac{n_1 - c_2^1}{n_1} \cdot n_2 \\ n_2 c_2^1 &> n_1 c_1^2. \end{aligned} \quad \text{(A.2)}$$

Condition (4.8) can be derived analogously. ∎

B. Proof of Lemma 6.1

Assume that $n_2 > c_1^2$ and thus $n_2 > c_1^0$ as well as $n_1 > c_2^0$ are satisfied, i.e. both (*i*) and (*ii*) are strict equilibria of G^m. From

$$n_1 - 1 + z n_2 - c_1^0 \geq n_1 - 1 + n_2 - c_1^2$$

one can derive the minimal z with

$$z = \frac{n_2 - c_1^2 + c_1^0}{n_2}$$

and obtains the bicentric prior probability

$$p_i = 1 - \frac{n_2 - c_1^2 + c_1^0}{n_2} = \frac{c_1^2 - c_1^0}{n_2}$$

for learning $l = 0$ in country 1. For country 2 the corresponding condition is

$$n_2 - 1 + n_1 - c_2^0 \geq n_2 - 1 + (1 - z) n_1,$$

i.e. the a priori-probability of player j in country 2 for learning $l = 0$ is

$$p_j = 1 - \frac{c_2^0}{n_1} = \frac{n_1 - c_2^0}{n_1}.$$

The condition for $s_i = (\rho_i, \delta_i) = (1, 0)$ versus $s_i = (0, 1)$ to be a best reply to the prior strategy combination is

$$\begin{aligned}
n_1 - 1 + n_2 - c_1^2 &< n_1 - 1 - c_1^0 + \sum_{k=0}^{n_2} k \binom{n_2}{k} \left(\frac{n_1 - c_2^0}{n_1}\right)^k \left(\frac{c_2^0}{n_1}\right)^{n_2 - k} \\
n_2 - c_1^2 &< -c_1^0 + \frac{n_1 - c_2^0}{n_1} \cdot n_2 \\
n_1 c_1^0 + n_2 c_2^0 &< n_1 c_1^2 \qquad \text{(B.1)}
\end{aligned}$$

whereas for $s_j = (\rho_j, \delta_j) = (1, 0)$ versus $s_j = (0, 0)$ as a best reply one must have

$$\begin{aligned}
n_2 - 1 + n_1 - c_2^0 &> n_2 - 1 \\
&\quad + \sum_{k=0}^{n_1} k \binom{n_1}{k} \left(\frac{n_2 - c_1^2 + c_1^0}{n_2}\right)^k \left(\frac{c_1^2 - c_1^0}{n_2}\right)^{n_1 - k} \\
n_1 - c_2^0 &> \frac{n_2 - c_1^2 + c_1^0}{n_2} \cdot n_1 \\
n_1 c_1^2 &> n_1 c_1^0 + n_2 c_2^0 \qquad \text{(B.2)}
\end{aligned}$$

since people in country 1 either learn $l = 0$ or $l = 2$. ■

C. Proof of Lemma 6.2

From

$$n_1 - 1 + z n_2 - c_1^0 \geq n_1 - 1$$

the prior probability of players i in country 1 to learn $l = 0$ is

$$p_i = 1 - \frac{c_1^0}{n_2}.$$

Similarly, for country 2

$$p_j = 1 - \frac{c_2^0}{n_1}$$

is players j's prior probability for learning $l = 0$. The condition for $s_i = (\rho_i, \delta_i) = (1, 0)$ versus $s_i = (0, 0)$ to be a best reply to $p = (p_i, p_j)$ is

$$
\begin{aligned}
n_1 - 1 &< n_1 - 1 - c_1^0 + \sum_{k=0}^{n_2} k \binom{n_2}{k} \left(1 - \frac{c_2^0}{n_1}\right)^k \left(\frac{c_2^0}{n_1}\right)^{n_2-k} \\
0 &< -c_1^0 + \frac{n_1 - c_2^0}{n_1} \cdot n_2 \\
n_1 c_1^0 + n_2 c_2^0 &< n_1 n_2. \qquad \text{(C.1)}
\end{aligned}
$$

The condition for $s_j = (\rho_j, \delta_j) = (1,0)$ versus $s_j = (0,0)$ to be a best reply to p is exactly the same. It follows from exchanging indices in (C.1). ∎

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Are Stable Demands Vectors in the Core of Two-Sided Markets? Some Graph-Theoretical Considerations

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Abstract. Stable demand vectors for games in coalitional form satisfy three requirements: 1) Players' demands are such that there are no leftovers in any coalition. 2) Each player's demand is such that he can find at least one coalition, including himself, that can satisfy the demands of its members. 3) No player i is dependent on player j in the sense that i always needs j to form a coalition satisfying its members' demands, while j does not need i in order to do so. Since the stability condition 3 focuses on pairs, we look at NTU games of pairs such as those arising from two-sided markets. For each demand vector we define a bipartite graph, and we use the König-Hall Theorem and its corollaries to study properties of those graphs. In particular, stable demands are shown to be in the core if the number of agents on each side of the market is not greater than four.

1. Introduction

The main purpose of this paper is to illustrate the use of graph – theoretical methods for the study of relations between the set of stable demand vectors and the set of core allocations in games arising from two-sided markets.

The definition of stable demand vectors for general games in coalitional form is based on the following intuition: Each player sets a demand for his participation in a coalition. Three stability criteria are imposed: 1. *Maximality* requires that the demand are maximal in the sense that no agent can raise his demand without excluding himself from any coalition that can satisfy the demand of its members. 2. *Feasibility* requires that each agent can find a coalition that includes him, and that satisfies the demand of its members. 3. *Independence* requires that no agent is dependent on another one in the sense that the first agent always needs the second to ensure feasibility of his demand, but the second agent can find a coalition that includes him and that satisfies its member's demands, but excludes the first agent. The terminology we use here is due to Selten (1981).

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Demand vectors exhibiting the above mentioned properties have been studied, among others, by Albers (1979) , Bennett (1983), Bennett and Zame (1988), Moldovanu and Winter (1994), and Reny and Wooders (1993). Bennett and Zame proved existence of stable demand vectors for a large class of NTU games in coalitional form. Moldovanu and Winter axiomatized the set of stable demand vectors using consistency axioms. The axiomatization is practical identical to the axiomatization of the core of NTU games, but the definition of reduced games is different, reflecting the fact that demand vectors need not be feasible for the grand coalition. We also note that stable demand vectors often appear in relation to equilibrium payoffs in non-cooperative games of coalition formation and payoff division (see Selten, 1981, Binmore, 1985, Moldovanu, 1992).

Since the Independence property focuses on pairs, it is natural to look at the implications of the three requirements described above for NTU games where the agents are divided in two disjoint sets (say, buyers and sellers), and where the effective coalitions are those involving exactly one buyer and one seller. Since many real-life markets are in fact bilateral, these types of games have drawn a huge amount of attention. In particular, Shapley and Shubik (1971) and Kaneko (1982) show that these games are balanced (for the TU and NTU case, respectively). Hence, the core is no empty, and, moreover, the core coincides with the set of competitive equilibrium allocations.

Each stable demand vector induces a graph where the vertices are the agents, and where a buyer and a seller are connected if their demands are compatible. By studying the properties of such graphs we can establish whether the given stable demand vector is in the core or not. The main tool is the celebrated König-Hall Theorem that establishes a necessary and sufficient condition for the existence of a perfect matching in a bipartite graph. The conditions on the induced graphs implied by the definition of stable demand vectors are, in general, rather weak. We first derive several properties of induced graphs that are sufficient for the respective stable demand vector to be in the core. We then use these results to show that stable demand are necessarily in the core if the number of agents on each side of the market is not greater than four. The result does not hold anymore for games with larger sets of agents on each side of the market.

The paper is organized as follows : In section 2 we introduce NTU games in coalitional form and stable demand vectors for these games. In section 3 we look at the graphs induced by stable demand vectors in NTU games of pairs, and study their relations to graphs induced by core allocations.

2. Games in Coalitional Form

Let $N = \{1,2,...n\}$ be a set of players. A coalition S is a non-empty subset of

N. $|S|$ denotes the cardinality of S. A payoff vector for N is a function $x: N \longrightarrow \mathbb{R}$. We denote by x^S the restriction of x to members of S. The restriction of $\mathbb{R}^S$ to vectors with non-negative coordinates is denoted by $\mathbb{R}^S_+$. The zero vector in $\mathbb{R}^S$ is denoted by 0^S. For $x, y \in \mathbb{R}^S$ we write $x \geq y$ if $x^i \geq y^i$ for all $i \in S$. Let K be a subset of $\mathbb{R}^S_+$. Then, intK denotes the interior of K relative to $\mathbb{R}^S_+$ and ∂K denotes the set $K \setminus \text{int}K$.

Definition 2.1: A *non-transferable utility (NTU) game in coalitional form* is a pair (N,V) where V is a function that assigns to each coalition S in N a set $V(S) \subseteq \mathbb{R}^S$ such that:

$$V(S) \text{ is a non-empty, closed, and bounded subset of } \mathbb{R}^S_+ \; ; \tag{2.1}$$

$$\forall i \in N, \; V(i) = 0^i \; ; \tag{2.2}$$

$$\text{If } y \in \mathbb{R}^S_+, x \in V(S) \text{ and } x \geq y \text{ then } y \in V(S) \; ; \tag{2.3}$$

$$\text{If } x, y \in \partial V(S) \text{ and } x \geq y \text{ then } x = y \; . \tag{2.4}$$

A *game with transferable utility (TU) in coalitional form* is a pair (N,v) where v is a function that assigns to each coalition S in N a real non-negative number v(S).

Condition 2.2 is a normalization. Condition 2.3 ensures that utility is freely disposable. A set V(S) satisfying 2.3 is said to be *comprehensive.* Condition 2.4 requires that the Pareto-frontier of V(S) coincides with the strong Pareto-frontier, and it implies that utility cannot be transferred at a rate of zero or infinity. A set V(S) satisfying 2.4 is said to be *non-leveled.*

We next define the two concepts that play the major role in our analysis:

Definition 2.2: Let (N,V) be an NTU game, and let $x \in \mathbb{R}^N$. x can be *improved upon* if there exists a coalition S and a vector $y^S \in V(S)$ with $y^i > x^i$ for all $i \in S$. The *core* of (N,V), C(N,V), is the set of all <u>$x \in V(N)$</u> that cannot be improved upon.

Definition 2.3: Let $d \in \mathbb{R}^N$. Denote:

$$\forall k \in N, \quad F_k(d) = \{S \mid S \subseteq N, k \in S, \text{and } d^S \in V(S)\} \tag{2.5}$$

A vector q is a *stable demand vector* if it has the following properties:

1) d cannot be improved upon (2.6)

2) $\forall i \in N,\ \exists S \subseteq N$ such that $i \in S$ and $d^S \in V(S)$ (2.7)

3) $\forall i,j \in N$, with $i \neq j$,
it is not the case that $F_i(d) \subsetneq F_j(d)$ (2.8)

3. Where 4 Are Few, But 5 Are Many

The fact that the stability condition 2.8 focuses on pairs suggests a study of "games of pairs" such as those arising from two-sided markets. In such games there are two distinct groups of players, say buyers and sellers, and the only relevant coalitional "pies" are those for singles, and those for coalitions consisting of exactly one buyer and one seller. Demands can be interpreted as reservation prices at which the agents are willing to buy and sell, respectively.

We first need some notation : Let B a non-empty set of buyers, and let S be a non-empty set of sellers. We assume that $B \cap S = \emptyset$. We assume here for simplicity that $|B| = |S| = n$. We denote agents by i, j, etc..., unless we want to be specific about their type, in which case we use the notation b_i, s_j, etc... To each agent $i \in N = B \cup S$ we associate the set $V(i) = 0^i$. To each pair $T = \{b_i, s_j\}$ we associate a set $V(T) \subset \mathbb{R}^N_+$, where V(T) satisfies the standard conditions for NTU games. In a two-sided market V(T) represents the set of utility vectors arising from feasible trades between the members of the pair. If Q is a pair of two buyers, or a pair of two sellers, then $V(Q) = 0^Q$. For a coalition S with more than two players let $\Pi(S)$ represent the set of all possible partitions of S in in mixed pairs and singletons . We define then V(S):

$$V(S) = \cup \{ \underset{T \in \pi(S)}{\times} V(T) \mid \pi(S) \in \Pi(S) \} \tag{3.1}$$

For the definition of a TU game of pairs we proceed in a similar manner : To each coalition T with no more than two players we associate a non-negative real number v(T). If T is a singleton, or if T is a pair of two buyers, or a pair of two sellers, then $v(T) = 0$. For a coalition S with more than two players we define v(S) as follows :

$$v(S) = \max \Big(\sum_{T \in \pi(S)} v(T) \mid \pi(S) \in \Pi(S) \Big) \tag{3.2}$$

The core of any game of pairs (TU or NTU) is non-empty (see Shapley and Shubik, 1971 , and Kaneko,1982) since these games are balanced. Moreover, for two-sided markets, the core coincides with the set of competitive equilibrium allocations.

We now recall several graph-theoretical concepts (see, for example, Berge

1976). An undirected *graph* G is a pair (X,E) where X is a set of *vertices* and E is a family of elements of the Cartesian product X × X called *edges.* Two vertices are *adjacent* if they are connected by at least one edge. Two edges are *adjacent* if they have at least one endpoint in common. The *degree* of a vertex x is the number of edges with x as an endpoint. A graph is *simple* if it has no edges of the form (x,x) and if no more than one edge joins any two vertices. A graph is *bipartite* if its vertices can be partitioned into two sets X_1 and X_2 such that no two vertices in the same set are adjacent. Given a simple graph $G = (X,E)$, a *matching* is a set $E_0 \subset E$ such that no two edges in E_0 are adjacent. A vertex x is *saturated* by a matching E_0 if an edge of E_0 contains x as an endpoint. A *perfect matching* is a matching that saturates all vertices of G.

For a game of pairs (N,V), and for a vector $d \in \mathbb{R}_+^N$ we define the graph G(N,V,d) as follows: the set of vertices is the set of agents (buyers and sellers) ; a buyer b_i and a seller s_j are connected by an edge if their demands are feasible, i.e., if $(d^{b_i}, d^{s_k}) \in V(b_i, s_j)$. Note that G(N,V,d) is a simple bipartite graph. Given a graph G(N,V,d), we define sets of *options* as follows:

$$OP_d(b_i) = \{ s_k \mid s_k \in S \text{ and } (d^{b_i}, d^{s_k}) \in V(b_i, s_k) \} \tag{3.3}$$

$$OP_d(s_j) = \{ b_k \mid b_k \in B \text{ and } (d^{b_k}, d^{s_j}) \in V(b_k, s_j) \} \tag{3.4}$$

(The set of options depends of course also on the game (N,V), but we omit this dependence whenever confusion cannot arise.) Clearly, the set of options for a an agent i is simply the set of vertexes adjacent to vertex i in G(N,V,d) , and the degree of the vertex i , $deg_d(i)$, is given by $|OP_d(i)|$.

We now use the graph–theoretical methodology to establish some relations between the core and the set of stable demand vectors for a game of pairs (N,V). To avoid a tedious case–differentiation, we look in the sequel only at stable demand vectors d such that $d^i > 0$ for all $i \in N$. Since $V(i) = 0^i$, we have for such demand vectors that

$$\forall i \in N,\ deg_d(i) \geq 1 \text{ in } G(N,V,d); \tag{3.5}$$

Proposition 3.1: Let (N,V) be an NTU game of pairs, and let d be a stable demand vector such that $\forall i \in N$, $deg_d(i) \geq 1$ in G(N,V,d). The induced graph G(N,V,d) has the following property: For any pair i,j of adjacent vertices

1. either $deg_d(i) = deg_d(j) = 1$, or
2. $deg_d(i) \geq 2$ and $deg_d(j) \geq 2$. (3.6)

Conversely, let $d \in \mathbb{R}_+^N$ such that $\forall\, b_i \in B$, $\forall\, s_j \in S$, $(d^{b_i}, d^{s_j}) \notin \text{int}V(b_i, s_j)$, and such that the graph G(N,V,d) satisfies conditions 3.5 and 3.6. Then d is a stable demand vector.

Proof: Assume that b_i and s_j are connected by an edge in G(N,V,d) , and that, without loss of generality, degd(b_i) $\geq$ 2. Hence $(d^{b_i}, d^{s_j}) \in V(b_i, s_j)$, and, moreover, there exists $s_k \neq s_j$ such that $(d^{b_i}, d^{s_k}) \in V(b_i, s_k)$. Assume that degd($s_j$) = 1. Then $F_{s_j} = (\{b_i,s_j\})$, while $F_{b_i} \supset (\{b_i,s_j\},\{b_i,s_k\})$. Hence $F_{s_j} \subsetneq F_{b_i}$, and this yields a contradiction to condition 2.8 in the definition of stable demand vectors.

The proof of the converse part is immediate by the definition of stable demand vectors. **Q.E.D.**

Proposition 3.2: Let (N,V) be an NTU game of pairs, and let $d \in \mathbb{R}^N_+$ such that $\forall\, b_i \in B,\ \forall\, s_j \in S$, $(d^{b_i}, d^{s_j}) \notin \text{int}V(b_i, s_j)$. Then $d \in C(N,V)$ if and only if the graph G(N,V,d) has a perfect matching.

Proof: A vector $d \in \mathbb{R}^N_+$ such that $\forall\, b_i \in B,\ \forall\, s_j \in S$, $(d^{b_i}, d^{s_j}) \notin \text{int}V(b_i, s_j)$ is in the core if and only if d is feasible for the grand coalition, i.e., if and only if $d \in V(N)$. The vector d is feasible for the grand coalition if and only there are n distinct pairs (each containing a buyer and a seller) such that the demands of the members of each pair (given by d) are feasible for that pair. Clearly, the graph G(N,V,d) has a perfect matching if and only if n such distinct pairs exist. **Q.E.D.**

Proposition 3.3: Let (N,V) be an NTU game of pairs, and let $d \in \mathbb{R}^N_+$ such that $\forall\, b_i \in B,\ \forall\, s_j \in S$, $(d^{b_i}, d^{s_j}) \notin \text{int}V(b_i, s_j)$. Then $d \in C(N,V)$ if and only if

for all sets $B' \subset B$, $\left| \bigcup_{i \in B'} OP_d(i) \right| \geq |B'|$.

Proof: The result follows immediately from Proposition 3.2 and from the König–Hall Theorem (see Berge, 1976). Note that the "only if" part is trivial. The interesting part is the "if" one. **Q.E.D.**

Having in mind Proposition 3.3, we observe that the special condition imposed on a graph G(N,V,d) whenever d is a stable demand vector (condition 3.6) is very weak. Accordingly, we cannot generally expect stable demand vectors to be in the core for games of pairs. However, such a result can be obtained if the number of agents on each side of the market is not too large. Denote by $[\lambda]$ the integral part of a real number λ. We first derive some preliminary results which yield sufficient conditions for the existence of perfect matchings in G(N,V,d).

Proposition 3.4: Let (N,V) be an NTU game of pairs, and let d be a stable demand vector such that $d^i > 0$ for all $i \in N$, and such that

$$\forall i \in N,\ deg_d(i) \leq 2 \qquad (3.7).$$

Then there exists a perfect matching for G(N,V,d), and $d \in C(N,V)$.

Proof: By condition 3.5, $deg_d(k) \geq 1$ for each vertex k in G(N,V,d). Let i be a vertex such that $deg_d(i) = 1$. Hence i is adjacent to a unique vertex j. By condition 3.6, we know that j must be adjacent to a unique vertex, hence j is only adjacent to i. Denote by N' the set of all vertexes k such that $deg_d(k) = 1$. The previous argument shows that we can find a matching E_0 that saturates N'. We now look at $N \setminus N'$. Using again condition 3.6, we obtain that no vertex in $N \setminus N'$ is adjacent to a vertex in N'. We now look at the graph $G(N \setminus N',V,d)$ which is the restriction of the graph G(N,V,d) to the set $N \setminus N'$. By construction, the degree of any vertex in $G(N \setminus N',V,d)$ is equal to its degree in G(N,V,d). By assumption, for any vertex k in $N \setminus N'$ we have $deg_d(k) = 2$. Hence $G(N \setminus N',V,d)$ is a graph where all vertexes have the same degree. By a corollary of the König–Hall Theorem, (see Berge, 1976) any simple bipartite graph has a matching that saturates all vertices with maximum degree. Hence there exists a matching E_1 that saturates $N \setminus N'$. Since no two elements of E_0 and E_1, respectively, have endpoints in common, the set $E_0 \cup E_1$ is a perfect matching for G(N ,V,d). By Proposition 3.2, $d \in C(N,V)$. **Q.E.D.**

Proposition 3.5: Let (N,V) be an NTU game of pairs, and let $d \in \mathbb{R}_+^N$ such that $\forall b_i \in B,\ \forall s_j \in S,\ (d^{b_i}, d^{s_j}) \notin intV(b_i, s_j)$. Moreover, assume that:

$$\forall i \in N,\ deg_d(i) \geq \left\lceil \frac{n+1}{2} \right\rceil . \qquad (3.8)$$

Then there exists a perfect matching for G(N,V,d), and $d \in C(N,V)$.

Proof: By Propositions 3.2, 3.3, it is enough to show that for all sets $B' \subset B$, $\left| \bigcup_{i \in B'} OP_d(i) \right| \geq |B'|$. The assertion is clearly satisfied for sets B' such that $|B'| \leq \left\lceil \frac{n+1}{2} \right\rceil$. Assume then that B' is such that $|B'| > \left\lceil \frac{n+1}{2} \right\rceil$. We now claim that $\left| \bigcup_{i \in B'} OP_d(i) \right| = n \geq |B'|$. To see that, assume, on the contrary, that there exists a vertex $j \in S$, such that j is not adjacent to any vertex $i \in B'$. This means that $OP_d(j) \cap B' = \emptyset$. We have then $|OP_d(j) \cup B'| = |OP_d(j)| + |B'| > 2 \cdot \left\lceil \frac{n+1}{2} \right\rceil \geq n = |B|$. This yields a contradiction to $(OP_d(j) \cup B') \subset B$. **Q.E.D.**

We now have:

Proposition 3.6: Let (N,V) be a game of pairs with four or less agents of each type, and let d be a stable demand vector such that $d^i > 0$ for all $i \in N$. Then $d \in C(N,V)$.

Proof : By condition 3.5, we know that, for each vertex k in G(N,V,d), $deg_d(k) \geq 1$. As in the proof of Proposition 3.4, we can find a matching E_0 that saturates the set N'of vertices k such that degd(k) = 1. By condition 3.6, there is no vertex in $N \setminus N'$ that is adjacent to a vertex in N'. We now look at the graph $G(N \setminus N',V,d)$ which is the restriction of the graph G(N,V,d) to the set $N \setminus N'$. By construction, the degree of any vertex in $G(N \setminus N',V,d)$ is equal to its degree in G(N,V,d). For any vertex k in $N \setminus N'$ we have $deg_d(k) \geq 2$. Since $2 = \left[\frac{4+1}{2}\right] \geq \left[\frac{|N \setminus N'|+1}{2}\right]$, we obtain by Proposition 3.5 that there exists a matching E_1 that saturates all vertices in $N \setminus N'$. As in the proof of Proposition 3.4, the set $E_0 \cup E_1$ is a perfect matching for G(N ,V,d). By Proposition 3.2, $d \in C(N,V)$. **Q.E.D.**

The next example shows that the bound on the number of agents of each type in Proposition 3.6 is tight.

Example 3.7: Let (N,v) be a TU game of pairs with five agents on each side of the market defined as follows:

$$v(b_i) = v(s_j) = 0 \ , 1 \leq i,j \leq 5 \tag{5.7}$$

$$v(b_i,s_j) = 2, \text{ for} \begin{cases} i = 1,2 \quad \text{and } j = 3,4,5 \\ i = 3,4,5 \text{ and } j = 1,2 \end{cases} \tag{5.8}$$

$$v(b_i,s_j) = 1, \text{ otherwise} \tag{5.9}$$

Then $d = (1,1,...1)$ is a stable demand vector, but $d(N) = 10 > v(N) = 9$. (four pairs can form obtaining 2, one pair obtaining 1). Thus d is not feasible and $d \notin C(N,v)$. **Q.E.D.**

If a payoff vector (and in particular a stable demand vector) is not in the core, then, at the given "prices" some players are "over-demanded" and some are "under-demanded". Respective adjustments should be made in the demands (over demanded players increase their demands, under-demanded players decrease). In Example 3.7 , sellers s_1,s_2 and buyers b_1,b_2 are over-demanded , while the other players are under-demanded. Kamecke (1988) constructs an algorithm where the above intuition is repeatedly used for making adjustments in the agents' payoffs. His algorithm arrives after finitely many steps to an allocation that is the most favorable core allocation

for one side of the market.

Finally, note that several possible extensions of the stability condition used in the definition of stable demand vectors (condition 3.6) become now apparent. Here we regarded as unstable situations in which one agent in a potential trade has at least one outside option, but his partner has none. Similarly, we could define new notions of stability requiring that the numbers of outside options available to each trader in a matched pair are, in some sense, close to each other (say, equal). Assume, for example, that a demand vector cannot be improved upon and that each agent has exactly k options (this could be such a "new" definition of stability). Then, by the corollary to the König–Hall Theorem that we used in the proof of Proposition 3.4, that demand vector must be in the core.

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The Consistent Solution for Non-Atomic Games*

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Abstract. We study here the relation between the consistent solution to large games, and economic equilibria. Using a non-atomic approximation, we show that homogeneous Pareto-optimal allocations are consistent if and only if they correspond to competitive equilibria.

Solution

We will consider here a market game with m types of players and n goods. Each player of type i $(1 \leq i \leq m)$ has an initial endowment

$$w^i = (w_1^i, ..., w_n^i)$$

of goods, and a utility function

$$u_i : R^n \to R$$

defined on bundles (n-tuples) of goods. We assume that each u_i is monotone non-decreasing in each variable, concave, and differentiable (at least at most places).

A **bundle** is simply an n-tuple $z = (z_1, ..., z_n)$. We will allow negative values (i.e. short sales are permissible), though in practice the utility functions can be used to make short sales (at least beyond some point) very disadvantageous. An **allocation** is an m-tuple

$$Z = < z^1, ..., z^m >$$

of bundles. (This allocation assigns the bundle z^i to each player of type i.)

A **coalition** is represented by a vector $X = (x_1, ..., x_m)$, where the x_i are non-negative integers: x_i is the number of players of type i in coalition X.

An allocation Z is **feasible** for coalition X if it satisfies

(1)
$$\sum_{i=1}^{m} x_i z_j^i = \sum_{i=1}^{m} x_i w_j^i$$

for each j $(1 \leq j \leq n)$.

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For a given coalition X, the usual definition of Pareto- optimality holds: Z is **Pareto-optimal for** X if it is feasible for X, and there is no Y, also feasible for X, giving at least as much utility to each type as Z does, and strictly more utility to at least one type. (Essentially, this means that no mutually profitable trade is possible starting from Z.) Assuming differentiability, the condition for a feasible Z to be Pareto-optimal is the existence of two vectors, $p = (p_1, ..., p_n)$ and $q = (q_1, ..., q_m)$, satisfying

(2)

$$u_{ij}(z^i) = q_i p_j$$

for each player type i and good j, where u_{ij} is the partial derivative $\frac{\partial u_i}{\partial z_j^i}$. Under the assumptions of monotone utility functions, the vectors p and q can be assumed positive (or at least non-negative). Moreover, since it is only the products $q_i p_j$ that matter, it is always possible to normalize one of the two vectors p and q.

In equation (2), the components p_j are **shadow prices**. The q_i represent the rate at which utility can be transferred between players: a player of type i can receive a small amount εq_i of utility if one of type k gives up the corresponding small amount εq_k (and this can be done by a physical transfer of goods worth ε units of money at the going shadow prices). The meaning of the shadow prices, together with Pareto-optimality, is the standard one: if the goods are for sale at the going prices, and players are holding the bundles Z, then no player can gain utility by a costless trade (i.e one which does not increase the shadow price- value of his bundle).

A Pareto-optimal allocation Z is a **competitive equilibrium** if, apart from (1) and (2), it satisfies

(3)

$$\sum_{j=1}^{n} p_j z_j^i = \sum_{j=1}^{n} p_j w_j^i$$

for each i.

A **consistent solution** to an n-person NTU game (N, V), as defined in [Maschler/Owen 1992] and characterized in [Hart/Mas Colell 1992], is a collection of payoff vectors $y(S) = (y_i(S))_{i \in S}$, one for each possible coalition $S \subset N$, which satisfies the conditions of Pareto-optimality (each $y(S)$ is Pareto-optimal in set $V(S)$) and **balanced contributions**: for each $S \subset N$, and each $i \in S$,

(4)

$$\sum_{\substack{k \in S \\ k \neq i}} h_i(S) \{y_i(S) - y_i(S-k)\} = \sum_{\substack{k \in S \\ k \neq i}} h_k(S) \{y_k(S) - y_k(S-i)\}$$

where $h(S)$ is the normal vector to the Pareto-optimal surface of $V(S)$ at point $y(S)$.

For our games, we can think of the consistent solution as, instead, a function defined over the set I_+^m of non-negative m- tuples of integers, which assigns, to each coalition (m-tuple) X, an allocation $Z(X)$. These allocations must all be feasible and Pareto-optimal (for the respective X). Condition (4) will now take the rather complicated form that, for each X, and each i such that $x_i \geq 1$,

(5)

$$\sum_{k\in M} x_k h_i(X)\{u_i(z^i(X)) - u_i(z^i(X - e^k))\} = \sum_{k\in M} x_k h_k(X)\{u_k(z^k(X)) - u_k(z^k(X - e^i))\}$$

where, as before, $h(X)$ is the normal vector to the pareto-optimal surface of $V(X)$ at point $u(Z(X))$, and where e^k is the vector (in R^m) whose kth component is 1, while all other components are 0.

Although the X are naturally restricted to being m-tuples of integers, we may, for very large games, assume that the X are, for all practical purposes, m-tuples of reals.[1] Thus, **for large games**, we assume that the consistent solution is a mapping from the space R_+^m into the space R^{mXn} of all allocations Z. We can then replace the increments by derivatives, so that (5) now takes the form

(6)

$$\sum_{k\in M} x_k h_i(X)\frac{\partial u_i}{\partial x_k}(z^i(X)) = \sum_{k\in M} x_k h_k(X)\frac{\partial u_k}{\partial x_i}(z^k(X)).$$

Now, the standard chain rule for differentiation,

$$\frac{\partial u_i}{\partial x_k} = \sum_{j\in N} \frac{\partial u_i}{\partial z_j^i} \cdot \frac{\partial z_j^i}{\partial x_k}$$

when combined with (2), takes the form

(7)

$$\frac{\partial u_i}{\partial x_k} = \sum_{j\in N} q_i p_j z_{jk}^i$$

where z_{jk}^i is the partial derivative, $\frac{\partial z_j^i}{\partial x_k}$.

We note, next, that the vector h is defined only up to multiplication by a scalar. Now the components of q (the vector of rates of motion along the Pareto-optimal surface), and those of h, the normal vector along this same surface, will be inversely proportional. Thus we can assume that each $h_i = \frac{1}{q_i}$, and equation (6) now takes the form

(8)

$$\sum_{k\in M} \sum_{j\in N} x_k p_j z_{jk}^i = \sum_{k\in M} \sum_{j\in N} x_k p_j z_{ji}^k$$

for each i such that $x_i > 0$.

The allocation mapping Z is homogeneous[2] (of degree 0) if, for every coalition X and any constant $k > 0$,

(9)
$$Z(kX) = Z(X).$$

Assuming differentiability[3], this is equivalent to saying that, for every X, every $i \in M$ and every $j \in N$,

(10)
$$\sum_{k \in M} x_k z^i_{jk} = 0.$$

We have thus defined three interesting properties which an allocation mapping (i.e. a mapping which assigns a Pareto-optimal allocation to each coalition) can have: it can be a competitive equilibrium mapping (equations 1-2-3), it can be consistent (1-2-8), and it can be homogeneous (1-2-9). The question, now, is as to the logical relationship among these properties. In particular, we would like to know whether consistent mappings are equilibrium mappings, and conversely. It turns out that, in the presence of homogeneity, the two are equivalent:

Theorem. Let Z be a differentiable, homogeneous allocation mapping. Then Z is consistent if and only if it is an equilibrium mapping.

Proof: Consider the feasibility condition (1). Differentiating both sides of this equation with respect to x_k, we obtain

$$\sum_{i \in M} x_i z^i_{jk} + z^k_j = w^k_j$$

or, interchanging i and k,

(11)
$$\sum_{k \in M} x_k z^k_{ji} = w^i_j - z^i_j.$$

Suppose now that Z is consistent, so that (8) holds. By the homogeneity condition (10), the left-hand side of equation (8) vanishes. On the other hand, the right-hand side of (8) can be written as

$$\sum_{j \in N} p_j \left\{ \sum_{k \in M} x_k z^k_{ji} \right\}$$

and, using (11), this reduces to

$$\sum_{j \in N} p_j (w^i_j - z^i_j)$$

Thus, equation (8) reduces to the statement that this last expression vanishes. But this is precisely equation (3), and so Z is an equilibrium mapping.

Conversely, suppose Z is an equilibrium mapping, so that (3) holds. Substituting (11) in (3), we have

$$\sum_{j \in N} \sum_{k \in M} p_j x_k z_{ji}^k = 0.$$

Now, from (10), we also have

$$\sum_{j \in N} \sum_{k \in M} p_j x_k z_{ji}^k = 0$$

and these two together give us (8). Thus Z is consistent.

The question of the relationship between consistent and equilibrium allocation mappings thus reduces to the question of their homogeneity.

Now, the conditions for equilibrium (1-2-3) are point-wise conditions, i.e. they hold (or fail to hold) at a single point regardless of what happens at other points. Moreover, it is easy to see that if an allocation Z is in equilibrium for coalition X, it is also in equilibrium for any coalition kX $(k > 0)$. Thus, given an equilibrium mapping (defined on all of R_{+}^{m}) it is always possible to obtain a homogeneous equilibrium mapping: it suffices to consider its restriction to some subdomain such as the unit simplex $\sum x_i = 1$, and expand this restriction to the entire space by equation (9). Thus, from an equilibrium mapping we can always obtain a consistent mapping.

On the other hand, given a consistent mapping, there is no guarantee (at least not at this time) that it will be homogeneous. (Note that [Owen 1994] shows that in general the consistent solution need not be unique, even for finite games.) Thus there may be consistent mappings which are not equilibrium mappings.

Of course, it is also important to remember that these are, essentially, non-atomic approximations to large but finite games. The question is whether the consistent allocations to the finite game will "eventually" converge to the consistent allocation for the non-atomic game. Our analysis leaves that question open, but seems to say that, **if it does converge**, then the allocation thus obtained will be an equilibrium allocation.

Footnotes

of mathematical analysis. Certainly, in physics, the answer seems to be in the affirmative: fluid dynamics treats both gases and liquids as continuous media rather than as consisting of very large numbers of "minuscule billiard balls", and the results obtained seem to fit well with observations. In economics, the idea was first developed in detail by Aumann and Shapley [1974] though the idea of a continuum of players (or economic traders) had been introduced earlier than that; vide the bibliography in [Aumann/Shapley 1974] for prior articles in this vein. Of course the question of the validity of such an approximation (large finite games replaced by non-atomic games) is a difficult one.

[2] The importance of (degree 0) homogeneity lies in the fact that, if a solution is homogeneous, then the economy can expand with no dislocations. In effect, it says that, so long as the economy expands **in scale** (i.e. the relative numbers of each type of trader remain constant), then the returns to each class of agent will be unchanged. In the non-homogeneous case, expansion of the economy, even **in scale**, would increase the returns to some agents while decreasing the returns to other agents. This would give rise to the standard cries of unfairness, etc. which characterize economic activities nowadays, and generaly create all sorts of difficulties.

[3] Starting with equation (7), we have introduced the partial derivatives z^i_{jk} as important factors in our analysis. Now, in the consistent solution, the utility payoffs $u_i(X)$ are, by equation (6), differentiable functions (almost everywhere) of the coalition components x_k. The question is whether we can assume that, in that same solution, the bundles z^i are differentiable functions of the x_k.

Certainly, in the most general case, we canot even expect the z^i to be continuous, let alone differentiable, functions of the x_k. Let us assume, however, that the utility functions u_i are strictly concave. In this case the values of u_i, being Pareto- optimal, determine the z^i uniquely, and it will follow (by an argument similar to that in [Owen 1995, pp. 72-73]) that z^i are continuous as functions of the x_k. In such case the differentiability (almost everywhere) is a reasonable assumption.

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Finite Convergence of the Core in a Piecewise Linear Market Game

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Abstract

We consider a pure exchange economy or "market" such that every player/agent has piecewise linear utilities. The resulting NTU–market game is a piecewise linear correspondence. Using a version of "nondegeneracy" for games in characteristic function form, we exhibit conditions to ensure the finite convergence of the core towards the Walrasian equilibrium.

SECTION 0 : Introduction and Notation

Equivalence theorems in General Equilibrium Theory and Game Theory state that for increasing or large sets of players/agents some solution concepts like the Walrasian equilibrium, the Core, or the Shapley value approximately coincide. Earlier, these results where formulated within the framework of replicated markets or games (e.g. DEBREU – SCARF [3]). Beginning with AUMANN's paper [1] , the measure space of agents/players turned out to be a very fruitful concept, see also HILDENBRAND [4] and MAS–COLELL [5] .

However, to some extent there is a third approach based on combinatorial or number–theoretical considerations related to the concept of a "non–degenerate" characteristic function in the sense of Cooperative Game Theory. Models of this type deal with a finite framework. There is a fixed number r of *types* (similar to the replica–model) of players and integer vectors of the grid $\mathbb{N}^r$ are interpreted as distributions of players over the types. The task is to describe certain areas in $\mathbb{N}^r$ and certain subgrids such that , within these areas the elements of the subgrids yield distributions of players such that a certain equivalence theorem holds true.

An overview over some applications of *nondegeneracy* and *homogeneity* as "surrogates" for nonatomicity is presented in [9] . More recent evidence is provided in PELEG–ROSENMÜLLER [6] and ROSENMÜLLER–SUDHÖLTER [10] , where it is shown that the *nucleolus* is as well a suitable object for an equivalence theorem in as much as, for sufficiently large sets of players in a homogeneous simple game, it coincides with the unique representation of this game.

The above–mentioned methods appear to work smoothly mainly in models which exhibit a "side–payment" or "transferable utility" character. Clearly, this is so since some version of "optimization" or "linear production" is always involved, at least implicitly. The present paper is meant to provide a first approach to the NTU–case. We want to study a pure exchange economy and the NTU–game it generates in a framework with

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finitely many types and piecewise linear utility functions. Can we exhibit areas in $\mathbb{N}^r$ such that the Core and the Walrasian Equilibrium coincide by suitably extending the definition of a *nondegenerate* game to the NTU–case and solving the appropriate combinatorial problem?

As yet the result is only a partial one: for certain classes of pure exchange economies the approach is successful, thus, we may formulate a "finite convergence"–theorem. However, since *nondegeneracy* and *piecewise linearity* of an exchange economy and the corresponding NTU–game viewed as a correspondence on distributions of players over the types can be established as consistent concepts, it may be that a departure point for more insight into the combinatorial structures of equivalence theorems has been reached.

The paper is organized as follows. SECTION 1 studies the NTU–market–game resulting from a pure exchange economy viewed as a correspondence from generalized profiles of coalitions into utility–space.It turns out that piecewise linearity of the utility functions of types renders this correspondence also to be piecewise linear. The regions of linearity are cones in $\mathbb{R}^r$ – the space of idealized distributions of players. Hence, in SECTION 2, we may study the behavior of a face of the correspondence within a region of linearity. Such a face is of course a polyhedron of lower dimension and the behavior of its normal as a function of profiles of coalitions is crucial.The extremals of the faces again can be identified as piecewise linear function (SECTION 3). Thus , gradually we approach the behavior of prices corresponding to faces of the NTU–game–correspondence. Eventually, in SECTION 4, we link Equilibrium–prices and Core–payoffs. SECTION 5 provides an extended example.

We now start out to provide the setup for our discussion. The piecewise linear pure exchange economy and the NTU–game derived from its data.

Let us consider a pure exchange economy with finitely many ***types*** of agents or players, for short such an economy shall be called a ***market***. Types are indicated by $\rho \in R = \{1,...,r\}$. The commodity space is $\mathbb{R}^m_+$, thus $j \in J = \{1,...,m\}$ refers to a commodity. Each player of type ρ commands an initial allocation $a^\rho \in \mathbb{R}^m_{++}$ (strictly positive).

We assume the preferences to be represented by a piecewise linear utility function which, for type ρ, is denoted by $u^\rho : \mathbb{R}^m_+ \longrightarrow \mathbb{R}$. More precisely, let us assume that there is a finite set L (not depending on $\rho \in R$ without loss of generality) and vectors $c^{\rho l} \in \mathbb{R}^m_+$ $(\rho \in R, l \in L)$ such that, for any $x \in \mathbb{R}^m_+$,

$$(1) \qquad u^\rho(x) = \min_{l \in L} c^{\rho l}x + d^{\rho l} = \min_{l \in L} h^{\rho l}(x), \text{ (where } h^{\rho l}(x) := c^{\rho l}x + d^{\rho l} \text{ for } x \in \mathbb{R}^m_+).$$

We assume $c^{\rho l} > 0$, $d^{\rho l} > 0$ $(\rho \in R, l \in L)$ thus, each $u^{\rho l}$ is positive, strictly monotone and, of course, continuous.

If we introduce the convex closed polyhedron $H^{\rho l} = \{x \in \mathbb{R}^m_+ \mid h^{\rho l}(x) \leq h^{\rho l'}(x) \; (l' \in L)\}$ for some $\rho \in R$ and $l \in L$, then $u^{\rho l}$ equals $h^{\rho l}$ within $H^{\rho l}$ and $\mathbb{R}^m_+$ is covered via

$\mathbb{R}^m_+ = \bigcup_{l \in L} H^{\rho l}$ for each $\rho \in R$. There is no loss of generality involved in assuming that each $H^{\rho l}$ has nonempty interior.

Let $k = (k_1,...,k_r) \in \mathbb{N}^r$ be a vector of integers. Then the resulting market $\bar{u}^k$ consists of k_1 players with utility u^1 and initial allocation a^1,..., k_r players with utility u^r and initial allocation a^r; that is, $\bar{u}^k$ is the "k–replica" of $\bar{u}^{(1,...,1)} = \bar{u}^e$ $(e = (1,..,1) \in \mathbb{N}^r)$.

Let $s = (s_1,...,s_r) \in \mathbb{N}^r_0 := \mathbb{N}^r \cup \{0\}$ (some coordinates allowed to be zero) be an integer (or 0) vector such that $s \leq k$. Then s is the ***profile*** of a coalition having s_1 players of type 1,..., s_r players of type r. Without offering an interpretation, we consider also "generalized profiles", i.e., vectors $t \in \mathbb{R}^r_+$. For any $t \in \mathbb{R}^r_+$ (and in particular for profiles), the (***aggregate***) ***initial endowment*** is

(2) $$a^t := \sum_{\rho \in R} t_\rho a^\rho$$

and the ***feasible*** allocations (assuming equal treatment) are given by

(3) $$\mathfrak{A}_t := \{X = (x^\rho)_{\rho \in R} \mid x^\rho \in \mathbb{R}^m_+, \sum_{\rho \in R} t_\rho x^\rho = a^t\}.$$

Switching to utility space we introduce the notation

$$u(X) = (u^1(x^1),...,u^r(x^r)), \quad t \otimes u(X) = (t_1 u^1(x^1),...,t_r u^r(x^r))$$

for $t \in \mathbb{R}_{++}$, $X \in \mathfrak{A}_t$. Finally consider, for $t \in \mathbb{R}_+$

(4) $$F(t) = \{u \in \mathbb{R}^r_+ \mid u \leq t \otimes u(X) \text{ for some } X \in \mathfrak{A}_t\},$$

then F is a correspondence that maps generalized profiles into subsets of $\mathbb{R}^r_+$, the elements of which can be considered as feasible utility vectors to a (generalized) coalition with profile t, coordinate ρ denoting the joint utility for all players of type ρ.

The restriction of F to all profiles, i.e., to $\{s \in \mathbb{N}^r_0 \mid s \leq k\}$ for some $k \in \mathbb{N}^r$ is denoted by $V^k = V^{k,u}$ and called the ***characteristic function*** of (the nonside–payment– or NTU–game corresponding to) $\bar{u}^k$.

SECTION 1: The Market Game as a Piecewise Linear Correspondence

Since all utilities are piecewise linear, the correspondence F should exhibit a similarly simple shape.We expect F(t) to be a convex compact polyhedron for fixed t; thus our first aim should be to describe the nature of the extremals. Next, with varying t, it turns out that F also behaves "piecewise linear" in the sense that within the interior of certain well defined cones in $\mathbb{R}^m$, the extremals are linear functions of t. This exposition is the aim of the first section.

For fixed type ρ the utility function u^{ρ} is linear within each of the convex polyhedra $H^{\rho l}$ as depicted in Figure 2., and these polyhedra provide a polyhedral covering of $\mathbb{R}^m$.If we consider an (equal treatment) allocation $X = (x^{\rho})_{\rho\in R}$, then each x^{ρ} is somehow located in some $H^{\rho l}$; thus refers to the system of polyhedra generated by u^{ρ} as explained above.

Next, consider a utility vector $\bar{u}$ which is an extremepoint of F(t) for some $t \in \mathbb{R}_{++}$. Suppose that this vector is generated by some $\bar{X} \in \mathfrak{A}_t$ via $\bar{u} = t \otimes u(\bar{X})$. Then we expect a tendency of the $\bar{x}^{\rho}$ ($\rho \in R$) to move towards the boundaries of the $H^{\rho l}$, roughly speaking. That is, next to the inevitable equation $\Sigma\, t_{\rho}x^{\rho} = a^t$, there will be "many" equations of the types $x^{\rho}_j = 0$ and $t_{\rho}h^{\rho l}(x^{\rho}) = u^{\rho}$ to be satisfied by $\bar{X}$ and $\bar{u}$ respectively; say for $j \in J_{\rho}$ and $l \in L_{\rho}$.

More precisely, let us first formulate the appropriate property for a system of sets of indices $\mathfrak{J} = (J_1,\ldots,J_r,L_1,\ldots,L_r)$.

Definition 1.1. Let $\mathfrak{J} = (J_1,\ldots,J_r,L_1,\ldots,L_r)$ be such that $J_{\rho} \subseteq J$ ($\rho \in R$) and $L_{\rho} \subset L$ ($\rho \in R$). $\mathfrak{J}$ is said to be an ***admissible system*** if the following equations are satisfied.

$$\text{(1)} \qquad \sum_{\rho=1}^{r} |J_{\rho}| \leq r(m-1),$$

$$|L_{\rho}| \geq 1\ (\rho \in R),\ \sum_{\rho=1}^{r} |J_{\rho}| + \sum_{\rho=1}^{r} |L_{\rho}| = mr+r-(m-\min_{\rho\in R} |J_{\rho}|)\,.$$

Now we may establish the connections between extreme points of F(t), admissible systems, and generating elements of $\mathfrak{A}_t$ as follows.

Lemma 1.2: Let $t \in \mathbb{R}^r_{++}$ and let $\bar{u} \in F(t)$ be a Pareto efficient ("P.E.") extreme point of F(t). Then there exists $\bar{X} \in \mathfrak{A}_t$ and an admissible system $\mathfrak{J} = (J_1,\ldots,J_r,L_1,\ldots,L_r)$ such that $(\bar{X},\bar{u})$ is the unique solution of the linear system of rm + r equations (in variables $(X,u) \in \mathbb{R}^{m\times r} \times \mathbb{R}^r$) given by

$$\text{(2)} \qquad x^{\rho}_j = 0\ (\rho \in R,\, j \in J^{\rho}),\ t_{\rho}\, h^{\rho l}(x^{\rho}) - u_{\rho} = 0,\ (\rho \in R,\, l \in L^{\rho}),\ \sum_{\rho\in R} t_{\rho}x^{\rho} = a^t$$

Proof: As $\bar{u}$ is Pareto optimal, it is easy to show that

$$\text{(3)} \qquad \mathfrak{A}^{\bar{u}}_t := \{X \in \mathfrak{A}_t,\ t \otimes u(X) = \bar{u}\}$$

is a nonempty, compact, convex polyhedron. Let $\bar{X}$ be an extreme point of $\mathfrak{A}^{\bar{u}}_t$ and put

$$J_{\rho} := \{j \in J \mid \bar{x}^{\rho}_j = 0\}\ \ (\rho \in R),\ \ L'_{\rho} := \{l \in L \mid t_{\rho}\, h^{\rho l}(\bar{x}^{\rho}) = \bar{u}_{\rho}\}\ \ (\rho \in R).$$

It is then verified by standard arguments that $(\overline{X},\overline{u})$ is the unique solution of the corresponding system (2) (– where L_ρ has been replaced by L'_ρ).

Now inspect the coefficient matrix of (2) (note that $h^{\rho l}(x) = c^{\rho l}x + d^{\rho l}$). Observe that the first and last group of equations yield a submatrix of rank

$$|J^1| + \ldots + |J^r| + m - \min_{\rho \in R} |J_\rho| \leq r\,m.$$

As the rank of the original matrix is mr + r, it is possible to complete this submatrix in order to obtain a square submatrix by choosing (at least r) rows from the middle group of (2) in such a way that for each ρ at least on row corresponding to a variable u_ρ appears. This procedure defines index sets $L_\rho \subseteq L'_\rho$ ($\rho = 1,\ldots,r$), $|L_\rho| \geq 1$. Note, that for each j there is ρ such that $j \notin J_\rho$; this justifies the first equation (1). The last equation (1) is obtained by counting rows. Note that $\overline{X}$ is a P.E. allocation which is extreme in $\mathfrak{A}_t^{\overline{u}}$. q.e.d.

Henceforth, a P.O. extreme point $\overline{u}$ of F(t) is called a ***t–vertex***, an extreme point $\overline{X} \in \mathfrak{A}_t^{\overline{u}}$ is called a ***corresponding node***.

Corollary 1.3: For every $t \in \mathbb{R}^r_{++}$, F(t) is a compact, convex polyhedron.
For, the number of admissible systems $\mathfrak{J}$ is finite and so is the number of vertices.
Given an admissible system $\mathfrak{J}$, consider for some $t \in \mathbb{R}^r_{++}$ the system of linear equations in variables η_ρ ($\rho \in R$), ξ^ρ_j ($\rho \in R$, $j \in J$) given by

(4) $\quad \xi^\rho_j = 0 \quad (\rho \in R, j \in J^\rho),\ c^{\rho l}\,\xi^\rho + \eta_\rho = -\,t_\rho\, d^{\rho l}\ (\rho \in R, l \in L^\rho),\ \sum_{\rho \in R} \xi^\rho = a^t$,

we have

Lemma 1.4: Let $\mathfrak{J}$ be admissible. Then the system (4) has a unique solution if and only if (2) has a unique solution.

Proof: Clearly, $(\overline{\xi}, \overline{\eta})$ is a solution of (4) if and only if $(\ldots, \overline{x}^\rho,\ldots,\ldots,\overline{u}) = (\ldots,\frac{\overline{\xi}^\rho}{t_\rho},\ldots,-\overline{\eta})$ is a solution of (2). Obviously, the square coefficient matrix of (4) does not depend on $t \in \mathbb{R}_{++}$. Hence, it is or is not nonsingular independently of t.

Definition 1.5: Let us call a system $\mathfrak{J} = (J_1,\ldots,J_r,L_1,\ldots,L_r)$ ***nonsingular*** if it is admissible and the coefficient matrix of (4) is nonsingular.

A nonsingular system obtained by a vertex $\overline{u}$ via some corresponding $\overline{X}$ node by means of the procedure indicated in the proof of Lemma 1.2 is also called ***corresponding*** (to $\overline{u}$ or to $(\overline{X},\overline{u})$).

Lemma 1.6: Let $\mathfrak{J}$ be a nonsingular system. Then there exist an $r \times r$ matrix $A = A^{\mathfrak{J}}$ and r $m \times r$ matrices $B^{\rho} = B^{\rho\mathfrak{J}}$ $(\rho = 1,...,r)$ such that, for $t \in \mathbb{R}_{++}$, the unique solution of (4) is given by

(5) $\qquad \bar{\eta} = -A\,t \quad , \quad \bar{\xi}^{\rho} = B^{\rho}\,t, \ (\rho \in R)$

Thus, in particular, $\bar{\eta}$ and $\bar{\xi}^{\rho}$ are linear in t. Of course, the unique solution of (2) is equivalently given by

(6) $\qquad \bar{u} = A\,t \ , \ \bar{x}^{\rho} = \frac{1}{t_{\rho}}\, B^{\rho}\,t, \ (\rho \in R)$

Proof: This follows by Cramers rule (cf.[12]).

We denote by int T the ***interior*** of a subset T of $\mathbb{R}^{r}_{+}$ and by CVCPH T its ***convex comprehensive hull***

Theorem 1.7: There exists a finite collection $\mathfrak{T} = \{T,...,T'\}$ of subsets of $\mathbb{R}^{r}_{+}$ with the following property.

1. $T \in \mathfrak{T}$ is a closed, convex polyhedral cone with apex $0 \in \mathbb{R}^{r}_{+}$ and nonempty interior.
2. $\mathbb{R}^{r}_{+} = \bigcup \{T \mid T \in \mathfrak{T}\}$.
3. For $T, T' \in \mathfrak{T}$ the cone $T \cap T'$ is lower-dimensional.
4. For $T \in \mathfrak{T}$ there exists a (finite) family $Q = Q_{T}$ of nonsingular systems such that, whenever $t \in T$, $t > 0$

(7) $\qquad F(t) = \text{CVCPH}\,\{A^{\mathfrak{J}}t \mid \mathfrak{J} \in Q\}$

i.e., F(t) is the *convex comprehensive hull* of vectors $A^{\mathfrak{J}}t$, $\mathfrak{J} \in Q$.

Proof: *1st Step*: Consider a nonsingular system $\mathfrak{J} = (J_1,...,J_r, L_1,...,L_r)$ and the corresponding solution $(\bar{X}, \bar{u})$ of (3). Clearly, $\bar{u} \in F(t)$ if $\bar{X} \geq 0$ and $h^{\rho l}(\bar{x}^{\rho}) \geq h^{\rho l'}(\bar{x}^{\rho})$ $(\rho \in R, l \in L_{\rho}, l' \in L-L_{\rho})$. This is equivalent to $\bar{\xi} \geq 0$ and $c^{\rho l}\,\bar{\xi}^{\rho} + t_{\rho}\,d^{\rho l} \geq c^{\rho l'}\,\bar{\xi}^{\rho} + t^{\rho}\,d^{\rho l'}$ $(\rho \in R, l \in L_{\rho}, l' \in L-L_{\rho},)$. Again, if $A = A^{\mathfrak{J}}$ and $B^{\rho} = B^{\rho\mathfrak{J}}$ $(\rho \in R)$ are determined by Lemma 1.6, we conclude that $\bar{u} \in F(t)$ if

(8) $\qquad B^{\rho}t \geq 0 \ \ (\rho \in R), (c^{\rho l}-c^{\rho l'})\,B^{\rho}t \geq t_{\rho}\,(d^{\rho l}-d^{\rho l'}) \ \ (\rho \in R, l \in L_{\rho}, l' \in L-L_{\rho}).$

Now, let us regard the rows of this system (which is linear in t) as to be represented by a matrix $D = D^{\mathfrak{J}}$ – which depends on $\mathfrak{J}$ only (and can be computed by means of the $c^{\rho l}$ and $B^{\mathfrak{J}}$).

From this, it follows that

(9) $\qquad F(t) = \text{CVCPH}\,\{A^{\mathfrak{J}}t \mid \mathfrak{J} \text{ nonsingular}, D^{\mathfrak{J}}t \geq 0\}$

holds true for $t \in R_{++}$.

2nd Step: The proof proceeds by omitting some of the nonsingular systems and rearranging the other ones until the requirements of the theorem are satisfied. See [12] for the details. q.e.d.

SECTION 2 : t–Faces

The previous section explains the behavior of the correspondence F(.): essentially it can be seen as the convex hull of finitely many linear functions in t – provided we restrict our observation to an appropriate subcone of $\mathbb{R}^r_{++}$. We would like to eventually obtain the same picture with respect to a certain face (r–1 – dimensional subpolyhedron in the boundary) of F(t). If necessary, we shall of course restrict ourselves to further subcones, however, it would be desirable to obtain a boundary face again as the convex hull of a finite number of linear functions.

First of all we shall have to clear up the relation of extremals in utility space $\mathbb{R}^r_{++}$ and the polyhedral decomposition in $\mathbb{R}^m_+$ – which is specified for each type ρ (cf. SEC. 1). We should expect that the nodes corresponding to extremals of the same face are located within the same $H^{\rho l}$.

Lemma 2.1.: Let $t \in \mathbb{R}^r_{++}$ and let $(\bar{u}^q)_{q \in Q}$ be a finite set of vertices of F(t) belonging to the same P.E. face. Then, for $\rho \in R$, there is $l = l(\rho) \in L$ with the following property: if, for each $q \in Q$, $\bar{X}^q$ is a corresponding node then, for $\rho \in R$, $\bar{x}^{q\rho} \in H^{\rho l}$ $(q \in Q)$. That is, any choice of extreme points of $\mathfrak{A}^{\bar{u}^q}_t$ $(q \in Q)$ yields points situated in a common $H^{\rho l}$.

It is sufficient to just sketch the following

Proof: If, for some $\rho \in R$, the $\bar{x}^{q\rho}$ are not adjacent to at least one $H^{\rho l}$, then consider for $\alpha_q > 0$, $(q \in Q)$, $\sum_{q \in Q} \alpha_q = 1$, the utility vector $\bar{u} = \sum_{q \in Q} \alpha_q \bar{u}^q$ which is an element of the face in consideration. Also, $\bar{X} := \Sigma \alpha_q \bar{x}^q$ satisfies $\bar{X} \in \mathfrak{A}_t$. However, since the $\bar{x}^{q\rho}$ are not adjacent to a common $H^{\rho l}$, we have

$$t_\rho\, u^\rho(\bar{x}^\rho) = t_\rho\, u^\rho\Big(\sum_{q \in Q} \alpha_q \bar{x}^{q\rho}\Big) > t_\rho \sum_{q \in Q} \alpha_q\, u^\rho(\bar{x}^{q\rho}) = \sum_{q \in Q} \alpha_q \bar{u}^q_\rho = \bar{u}_\rho,$$

while, for all remaining $\rho' \in R$ we have at least $\geq$. This contradicts the fact that $\bar{u}$ is P.E.

Let us use the notation *t–face* for a P.E. face of F(t) which has dimension r–1. A t–face is, of course, the convex hull of all the t–vertices it contains. F(t) is the comprehensive convex hull of all its t–vertices. We say that $x, y \in \mathbb{R}^m_+$ are ***adjacent*** (referring to some $\rho \in R$) if there is some $l \in L$ such that $U^{\rho l}$ contains both points.

Corollary 2.2.: 1. If $\bar{u}$ is a t–vertex, then all corresponding nodes are adjacent.

2. Let $(\bar{u}^q)_{q\in Q}$ be (all) t–vertices belonging to the same face, say $F^Q(t)$, and let $\bar{X}^q$ be a corresponding node ($q \in Q$). Suppose, for each q, $L^q_\rho \subseteq L$ is given by Lemma 1.1. ($\rho = 1,\dots,r$). Then, for every $\rho \in R$,

(1) $$\bigcap_{q\in Q} L^q_\rho =: L^Q_\rho \neq \emptyset .$$

Thus, the $\bar{x}^{q\rho}$ ($\rho \in R$) are adjacent.

3. $\mathfrak{A}^Q_t := \{X \in \mathfrak{A}_t \mid u(X) \in F^Q(t)\}$ is a compact convex polyhedron; the extreme points are the extreme points of $\mathfrak{A}^{\bar{u}^q}_t$ ($q \in Q$), hence corresponding nodes. The ρ–coordinates of these nodes are adjacent.

4. If, for $\rho \in R$, we choose $\ell = \ell(\rho) \in L^Q_\rho$, according to Lemma 2.1., then

(2) $$F^Q(t) = \{t \circledast \bar{h}(X) := (t_1 h^{1\ell(1)}(x^1),\dots,t_r h^{r\ell(r)}(x^r)) \mid X \in \mathfrak{A}^Q_t\}$$

The choice of an index set Q in order to identify a face may seem to be an arbitrary one. We shall make an attempt to justify this procedure in SEC. 3.

The next remark shortly describes the way, the normal of a face depends on the parameter t.

Remark 2.3.: Let $t \in \mathbb{R}^r_{++}$ and let $(\bar{u}^q)_{q\in Q}$ be a set of r t–vertices spanning a t–face, say $F^Q(t)$, of F(t). In view of Lemma 1.1 and 1.5 there is, for any $q \in Q$, an $r\times r$–matrix A^q such that $\bar{u}^q = A^q t$ ($q \in Q$) holds true. Let $\bar{\lambda} = \bar{\lambda}^{Q,t}$ denote the *normal* of the t–face $F^Q(t)$; this normal can be chosen to be nonnegative and hence can be normalized to satisfy $\bar{\lambda}_1 + \dots + \bar{\lambda}_r = 1$. If so, then $\bar{\lambda}$ may be obtained as the solution of the linear system of equations

$$\lambda(\bar{u}^q - \bar{u}^q) = 0 \ (q \in Q, q \neq p) \quad \sum_{\rho\in R} \lambda_\rho = 1$$

where p is some fixed element of Q.That is

(3) $$\lambda(A^q - A^p)t = 0, \quad \sum_{\rho\in R} \lambda_\rho = 1$$

Using Cramers rule and expanding determinants we find at once r polynomials in $t_1,\dots,t_r$ of degree r–1, say D_ρ ($\rho = 1,\dots,r$) such that

$$\bar{\lambda}_\rho(t) = \frac{D_\rho(t)}{D_1(t) + \dots + D_r(t)}$$

Next we would like to discuss the behavior of *price vectors* corresponding to some face of F(t). To this end, pick $t \in \mathbb{R}_{++}$ and consider some face, say $F^Q(t)$. Let $\bar{u} \in F^Q(t)$ and let $\bar{X} \in \mathfrak{A}_t$ be a corresponding node, thus $\bar{u} = t \circledast u(\bar{X})$ holds true. Let $\bar{\lambda}$ denote the normal at $F^Q(t)$in $\bar{u}$.

Since $\bar{u}$ is Pareto efficient, we know that $\Sigma \lambda_\rho \bar{u}_\rho = \Sigma \lambda_\rho t_\rho u^\rho(\bar{x}^\rho) = \max \{ \Sigma \lambda_\rho t_\rho u^\rho(x^\rho) \mid X \in \mathfrak{A}_t \}$, and by a standard Kuhn – Tucker – argument there is a joint tangential hyperplane for the graphs of all functions $\lambda_\rho u^\rho(\ )$ at $\bar{x}^\rho$. (The constraints defining $\mathfrak{A}_t$ include the t_ρ – therefore the t_ρ do not appear in the description of these hyperplanes.)

As gradients of these hyperplanes all gradients of $\lambda_\rho u^\rho(\ )$ at $\bar{x}^\rho$ are feasible. With $\bar{x}^\rho$ in the interior of some $H^{\rho l}$, they are uniquely defined (and equal $\lambda_\rho c^{\rho l}$) – and at the boundaries of the $H^{\rho l}$ we may take the corresponding convex combinations.
The joint tangential hyperplanes may be viewed as graphs of the linear functions $\lambda_\rho u^\rho(\bar{x}^\rho) + \bar{p}(y - \bar{x}^\rho)$ $(y \in \mathbb{R}^m_+)$ satisfying $\lambda_\rho u^\rho(\bar{x}^\rho) + \bar{p}(y - \bar{x}^\rho) \geq \lambda_\rho u^\rho(y)$ $(y \in \mathbb{R}^m_+)$.

Because of the above–mentioned situation, the *price vector* $\bar{p}$ for each ρ has to be a convex combination of certain $\lambda_\rho c^{\rho l}$ – and the set of indices l depends on Q only.

Because these reflections are still vague, we supply a more formal description in [12].

SECTION 3 : Q – Faces

Consider a cone T as specified by Theorem 1.6. With T, for any t we know that F(t) is the convex, comprehensive hull of finitely many points of the form $A^{\mathfrak{I}}t$; here $\mathfrak{I}$ denotes a nonsingular system in the sense of SEC. 1. However, if $\mathfrak{I}$ is nonsingular and $A^{\mathfrak{I}}t$ is feasible then, nevertheless, the latter is not necessarily Pareto efficient. Also, if $(\mathfrak{I}^q)_{q \in Q}$ is a family of nonsingular systems, then the points $A^q t = A^{\mathfrak{I}^q} t$ do not necessarily constitute a t–Face.

In order to specify a family of nonsingular systems $Q \subseteq \{\mathfrak{I} \mid \mathfrak{I}$ is nonsingular$\}$ we identify indices $q \in Q$ and systems $\mathfrak{I} = \mathfrak{I}^q$.

Theorem 3.1.: Let $T \subseteq \mathbb{R}^r_+$ be a cone as given by Theorem 1.6, and let Q be the corresponding family of nonsingular systems. Then there is a finite collation $\hat{\mathfrak{T}} = \{\hat{T}, \hat{T}', \ldots\}$ of subsets of T with the following property.

1. $\hat{T} \in \hat{\mathfrak{T}}$ is a closed cone with apex 0 and nonempty interior.
2. $T = \bigcup \{\hat{T} \mid \hat{T} \in \mathfrak{T}\}$
3. For $\overset{o}{T}$, $\hat{T}' \in \hat{\mathfrak{T}}$ the cone $\hat{T} \cap \hat{T}'$ is lower–dimensional.
4. For any $\hat{T} \in \hat{\mathfrak{T}}$ there exists a finite family $\underline{\underline{\hat{Q}}} = \{\hat{Q}\}$ of subsets of Q such that, for $t \in \hat{T}$, the sets

   $$F^{\hat{Q}}(t)\ \mathrm{CVH}\ \{A^q t \mid q \in \hat{Q}\}$$

   are exactly the t–faces of F(t).

In other words, within every $\hat{T}$ the t–faces depend essentially only on the choice of a system $\hat{Q} \subseteq Q$. Hence, if we pick $\hat{Q} \in \underline{\underline{\hat{Q}}}$, then for any $t \in \hat{T}$ we know that $F_{\hat{Q}}(t)$ is a t–face.

The proof is slightly tedious and similar to the one given for Theorem 1.7. We refer the reader to [12] for the details.

In the following we want to deal with those situations only that yield a unique price vector for every face of the correspondence F at varying t. Clearly, inside a cone T as specified by Theorem 3.1, faces may be identified with index–sets Q, and hence the required property is a property of the cone – and not of varying t.

Definition 3.2: $\vec{u}^e$ has *finite character* if, for every T as given by 3.1. and every Q specifying a face $F^Q(t)$ for all $t \in T$, the price vector is uniquely defined, i.e., $P^Q(\overline{\lambda})$ is a singleton, depending on Q only.

This definition is very much "ad hoc" and it will have to be enhanced by pointing to appropriate requirements for $\vec{u}^e$ to satisfy such that finite character is ensured. This we will postpone to some other treatment. However, *there are* classes of markets satisfying this condition. An obvious candidate is the class of ***side–payment*** or ***TU–markets*** as the normal λ equals $e = (1,...,1)$ constantly – and indeed, this class has been shown to yield a finite convergence theorem based on nondegeneracy – see [8].

Another class can be obtained by "generically" requiring that, for any T and Q the resulting face $F^Q(t)$ enjoys a (relatively) interior point, say $\bar{u}$, such that a corresponding element $\bar{X} \in \mathfrak{A}_t$ provides at least one $\bar{x}^\rho$ within the interior of some $H^{\rho l}$. Indeed, by counting equations, it is easily seen that $mr \geq m + r$ is sufficient to ensure that generically every face of F has dimension r–1. (The mapping
$\{ X \in \mathbb{R}^{mr} \mid X = (x^\rho)_{\rho\in R}, \sum_\rho t_\rho x^\rho = \sum_\rho t_\rho a^\rho \} \rightarrow \{ t_1c^1x^1, \ldots ,t_rc^rx^r \}$ must have full rank, which is generically ensured by $mr - m \geq r$).

On the other hand, if all elements of a face $F^Q(t)$ yield corresponding $X \in \mathfrak{A}_t$ such that *every* x^ρ in at least *two* $H^{\rho l}$, then these elements satisfy r + m equations; thus if $mr - (m + r) < r - 1$, then the Pareto face cannot have full dimension. Therefore, by asking for $r + m + (r - 1) > mr > m + r$ we ensure the finite character of the market.

With finite character we will essentially be able to demonstrate finite convergence of the core towards the Walrasian equilibrium – this is the topic of the next section.

SECTION 4: Finite Convergence of the Core

Within this section we are now going to collect our structural results in order to prove a "finite convergence" property of the core towards the Walrasian equilibrium. Presently, we can only succeed for core–payoffs that are within the relative ϵ–interior of some Pareto–Face of F(t), say $F_Q(t)$. However, similar to the situation in [7] and [8], it is possible to exhibit regions in $\mathbb{N}^r$ (to be interpreted as areas of distributions of players over the types) such that the core and the Walrasian equilibrium coincide.

Within some cone $\hat{T}$ as defined by Theorem 3.1 a face $F^Q(T)$ is essentially defined by a system Q (of index sets) and by the resulting extremals of the form $A^q t$ ($q \in Q$). Denote by $K^Q(t)$ the *cone with vertex* 0 *that is spanned by* $A^q t$, i.e.

(1) $$\dot{K}^Q(t) = \{ \sum_{q \in Q} \alpha_q A^q t \mid \alpha_q \geq 0 \ (q \in Q)\}.$$

If $u \in \mathbb{R}^r_+$ is a vector satisfying $u \in K^Q(t)$, $u \notin F(t)$, then u is located in front of $F^Q(t)$, this cab be separated from F(t) by the hyperplane containing $F^Q(t)$ – which is defined by the normal λ.

Lemma 4.1: Let $\bar{u}$ be of finite character. Fix Q and consider $t \in \hat{T}$ (cf. 3.1) such that $F^Q(t)$ is a face of F(t), let $\bar{p}$ be the corresponding price vector (Definition 3.2). Let $\bar{X} \in \mathcal{O}_t(\bar{u})$ be such that $\bar{u} = t \otimes u(\bar{X}) \in F^Q(t)$. Then, for any $s \in \mathbb{R}^r_+$ such that $s \otimes u(\bar{X}) \in K_Q(s)$, $s \otimes u(\bar{X}) \notin F(s)$, it follows that $\bar{p} \sum_{\rho \in R} s_\rho(\bar{x}^\rho - a^\rho) \geq 0$ holds true.

Proof: Let $\bar{\lambda}$ be the normal to $F^Q(t)$ and $\tilde{\lambda}$ the one to $F^Q(s)$. Then, for suitable $\rho \in \mathbb{R}$ and $\bar{c} \in \mathbb{R}^m_{++}$ the corresponding prices $\bar{p}$ and $\tilde{p}$ satisfy $\bar{p} = \bar{\lambda}_\rho \bar{c}$, $\tilde{p} = \tilde{\lambda}_\rho \bar{c}$ (Definition 3.2), Hence

(2) $$\tilde{p} = \text{const}\ \bar{p}$$

with a certain positive constant. From this we deduce the following line of inequalities for any $\tilde{X} \in \mathcal{O}_s(\bar{u})$ with $s \otimes u(\tilde{X}) \in F^Q(s)$:

$$\sum_{\rho \in R} \tilde{\lambda}_\rho s_\rho u^\rho(\bar{x}^\rho) \geq \sum_{\rho \in R} \tilde{\lambda}_\rho s_\rho u^\rho(\tilde{x}^\rho) \ldots$$

(since $\tilde{\lambda}$ separates $s \otimes u(\bar{X})$ from $F^Q(s)$)

$$\cdots = \sum_{\rho \in R} s_\rho (\tilde{\lambda}_\rho u^\rho(\tilde{x}^\rho) - \tilde{p}(\tilde{x}^\rho - a^\rho)) \ldots$$

(since $\tilde{X} \in \mathcal{O}_s(\bar{u})$)

(3)

$$\ldots \geq \sum_{\rho \in R} s_\rho (\tilde{\lambda}_\rho u^\rho(\bar{x}^\rho) - \tilde{p}(\bar{x}^\rho - a^\rho))$$

(by the Kuhn–Tucker–argument, see SECTION 2.)

$$= \sum_{\rho \in R} \tilde{\lambda}_\rho s_\rho u^\rho(\bar{x}^\rho) - \tilde{p} \sum_{\rho \in R} s_\rho(\bar{x}^\rho - a^\rho).$$

Hence, using (2) and (3)

$$\bar{p} \sum_{\rho \in R} s_\rho(\bar{x}^\rho - a^\rho) = \text{const}\ \tilde{p} \sum_{\rho \in R} s_\rho (\bar{x}^\rho - a^\rho) \geq 0.$$ q.e.d.

Next, for small $\epsilon > 0$, let $F^Q_\epsilon(t)$ denote the (closed) "ϵ–interior" of $F^Q(t)$; more precisely

(4) $$F^Q_\epsilon(t) = \{ \sum_{\rho \in Q} \alpha_q A^q t \mid \sum_{q \in Q} \alpha_q = 1, \alpha_q \geq \epsilon \quad (q \in Q)\}$$

This is a compact convex polyhedron with extremals, say, $\bar{u}^{q\epsilon} \in F^Q_\epsilon(t)$ $(q \in Q)$.

Lemma 4.2: Fix $\hat{T}$ according to Theorem 3.1. Let $\epsilon > 0$. For every $t \in \hat{T}$, there is a closed cone $C^Q_\epsilon(t)$ such that for all $\bar{u} = t \otimes u(\bar{X}) \in F^Q_\epsilon(t)$ it follows that

(a) t is located within the interior $C^Q_\epsilon(t)$; (b) if $s \in C^Q_\epsilon(t)$, then $s \otimes u(\bar{X}) \in K^Q(s)$.

Proof: Let $t \in \hat{T}$. First of all, fix $\bar{u} = t \otimes u(\bar{X}) \in F^Q_\epsilon(t)$. Note that $t \otimes u(\bar{X}) = \sum_{q \in Q} \alpha_q A^q t$ with suitable "convexifying" coefficients α. The ρ'th coordinate is

$$t_\rho u^\rho(\bar{x}^\rho) = \sum_{q \in Q} \alpha_q A^q_{\rho \cdot} t .$$

$(A_{\rho \cdot}$ is the ρ'th row of a matrix A); and hence the ρ'th coordinate of $s \otimes u(\bar{X})$ is

(6) $$s_\rho u^\rho(\bar{u}^\rho) = \frac{s_\rho}{t_\rho} \sum_{q \in Q} \alpha_q A^q_{\rho \cdot} t.$$ (Note that this is linear in s!). Consider

$$C := \{s \in \mathbb{R}^r_{++} \mid s \otimes u(\bar{X}) \in K^Q(s)\}.$$

Clearly, $t \in C$ and as $t \otimes u(\bar{X})$ is in the interior of $F^Q(t)$ (hence in the interior of $K^Q(T)$), C is seen to be a (closed) cone containing t in it's interior. (In fact, C is described by finitely many algebraic hypersurfaces.)

Now, let C^q $(q \in Q)$ denote the cones generated by taking the extreme points $\bar{u}^{q\epsilon}$ $(q \in Q)$ of $F^Q_\epsilon(t)$ and performing the above procedure. Put

(7) $$C^Q_\epsilon(t) := \bigcap_{q \in Q} C^q.$$

Then again, t is within the interior of the closed cone $C^Q_\epsilon(t)$.

Finally, let again $\bar{u} = t \otimes u(\bar{X})$ be arbitrary in $F^Q_\epsilon(t)$; we know that

$$t \otimes u(\bar{X}) = \bar{u} = \sum_{q \in Q} \alpha_q \bar{u}^{q\epsilon} = \sum_{q \in Q} \alpha_q \, t \otimes u(\bar{X}^{q\epsilon})$$

with suitable α_q and $\bar{X}^{q\epsilon}$ $(q \in Q)$. The ρ'th coordinate of $s \otimes u(\bar{X})$ is, therefore,

$$s_\rho u^\rho(\bar{x}^\rho) = \frac{s_\rho}{t_\rho} \sum_q \alpha_q t_\rho u(\bar{x}^{q\epsilon\rho}) = \sum_q \alpha_q s_\rho u(\bar{x}^{q\epsilon\rho}),$$ i.e.

(8) $$s \otimes u(\bar{X}) = \sum_{q \in Q} \alpha_q \, s \otimes u(\bar{X}^{q\epsilon})$$ (essentially the linearity of (6) in s is used!).

Therefore, if $s \in C_\epsilon^Q(t)$, then $s \otimes u(X^{q\epsilon}) \in K^Q(s)$ (by (7)), and $K^Q(s)$ is convex, (8) implies that $s \otimes u(\bar{X}) \in K^Q(s)$, q.e.d

Remark 4.3: Note that $C_\epsilon^Q(t)$ is "algebraic", i.e., its boundary is constituted by a finite number of algebraic hypersurfaces. Also, $C_\epsilon^Q(t)$ can be chosen to exhibit some property of being "positively homogeneous", i.e., if $\alpha > 0$ then $C_{\epsilon/\alpha}^Q(\alpha t) = C_\epsilon^Q(t)$.

Definition 4.4: Let $\mathbb{N}^r \ni k \in \hat{T}$. k *generates a full cone* (w.r.t. ϵ, Q) if there are r linearly independent vectors ("profiles") $s \in \mathbb{N}^r$ such that

$$\text{(a) } s \leq k, \text{ (b) } s \in C_\epsilon^Q(k), \text{ (c) } k - s \in C_\epsilon^Q(k),$$

is satisfied.

Using profiles $k \in \mathbb{N}^r$ we now switch to markets u^k with a certain distribution $k = (k_1,\dots,k_r)$ of players over the types: this setup we started out with in SECTION 0. We want to compare the Core $\mathcal{C}(\bar{u}^k)$ and the Walrasian allocations of $\bar{u}^k$.

Theorem 4.5: Let $k \in \mathbb{N}^r$ generate a full cone. If $\bar{X} \in \mathcal{C}(\bar{u}^k)$ is such that $k \otimes u(\bar{X}) \in F_\epsilon^Q(k)$, then $\bar{X}$ is Walrasian.

Proof: Fix $\bar{X}$ with the required properties. Since $\bar{X} \in \mathcal{C}(\bar{u}^k)$, we know that $s \otimes u(\bar{X})$ is not in the interior of $F(s) = V(s)$. If, in addition, $s \in C_\epsilon^Q(k)$ holds true, then we have all the conditions of Lemma 4.1. Therefore, there are r linearly independent and integer vectors $s \in \mathbb{N}^r$ satisfying the conclusion of Lemma 4.1, i.e.,

(10) $$\bar{p} \sum_{\rho \in R} s_\rho (\bar{x}^\rho - a^\rho) \geq 0,$$

here $\bar{p}$ is the price–vector corresponding to Q. Since for any such s, $k - s$ has the same property ("full cone"–definition) we have as well

(11) $$\bar{p} \sum_{\rho \in R} (k_\rho - s_\rho)(\bar{x}^\rho - a^\rho) \geq 0.$$

However, as $\bar{X} \in \mathcal{A}_k(\bar{u})$, we have

(12) $$\sum_{\rho \in R} k_\rho (\bar{x}^\rho - a^\rho) = 0$$

and hence it follows from (10) and (11) that

(13) $$\bar{p} \sum_{\rho \in R} s_\rho (\bar{x}^\rho - a^\rho) = 0.$$

Since our assumption is that there are r linearly independent vectors s satisfying (13), we conclude that

(14) $$\bar{p}(\bar{x}^\rho - a^\rho) = 0 \quad (\rho \in R).$$

It is well known that this implies that $(\bar{p}, \bar{X})$ is a Walrasian equilibrium. q.e.d.

Theorem 4.6: (An equivalence theorem). Let $\bar{u}^e$ be of finite character and let $\mathring{T} \subseteq \mathbb{R}^r_{++}$ be defined by Theorem 3.1. Then, for any $\epsilon > 0$ there exists a set $H_\epsilon \subseteq \mathring{T}$ with the following properties:

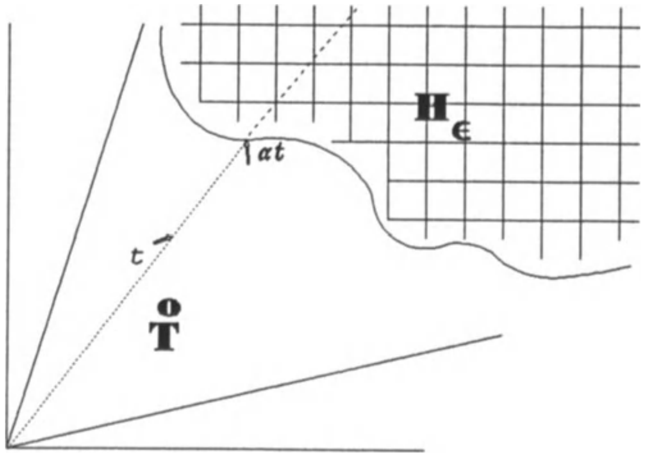

Figure 1

(a) If $t \in \mathring{T}$, then there is $\alpha > 0$ such that $\alpha t \in H_\epsilon$.

(b) ∂H_ϵ is an algebraic hypersurface.

(c) If $k \in \mathbb{N}^r$, $k \in H_\epsilon$, and $\bar{X} \in \mathcal{C}(\bar{u}^k)$ is such that $k \otimes u(\bar{X}) \in F^Q_\epsilon(k)$, then $\bar{X}$ is Walrasian.

Thus, for sufficiently large $k \in \mathbb{N}^r$, all core allocations that yield payoffs which are in the ϵ-Pareto efficient face, are Walrasian.

Proof: For $t \in \mathring{T}$, $C^Q_\epsilon(t)$ is a closed cone with nonempty interior (and "algebraic"). In view of Remark 4.3, $C^Q_\epsilon(t)$ increases ("linearly") with t. Therefore the set

(15) $$E^Q_\epsilon(t) := C^Q_\epsilon(t) \cap \{s \mid s \leq t\} \subseteq \mathbb{R}^r_{++}$$

contains a convex closed body, the volume of which increases ("linearly") in t. By MINKOWSKI's ("second") Theorem (see CASSELS [2], and compare the argument in ROSENMÜLLER [7], [8],) the set $E^Q_\epsilon(t)$ described by (15) contains r linearly independent integer vectors s once the volume exceeds a certain constant, say c_Q. Define $H^\epsilon = \{t \in \mathring{T} \mid \text{volume}(E^Q_\epsilon) \geq c_Q\}$. Then H^ϵ has properties (a) and (b). Property (c) follows from Theorem 4.5, since $k \in H^\epsilon$ implies that k generates a full cone. q.e.d.

SECTION 5 : Examples

Example 5.1: Let m = 2, r = 2 and consider the utility functions $u^1 : \mathbb{R}^2_+ \to \mathbb{R}$,
$u^1(x) = \min\{x_1+2x_2, 2x_1+x_2\}$ $(x \in \mathbb{R}^2_+)$ for type 1as well as $u^2 : \mathbb{R}^2_+ \to \mathbb{R}$,
$u^2(x) = x_1+x_2$ $(x \in \mathbb{R}^2_+)$ for type 2.
Also, we fix initial allocations $a^1 = (2,1)$, $a^2 = (1,1)$ for both types; hence, for $t \in \mathbb{R}^2_{++}$ the "ideal" initial endowment is

(1) $$t \otimes a = a^t = t_1 a^1 + t_2 a^2 = (2t_1+t_2, t_1+t_2).$$

The Pareto optimal allocations are represented by the following sketch

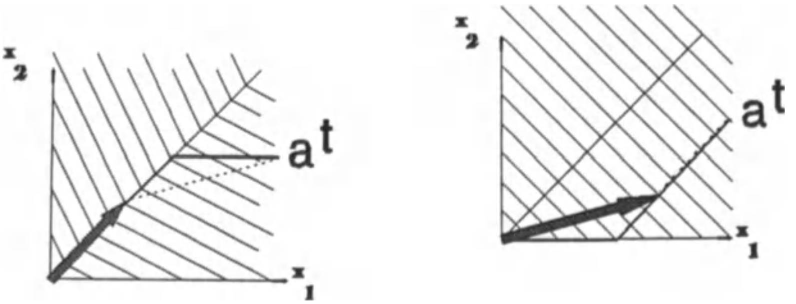

Figure 2

It follows that we have three allocations yielding extreme Pareto efficient points in utility space, to write:

$t \otimes \overline{X}^R = (t_1 \overline{x}^{R1}, t_2 \overline{x}^{R2}) = (a^t,0)$, $t \otimes \overline{X}^M = (t_1 \overline{x}^{M1}, t_2 \overline{x}^{M2}) = ((a^t_2,a^t_2); (a^t_1-a^t_2,0))$,
$t \otimes \overline{X}^T = (t_1 \overline{x}^{T1}, t_2 \overline{x}^{T2}) = (0,a^t)$

((R,M,T) for "right, middle, top"); inserting (1) yields

$t \otimes \overline{X}^R = ((2t_1+t_2, t_1+t_2), 0)$, $t \otimes \overline{X}^M = ((t_1+t_2, t_1+t_2); (t_1,0))$,
$t \otimes \overline{X}^T = (0; (2t_1+t_2, t_1+t_2))$

The corresponding points in utility space are easily computed; observe that $u(t\otimes x) = t\otimes u(x)$ (since $u = (u^1,u^2)$ is homogeneous). Also, we may substitute $u^1(x) = h^{11}(x) = x_1+2x_2$ as any P.E. extreme point yields $h^{11}(x) \le h^{12}(x)$. Thus, we come up with the following three P.E. extreme points of V(t):

$$\overline{u}^R = (4t_1+3t_2,0),\quad \overline{u}^M = (3t_1+3t_2,t_1),\quad \overline{u}^T = (0,3t_1+2t_2).$$

For $t_1 < 2t_2$, the situation is represented by the following sketch. Note that the normal vectors of the P.O. surface of V(t) are $\lambda^T = (2,3)$ and $\lambda^R = (1,1)$ (independent on t and without normalization.)

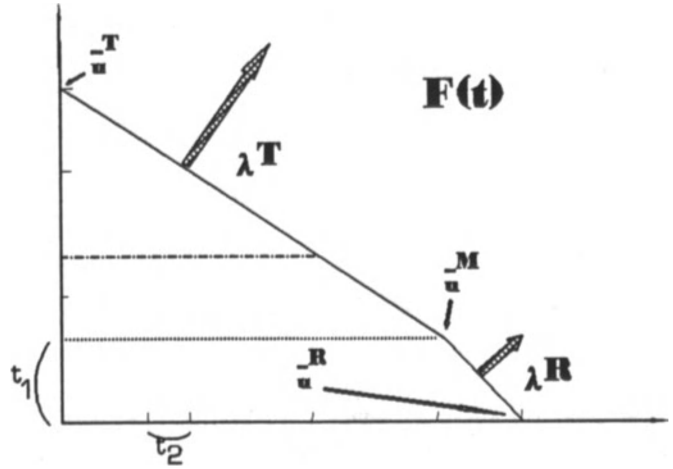

Figure 3

The Walrasian equilibrium is obtained in a slightly modified "Edgeworth–Box" (featuring the aggregated allocations for types)

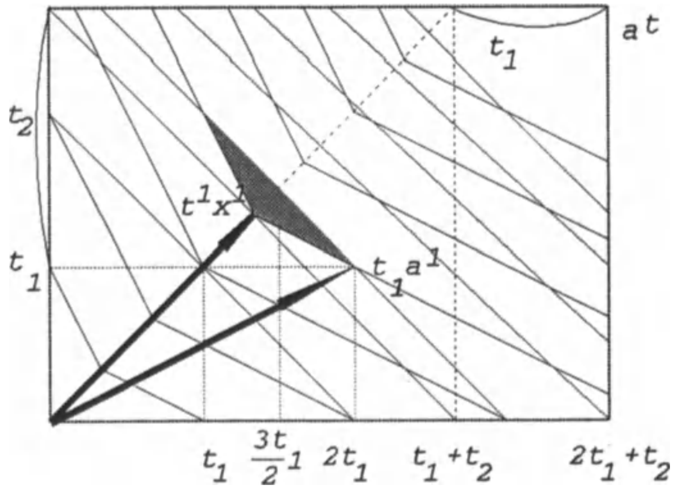

Figure 4

The equilibrium price is $(\frac{1}{2}, \frac{1}{2})$ and the corresponding allocations are $t_1 \bar{x}^1 = (\frac{3t_1}{2}, \frac{3t_1}{2})$ and $t_2 \bar{x}^2 = (t_2,t_2) + (\frac{t_1}{2}, -\frac{t_1}{2})$. The corresponding utilities are $^{(t)}\bar{u} = (\frac{9}{2}t_1, 2t_2)$

For $t_1 > 3t_2$ the situation is similar (although the "type–oriented Edgeworth Box" is not quite suitable) and the Equilibrium is given by $t_1 \bar{x}^1 = (2t_1-2t_2, t_1+t_2)$ and $t_2 \bar{x}^2 = (3t_2, 0)$ with price $(\frac{1}{3}, \frac{2}{3})$ and utility vector ${}^{(t)}\bar{u} = (4t_1, 3t_2)$.

Finally, if $2t_2 < t_1 < 3t_2$ then the price depends on t, $p = (\frac{t_1-t_2, t_2}{t_1})$ and we have $t_1 \bar{x}^1 = (t_1+t_2, t_1+t_2)$ $t_2 \bar{x}^2 = (t_1, 0)$ with utility vector ${}^{(t)}\bar{u} = (3t_1 + 3t_2, t_1)$.
Note that $(\bar{x}^1, \bar{x}^2)$ is a (rational) function of t while ${}^{(t)}\bar{u}$ is used for
${}^{(t)}\bar{u} = (t_1\, u^1(\bar{x}^1), t_2\, u^2(\bar{x}^2)) = (u^1(t_1\, \bar{x}^1), u^2(t_2\, \bar{x}^2))$.

Now for the Core. We are to consider only $\mathcal{C}(V(t))$ (i.e. a concept in utility space). Hence, we are interested in vectors $\hat{u} \in \mathbb{R}^2$ such that $t \otimes \hat{u} = (t_1\, u_1, t_2\, u_2) \in V(t)$ while $s \otimes \hat{u}$ is not in the relative interior of V(s). Let us first discuss the continuous case ($s, t \in \mathbb{R}^2_{++}$). Assume for the moment that $t \otimes \hat{u}$ is on the upper part of the P.O.–surface; thus $\lambda^T\, t \otimes \hat{u} = \lambda^T\, {}^{(t)}b$ where ${}^{(t)}b$ is the "corner point" $(3t_1+3t_2, t_1)$ Now, if $0 < s < t$ satisfies $\lambda^T\, s \otimes \hat{u} < \lambda^T\, {}^{(s)}b$ then the profile $0 < t-s <$ satisfies$\lambda^T(t-s) \otimes \hat{u} > \lambda^T(t-s) \otimes \hat{u}$.

In a small neighborhood of $\frac{t}{2}$ we shall find continuously many such profiles s for which, in addition, $\lambda^T\, s \otimes \hat{u} \leq \lambda^T\, {}^{(s)}b$ is equivalent to $\lambda^T\, s \otimes \hat{u} \in V(t)$. It follows that, for such t we must have $\lambda^T\, s \otimes \hat{u} = \lambda^T\, {}^{(s)}b$ for continuously many s; i.e. $\lambda_1^T\, s_1\, \hat{u}_1 + \lambda_2^T\, s_2\, \hat{u} = \lambda_1^T\, (3s_1+3s_2) + \lambda_2^T\, s_1$.

In view of the prevailing linearity we may actually insert s = (1,0) and s = (0,1) in order to find the unique solution

$$\hat{u} = \left[\frac{\lambda^{T(1,0)}b}{\lambda_1^T}, \frac{\lambda^{T(0,1)}b}{\lambda_2^T}\right] = (\frac{9}{2}, \frac{6}{3}).$$

Now, for $t_1 < 2t_2$, $t \otimes \hat{u} = (\frac{9t_1}{2}, \frac{6t_2}{3})$ is in fact an element of V(t) and equals the equilibrium payoff.

Now we switch to the discrete case.

Let $T^T := \{t \in \mathbb{R}^2_{++} \mid t_1 < 2t_2\}$, $E_t^T := \{s \in \mathbb{R}^2_{++} \mid s \leq t, s \in T^T, t-s \in T^T\}$.

We should say that the payoff ${}^{(t)}\hat{u}$ is *nondegenerate* with respect to E_t^T, if it is the unique solution of the linear system in variables y_1, y_2 given by $\lambda^T\, s \otimes y = \lambda^T\, {}^{(s)}b$ ($s \in E_t^T \cap \mathbb{N}^2$) that is, if there are 2 linear independent profiles in E_t^T.

In this case, $\mathcal{C}(V(t)) = \{t \circledast \overline{u}\}$ and the core collapses towards the equilibrium.

Clearly the set

$$\{t \in \mathbb{N}^2 \mid \hat{u} \text{ n.d.w.r.t. } E_t^T\}$$

is the system of distributions such that $\mathcal{W} = \mathcal{C}$ is true and, as the number of linearly independent profiles depends on the volume of the compact convex polyhedron E_t^T we expect the region where $\mathcal{W} = \mathcal{C}$ (and $t_1 < 2t_2$) to be of the following shape.

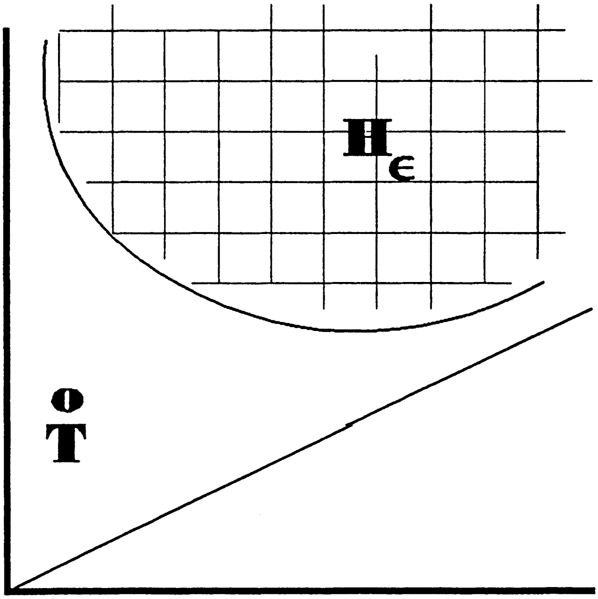

Figure 5

For $t_1 > 3t_2$, the argument is quite similar, except that $\hat{u} = (\frac{4}{1}, \frac{3}{1})$ and $t \circledast \hat{u} = (4t_1, 3t_1)$ is a point on the south–east part of the Pareto surface of V(t).

For $2t_2 < t_1 < 3t_3$ the argument is slightly different.

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Credible Threats of Secession, Partnership, and Commonwealths*

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Abstract. We define a concept of a credible threat to secede; a group of players can credibly secede if, by secession, the group can harm a subset of the complementary coalition while maintaining its own payoff and the payoff of the remainder of the complementary coalition. The set of efficient payoffs that are immune to secession constitutes a subset of the core and may be empty even when the core is nonempty. To solve this problem and to explain the formation of not-necessarily-self-sufficient groups within larger organizations, we introduce the concept of a commonwealth – a payoff in the core and a partition of players into states so that all members of a state are mutually dependent upon each other and no state can credibly secede. We show that the commonwealth core is equivalent to the partnered core. That is, when secession is constrained to be carried out only by partnerships the two concepts coincide. As a corollary, the commonwealth core is nonempty whenever the core is nonempty.

1. Commonwealths and Credible Threats of Secession

There are many examples of organizations consisting of affiliations of smaller units within the organization. Such examples include the Y.M.C.A, the Girl Guides of America, the U.S., Canada, Spain, the U.N., etc. The units within the organization may be described by the terms branches, clubs, states, provinces, and member nations, to name a few. Typically, organizations have the property that exchanges and cooperation must take place between units to realize all gains to collective activities; units within the organization are not necessarily self-sufficient. To fix ideas, we shall refer to the units within

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an organization as "states." Organizations having certain stability properties will be called "commonwealths." Although our analysis carries over to any organization made up of any number of units, these terms and their usual meanings may best reflect the ideas we have in mind.

Certain options are available to states within an organization, including the option to secede. This paper studies the stability of an organization when the possibility of secession is available to the states within it. Indeed, part of our purpose is to explain the division of a population of players into distinct states, even when the states are not necessarily self-sufficient. Although the model we employ is that of a (cooperative) game with side payments, our notion of a credible threat to secede is derived from noncooperative game theory, and in particular the writings of Selten (1965, 1975).

A payoff for a game with side payments is said to be *immune to secession* if it is efficient and if no group of players can *credibly threaten* to separate itself from the others. This means that no group of players can realize their part of the payoff for themselves and, by leaving, harm some or all of the remaining players. The requirement that the act of departing harms those who remain makes the possibility of leaving a threat and the fact that the departing group can realize its part of the payoff renders the threat credible.[1]

Recall that a payoff in the core of a game has the property that it is immune to improvement by coalitions; no group of players can withdraw and realize a payoff preferred by all members of the group. It is quite different to say that a payoff has the property that no coalition can secede and simultaneously impose costs on some subset of the complementary coalition. If a group of players were in such a position, then that group, in the hope of achieving a larger share of the total payoff, may threaten to leave the total player set and cooperate only with some subset of players. Thus, a payoff in the core may not be immune to credible secession. This difficulty motivates our definition of a commonwealth and a commonwealth core.

An elementary argument establishes that the set of payoffs that are immune to secession is contained in the core (Proposition 1). However, even for games with nonempty cores there may be no payoffs that are immune to secession. Consider, for example, the one buyer and two sellers game, where the buyer has a reservation price of one for an object and each seller has a reservation price of zero. The core of this game is nonempty, and gives all the surplus to the buyer. There are no feasible payoffs that are immune to secession. For any payoff giving the buyer less than all the surplus, there is a seller who, along with the buyer, can credibly threaten to secede. If the buyer receives all the surplus however, the two sellers can credibly threaten

[1] There are other possible definitions of credible threats. For example, one might say that a threat is credible if, were the threat carried out, it would impose a larger cost on the threatening group than on the group that initiated the threat. Such a notion of credibility, however, would be distinctly different than the standard notion of credibility of noncooperative games introduced by Selten and adopted here.

to secede. This motivates our definition of a commonwealth and the commonwealth core.

We define a payoff and a partition of the total set of players of a game into states to be a "commonwealth" if (a) the payoff is in the core of the game, (b) each state consists of a partnership, that is, the members of a state all "need" each other and (c) no state can credibly threaten to secede and by doing so, impose costs on another state. A payoff is in the commonwealth core if there is a commonwealth with that payoff. The motivation for the commonwealth core stems from the ideas that individuals who have a mutual need for one another naturally align themselves into decision-making states and that the commonwealth's stability requires not only that the payoff be in the core but also that no state can credibly threaten to secede.[2]

In this paper we show that for games with side payments the commonwealth core coincides with the partnered core introduced in Reny, Winter, and Wooders (1993). A payoff for a game is partnered if it admits no asymmetric dependencies. An asymmetric dependency arises when one player needs the cooperation of another to realize his share of the payoff but the second player does not require the cooperation of the first.

Finally, it is important to note that a state in a commonwealth is not necessarily self-sufficient. Achieving efficient outcomes may necessitate cooperation among states. But this cooperation is more like the cooperation of an anonymous market (where there are many buyers and sellers) than the cooperation between members of the *same* state each of whom requires each of the others to obtain his part of the payoff.

2. Games

A *game (in characteristic form)* is a pair (N, v) where $N = \{1, ..., n\}$ is a finite set of *players* and v is a function from 2^N to $\mathbf{R}_+$ with $v(\emptyset) = 0$. A (possibly empty) subset S of N is called a *coalition*. We shall assume throughout that (N, v) is superadditive (i.e. $v(S)+v(T) \leq v(S \cup T), \forall S, T \subseteq N$ with $S \cap T = \emptyset$).

A *payoff* for a game (N, v) is a vector x in $\mathbf{R}^{\mathrm{N}}$. Fix x in $\mathbf{R}^{\mathrm{N}}$. For $S \subseteq N$, define $x(S) = \sum_{i \in S} x_i$. The payoff x is *feasible* if $x(N) \leq v(N)$ and *efficient* if $x(N) = v(N)$. The payoff x is in the *core* of (N, v) if x is efficient and $x(S) \geq v(S)$ for all coalitions $S \subseteq N$.

[2] The notion of the commonwealth core was stimulated by discussions of Quebec and Cataluna, neither of which would necessarily be better off after secession. That the secession of these states from their countries (Canada and Spain, resp.) may impose costs on other states may provide an explanation for the continuing discussions, in both cases, of secession.

3. Threats of Secession

Let (N, v) be a game with side payments and let x be an efficient payoff. We say that a coalition $S \subset N$ can *credibly threaten to secede* if for some (possibly empty) coalition T, $x(S \cup T) \leq v(S \cup T)$ and for some coalition Q disjoint from $S \cup T$, $v(Q \cup R) < x(Q \cup R)$ for all coalitions R disjoint from S. Thus, by seceding, S hurts coalition Q regardless of how players in $N - S$ might subsequently cooperate; therefore secession is a threat. The threat is credible since together with T (disjoint from Q), S (and T) can realize its payoff. If no coalition can credibly threaten to secede, we say that x is *immune to secession.* A payoff x is *cohesive* if it is efficient and immune to secession.

Proposition 1. *If (N, v) is a superadditive game, then every cohesive payoff is a core payoff.*

Proof. Let x be cohesive and suppose that x is not in the core. Since x is efficient there is then a coalition S with $x(S) < v(S)$. By superadditivity and efficiency, $x(S) + x(N - S) = v(N) \geq v(S) + v(N - S)$. Rearranging, we obtain $x(S) - v(S) \geq v(N - S) - x(N - S)$. Since $x(S) - v(S) < 0$ it follows that $v(N - S) - x(N - S) < 0$. Taking $T = \emptyset$ and $Q = N - S$, this contradicts the property that x is cohesive. ■

As illustrated in the introduction, there are games with nonempty cores having no payoffs that are cohesive.

4. Partnership

Let N be a finite set of players and let $\mathcal{P}$ be a collection of subsets of N. For each i in N let

$$\mathcal{P}_i = \{S \in \mathcal{P} : i \in S\}.$$

We say that $\mathcal{P}$ has *the partnership property* (for N) if for each i in N the set $\mathcal{P}_i$ is nonempty and for each pair of players i and j in N the following requirement is satisfied:

$$\text{if } \mathcal{P}_i \subset \mathcal{P}_j \text{ then } \mathcal{P}_j \subset \mathcal{P}_i.$$

That is, if all the coalitions in $\mathcal{P}$ that contain player i also contain player j then all the coalitions that contain j also contain i.[3]

The payoff x for the game (N, v) is called a *partnered payoff* if the collection of coalitions supporting x (i.e. $\{S \subseteq N : x(S) \leq v(S)\}$) has the partnership property. The *partnered core* is the set of all partnered payoffs in the core of (N, v). Reny, Winter and Wooders (1993) show that the relative interior of the core of a game with side payments is contained in the partnered core.

[3] For the reader familiar with these works, we note that partnered collections of coalitions are called "separating collections" in Maschler and Peleg (1966, 1967), and Maschler, Peleg, and Shapley (1971). See Section 7 for further discussion.

5. Commonwealths and the Commonwealth Core

Let (N, v) be a game with side payments and let x be a feasible payoff. We'll say that i *needs* j *for* x if for all $S \subseteq N$, $x(S) \leq v(S)$ and $i \in S$ together imply that $j \in S$. If both i needs j, and j needs i for x, we'll say that *i and j are partners (for x)* (or that i is partnered with j (for x)). Clearly, for a fixed payoff x, the relation "is partnered with (for x)" is an equivalence relation on the player set. Call each equivalence class a *partnership (relative to x)*. Let $\mathcal{N}_x = \{N_1, ..., N_K\}$ denote the (unique) partition of N into *states* in which each state N_k consists of a partnership relative to x. We say that *state N_k can credibly threaten to secede from $\mathcal{N}_x$* if N_k can credibly threaten to secede in the sense of Section 3 but where the sets T and R are restricted to being unions of states (rather than arbitrary coalitions of players), and Q is restricted to being a single state. If no state N_k can credibly threaten to secede, then $\mathcal{N}_x$ is called a *commonwealth partition* (*relative to x*). The pair $(\mathcal{N}_x, x)$ is a *commonwealth* (for the game (N, v)) if x is in the core of the game and $\mathcal{N}_x$ is a commonwealth partition.

Let (N, v) be a game and let x be a payoff. We say that x is in the *commonwealth core* if there is a partition $\mathcal{N}_x$ such that the pair $(\mathcal{N}_x, x)$ is a commonwealth.

Proposition 2. *Let (N, v) be a superadditive game. Then a payoff x is in the commonwealth core if and only if it is in the partnered core. In addition, the commonwealth core is nonempty if and only if the core is nonempty.*

Proof: Let x be in the partnered core and suppose that $(\mathcal{N}_x, x)$ is not a commonwealth. Since x is in the core, it must be the case that some state N_1, say, can credibly threaten to secede from $\mathcal{N}_x$. That is, (i) there is a union of states, T, such that $x(N_1 \cup T) \leq v(N_1 \cup T)$; and (ii) there is another state, N_2, say, disjoint from N_1 and T, such that $v(N_2 \cup R) < x(N_2 \cup R)$ for all unions of states R disjoint from N_1. Now, suppose that $i \in N_2$. Choose S such that $i \in S$ and $x(S) \leq v(S)$ (possible since x is partnered). Since by definition of $\mathcal{N}_x$ each state is a partnership, S must be a union of states. Therefore we also have $N_2 \subseteq S$. Consequently, we have $x(N_2 \cup S') \leq v(N_2 \cup S')$, where $S' \equiv S - N_2$ is a union of states. Hence by (ii) it must be the case that $N_1 \subseteq S$. But since S was an arbitrary coalition containing $i \in N_2$, this implies that i (indeed every player in N_2) needs every player in N_1. However by (i), no player in N_1 needs any player in N_2. But this contradicts the fact that x is partnered. Hence $(\mathcal{N}_x, x)$ is a commonwealth and so x is in the commonwealth core.

Conversely, suppose that x is in the commonwealth core. Hence, $(\mathcal{N}_x, x)$ is a commonwealth. Suppose by way of contradiction that x is not partnered. Then there is a pair of players 1 and 2, say, such that 1 needs 2 (for x) but 2 does not need 1. Consequently, 1 and 2 are in different states, say N_1, and N_2, respectively. Moreover, since every state is a partnership, this implies that every player in state N_1 needs every player in state N_2, while no player in state

N_2 needs any player in state N_1. Thus we may say quite unambiguously that state N_1 needs N_2 to realize its payoff $x(N_1)$, while N_2 does not need N_1 to realize $x(N_2)$. Therefore there exists a coalition S containing N_2 and disjoint from N_1 such that $x(S) \leq v(S)$. Moreover, since every state is a partnership, S is a union of states. But this implies that N_2 can credibly threaten to secede from $\mathcal{N}_x$ (since $x(N_2 \cup S') \leq v(N_2 \cup S')$ where $S' \equiv S - N_2$ is a union of states, and $x(N_1 \cup R) > v(N_1 \cup R)$ for all states (in fact all coalitions) R disjoint from N_2 since N_1 needs N_2) contradicting the fact that $(\mathcal{N}_x, x)$ is a commonwealth. Since the relative interior of the core is in the partnered core, (see Albers (1974), Bennet (1983), or Reny, Winter and Wooders (1993)), it follows that the commonwealth core is nonempty whenever the core is. ∎

6. Other Literature Relating to the Partnership Property

The partnership property of collections of sets first appeared in Maschler and Peleg (1966,1967) and Maschler, Peleg, and Shapley (1971) in their study of the kernel of a cooperative game. The concern of Maschler and Peleg was "separating out" players, Maschler and Peleg call a partnered collection of sets a *separating collection.* Two players i and j are *inseparable* if, in our terminology, they are partners. Since the partnered core was motivated by a desire to focus on coalition formation we adopt the term "partnership" of Albers (1974,1979), Bennett (1983) and Bennett and Zame (1988).[4] The terms introduced by Maschler and Peleg are also appealing, especially from the viewpoint of core solutions, since we may think of players who are partners as inseparable.

It is important to note that a partnership is not at all the same concept as a coalition in a coalition structure. In the typical literature on coalition structure games, such as Aumann and Dreze (1974) or Kaneko and Wooders (1982), each coalition is self-sufficient, that is, $v(S) = x(S)$. The approach of this literature does not address why we may have organizations consisting of distinct sub-units, such as states, that are not necessarily self-sufficient. In Reny, Winter, and Wooders (1993) and Reny and Wooders (1993) we provide examples showing the differences between coalition structures and partnerships. Another simple example is the one-buyer, two-seller game described in the introduction. The partnerships associated with the only payoff in the core, giving all the surplus to the buyer, are the 1-member partnerships.

Other motivation for our work comes from the literature of noncooperative games and competitive equilibrium. The importance of the credibility of threats was emphasized in Selten (1965), (1975). The concept of partnership

[4] Bennett (1983) is a rich source of examples of partnered payoffs for games with side payments.

in noncooperative theory first appeared in Selten (1981). There, a noncooperative foundation was given for partnered "expected" payoffs, as the outcome of a bargaining process. This research has been continued in Winter (1989), Bennett (1981) and other recent papers. As noted in the introduction, the fact that the competitive equilibrium is not partnered is a weakness of the concept. Bennett and Zame (1988), however, show that when preferences are strictly convex the competitive payoff is partnered[5]. Page and Wooders (1994) introduce the partnered core of an economy and show that, even though the competitive equilibrium may not be partnered, convexity of preferences ensures nonemptiness of the partnered core. In addition, Page and Wooders (1994) show that with additional conditions on the economic model, the Page-Werner condition of no unbounded arbitrage is necessary and sufficient for nonemptiness of the partnered core of an economy and the partnered competitive equilibrium.

Finally, we remark that Reny and Wooders (1993) show that the partnered core of a balanced game without side payments is nonempty and thus provide a refinement of Scarf's Theorem (1967) on the nonemptiness of the core. Ongoing research studies the notions of credible threats to secede and the commonwealth core of games without side payments.

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Rules for Experimenting in Psychology and Economics, and Why They Differ

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Abstract. This chapter discusses methodological differences in the way economists and psychologists typically conduct experiments. The main argument is that methodological differences spring from basic differences in the way knowledge is created and cumulated in the two fields-- especially the important role of simple, formal theory in economics which is largely absent in psychology.

Keywords. Experimental economics, psychology, bounded rationality.

1 Introduction

There are many ways to conduct experiments in social sciences. This chapter contrasts two philosophies or approaches, referred to as the "experimental economics" (E) and "experimental psychology" (P) approaches. The distinction is not sharp, of course, but even fuzzy distinctions can be helpful in framing debates and clarifying misunderstanding. The main focus in this chapter is the difference between E and P experiments in research in decision theory and game theory.

There is substantial bickering between these camps about the right way to conduct experiments. My central argument in this chapter is that many crucial differences in experimental style are created by underlying differences in working assumptions about human behavior and about how knowledge is best expressed, not by underlying differences in areas of substantial interest. My hope is that understanding this point can sensitize both sides (and outsiders or newcomers) to differences in these philosophies, without particularly advocating one over the other, in order to defuse some sensitive aspects of the debate between E and P.

[1] Comments from participants at the Big Ten Accounting Doctoral Consortium, Minneapolis 5/13-15/94, Chip Heath, Wulf Albers, and Werner Güth, were helpful. Many conversations over the years -- particularly with Daniel Kahneman, Charlie Plott, and Richard Thaler, and numerous coauthors -- have shaped the contents of this paper. Discussions at the Russell Sage Foundation during my 1991-92 visit there were especially formative.

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The chapter has several sections. Next I argue that economics and psychology are more sharply distinguished by style than by substance. In section 2 we make the leap to experimental style, describing key differences in P and E experimental styles and showing how they follow from differing presumptions in the underlying disciplines. Section 3 gives a specific example to further illustrate the argument. In section 4 I express some preferences for certain stylistic features, add caveats, and mention the exceptional methodological style of Reinhard Selten.

This paper draws clearly on many aspects of Smith's classic 1976 and 1982 papers which guided experimental work in economics for the last twenty years. I was also inspired by a thoughtful unpublished 1988 paper by Daniel Kahneman which contrasted P and E methods. Similar ground is covered by Cox & Isaac (1986). Descriptions of now-standard methodology in experimental economics can be found in Hey (1991), Davis & Holt (1992), and Friedman & Sunder (1994). Chapters in Kagel & Roth (1995) review discoveries in experimental economics.

1.1 Substance vs. Style: Economics and Psychology

I claim differences in E and P experiments arise from differences in style of the two fields. By substance I mean, "what is the basic question being asked?" By style, I mean, "What constitutes an answer to the question? How are answers expressed?"

As a starting point for discussion, let's suppose psychologists ask:

P: How do people think & behave?

and economists ask

E: How are scarce resources allocated?

At first blush, this difference in fundamental "why" questions seems to sharply divide the two fields along substantive lines-- e.g., psychologists study human thinking, and economists study resource allocation.

But the two questions also express inherent methodological differences. Answering the P question demands observation of individual subjects in controlled settings (experiments). The E's "allocation of scarce resources" hints strongly at an underpinning from mathematical optimization. My claim is that the substantive difference is actually small, and the stylistic difference large.

1.2 How Much Substantial Overlap is There?

Take any one of several concrete examples: How do people decide how much time to spend with their families, instead of working? Why do people discriminate against others who are unlike them? Who do people vote for? When do people retire? How do people choose friends? Spouses? Why do people break the law? How do people invest their wealth? Choose jobs or education? Why do people become addicted to drugs and acquired tastes?

Virtually all these questions can be posed as P or E questions, and have been studied by social scientists of both types (and also by sociologists, whose work is ignored in this chapter for brevity). To further illustrate that the differences between P and E are not entirely substantial, Table 1 illustrates some of the similarities in topics of substantial interest across the two fields. Many interesting topics-- altruism, learning, taste formation-- have been studied by people in both fields. The key difference is perhaps not the substantive topics psychologists and economist study, but the way theories are expressed and data are gathered.

Table 1: Similarities in substantial topics in P and E

Psychology	Economics
GUIDING QUESTIONS	
How do people think & behave. Why?	How are scarce resources allocated?
SPECIFIC TOPICS	
reciprocity norms	"gift exchange" labor models
prisoner's dilemma	externalities
expectancy theory	expected utility (EU & SEU)
altruism	altruism (bequests to children)
impulsivity	impatience (discounting)
language & meaning	language & meaning (cheaptalk)
learning	learning (least-squares, population learning)
taste formation	taste modelling (addiction, participation externalities)
symbolic consumption	signaling
conformity/herd behavior	info. cascades, rational conformity

1.3 Stylistic Differences in E and P

I think the fundamental stylistic differences are two: (i) E's strongly prefer mathematical formalisms. And since formalisms become unwieldy and inelegant as they get too complex, E's like simple formal models. (ii) E's like predictions that are surprising, and testable using naturally-occurring ("field") data. Note how features (i) and (ii) work together. The desire for parsimony enables many E's to justifiably ignore whether assumptions are realistic (if alternative assumptions are "too" complicated), and hence place more value on naturally-occurring data than on experimental refutations.

In contrast, P's tend to prefer verbal models or principles and eschew sophisticated mathematical expressions. Since these principles need not fit together

in mathematically coherent ways, models can resemble lists of effects or critical variables, and can be expressed (and discovered) by experimental demonstrations. Field tests or demonstrations are consequently rare.

My argument for stylistic difference implies that a single substantive question can be translated from the P to E mode, or vice versa.

For example, translating from P to E, it is a simple matter to translate a loose psychological account of, say, how financial incentives influence performance into economic terms: The "scarce resource" is attention or thinking effort, higher incentives produce more thought (the supply curve for thinking is upward-sloping) and more thought improves performance (thought is a factor or production in generating performance), as in Smith and Walker (1993). But doing so has no special interest to many Ps, because the resulting model will of course be oversimplified ("reductionist") and plainly at odds with at least some experimental observations.

Similarly, one can ask an E question--do people play the subgame perfect solution to a sequential bargaining problem?-- in a P way, by observing whether subjects in a laboratory search for information in the way that is necessary to calculate perfect equilibrium using backward induction (Camerer, Johnson, Sen & Rymon, 1993). The answer to that question is "No", but that answer does not deter an E from continuing to assume the answer is yes, if mathematical simplicity is sufficiently important and no equally parsimonious replacement assumption emerges from the experimental findings.

A helpful and charitable way to think of the E and P camps is as competing cultures, religions, or schools of artistic expression. Psychologists are realists who paint detailed landscapes and strive to make them "lifelike". (A good psychology experiment is a drama that makes a point about human nature.) Economists are abstract expressionists who value pictures that expresses a feature of the world with minimal clutter. Naturally, the P's criticize the E's for clinging to unrealistic assumptions, and the E's criticize the P's for lacking formal theory, or parsimony.

2 Experimental Styles in E and P

My central claim is that general stylistic differences in how knowledge is generated in the E and P fields accounts for many of the major differences in how experiments are conducted.

Table 2 lists some important differences in experimental styles. Most of the differences spring from basic differences in media for expression of knowledge and how the interplay of theory and data work in the two fields.

For E's, the medium for expression of knowledge is, primarily, a body of formalism and modelling principles. A body of facts buttresses the formalism and sustains faith in it, but is clearly subservient because ample evidence against a formal principle is often insufficient to create widespread rejection and search for replacements. (E.g., the experimental evidence against expected-utility maximization

Table 2: Differences in P and E Experimental Style

Psychology (P)	Economics (E)
MEDIUM FOR EXPRESSION OF KNOWLEDGE	
Body of facts (1st)	Formal model (1st)
Informal interpretation	Body of facts
INTERPLAY OF THEORY & DATA	
Experimental results are cloth from which theory is woven	Experiments operationalize and test general theory in specific (artificial) setting
CONTEXT/LABELS	
Rich context preferred: engages subjects more lifelike	Rich context avoided: creates nonpecuniary utility irrelevant from theory's view
SUBJECT POOLS	
Gender, age, etc. often recorded	Theory predicts no effects (or abstractness of setting swamps subject pool differences)
TYPES OF DATA	
Ratings, response times, subject self-reports, etc.	Choices. Self-reports not informative.
REPEATED TRIALS?	
No need. Some of life is like first trial	Yes. Theory predicts "last" (equilibrium) trial
INCENTIVES OF SUBJECTS	
Volunteer subjects are well-motivated to "try hard"	Financial motivation best.
DATA ANALYSIS & REPORTING	
Assumes friendly reader.	Assumes skeptical reader.
Reduced-form.	More raw data.
Sometimes F's or p-values only.	(Reader can reanalyze).
Don't report "failures".	No "failures" in a well-designed (competing-theory) experiment.
DECEPTION?	
Sometimes, to create unnatural situations or break confounds.	Rarely. (Experimenter credibility is a public good)

is now forty years old and still piling up, but textbooks and most applied work continue to assume it.) The interplay of theory and data occurs as good experiments test theory or provoke its development by exploring behavior in an interesting domain where theoretical principles haven't been developed. For P's, theory is sheer cloth wrapped around a body of prominent experimentally-observed results. Explanations that don't fit new observations are jettisoned more quickly than old facts are.

Several properties of good experimental design follow from these basic differences.

Context or labels: In experimental E, an abstract context (or bland verbal labels) is sufficient to test theory; it is considered unnecessary to make the experimental context realistic since most theory does not predict that results will depend on verbal labels. (Indeed, labelling choices may even be undesirable since it may interfere with attempts to clearly control preferences by paying financial incentives.) By contrast, in experimental P a rich context is often used to motivate subjects to behave realistically and to provide a concrete illustration of an underlying (mathematical) principle. In addition, most psychologists feel that abstract, symbolic information is processed differently than concrete, lifelike stimuli. Hence, learning about subjects' intuitive grasp of statistics using only dice and coins could be misleading as a guide to how subjects reason about baseball statistics or categorize people into stereotypes.

Subject pools: Economic theories usually do not predict any reason why different pools of subjects will behave differently, so E's rarely collect data on the demographics of subjects or test whether behavior varies with gender, age, education, etc. (though see Ball & Cech, 1990, for a review). Indeed, nonhuman subjects such as rats, pigeons, et al have been productively used in experiments on consumer choice theory, theories of labor supply, and expected utility (see Kagel, Battalio& Green, 1995). A long (now quiet?) tradition of interest in personal differences by some psychologists leads most psychologists to at least record personal information, if not theorize about differences due to age, gender, educational background, etc.

Types of data: Since economic theory usually does not predict what cognitive process will lead to an outcome (say, competitive equilibrium in a market or a game-theoretic equilibrium), E's usually do not collect types of data other than choices. In contrast, data like Likert rating scales of attractiveness, response times, self-reports by subjects of what they were thinking, post-experiment justifications for their choices, etc. are widely used in psychology and theories often make predictions about these variables. (Theory-minded E's would not know what to do with such data; though some E's interested in decision error have begun to use response times.)

Repetition of trials: Economic theories are usually asserted to apply to equilibrium behavior, after initial "confusion" disappears and (unspecified) equilibrating forces have had time to operate. So E's generally conduct an experiment for several repeated trials under "stationary replication"-- destroying old goods and refreshing endowments, in order to recreate the decision situation in exactly the same way each time (except for anything that was learned from the

previous experience, in a "Groundhog Day" design[2]). Then special attention is paid to the last periods of the experiment (e.g., often the last half of the trials are used as data to test an hypothesis) or to the change in behavior across trials. Rarely is rejection of a theory using first-round data given much significance. P's, in contrast, sometimes run a single trial for each stimulus if learning is of no special interest.

Incentives of subjects: A crucial property of most E experiments is that the consequences of their choices are made clear (or "salient") to subjects, and they are always given financial incentives. The standard explanation for insisting on salience of this sort, and financial rewards (rather than say points, course credit, hypothetical dollars, etc.) is articulated by Smith (1976, p. 277):

> "Individuals may attach game value to experimental outcomes...Because of such game utilities it is often possible in simple-task experiments to get satisfactory results without monetary rewards by using instructions to induce value by role-playing behavior...But such game values are likely to be weak, erratic, and easily dominated by transactions costs, and subjects may be readily satiated with `point' profits."

The insistence on paying subjects is absolute among experimental E's even though most, in their hearts and in hallway discussions, are agnostic about whether results would differ substantially if subjects' payments were hypothetical.[3] The insistence on money seems to be a fetish arising from the frequently-made assumption that people are self-interested and largely motivated by money. My view is that more comparisons of high- and low-incentive conditions (including hypothetical payment) are needed to test Smith's assertions. Probably the result of such studies would be that in some situations, where the task is either particularly hard or simple (or intrinsic motivation is high), paying performance-based incentives makes little difference. In tasks which get boring quickly (e.g., "probability matching"), or where performance is of intermediate responsiveness to effort, paying subjects could make a lot of difference. Also, many studies comparing hypothetical and actual

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[2] In the movie "Groundhog Day" a hapless reporter is forced to repeat the same 24-hour sequence of interactions over and over, with everything unchanged except his own memory of the outcomes of his actions in the previous day. The endless looping of experience, while boring, turns out to be ideal for learning since different actions can be tried while everything else is held equal. This is precisely the "stationary replication" design widely used in experimental economics.

[3] It is virtually impossible (I know of no examples in the last ten years or so) to publish experimental results in a leading economics journal without paying subjects according to their performance. Paying subjects often costs substantial sums of money, which acts like a very high submission fee, and may particularly limit the ability of younger investigators or those in less well-funded institutions to generate widely-read results. Also, paying subjects or using incentive-compatible mechanisms (like the "lottery ticket" procedure for inducing risk tastes, or the Becker-DeGroot-Marschak procedure for selling prices) can complicate an experiment or produce demand effects.

payments have found that the frequency of outliers is reduced by paying real money, which may be particularly important in outlier-sensitive situations, like minimum-action coordination games, or markets for long-lived assets which can exhibit price bubbles.

In contrast, experimental P's usually do not pay subjects (though some do); instead they rely on the intrinsic motivation of subjects who volunteered out of curiosity, and whose attention is sustained by pride and eagerness to do a good job. (Unfortunately, in my view, many subjects in introductory P classes are forced to participate in experiments as part of course credit; they may indeed be readily satiated or only erratically motivated by points, as Smith feared.)

Data reporting and analysis: Styles of data analysis are different in E and P. Experimental E's generally follow the useful convention in reporting other kinds of empirical economics (e.g., econometric results), report lots of raw data, using clever graphical methods and often testing hypotheses in statistically sophisticated ways (see, e.g., El-Gamal & Palfrey, in press, on Bayesian methods and "optimal design"). Then readers, knowing as much about the data as space permits, can form their own opinions about the results rather than blindly accepting the author's.

In my opinion, data reporting and analysis standards are simply lower in many domains of experimental P (particularly social psychology). P's often report highly aggregated data, using embarassingly simple bar charts, with outliers trimmed capriciously and analyzed with crude statistical tests that are usually parametric and often badly misspecified. (F-tests are routinely used for differences in variances of grouped data that are clearly non-normal; tests assuming continuous variables are often applied to discrete data like 5- or 7-point scale ratings.) There also appears to be more of a "file drawer" problem in psychology-- only surprisingly large effects are reported, ensuring that exact replications will show smaller effects due to regression-toward-the-mean. (Some psychologists publish only a third or fourth of the data they collect, which suggests the magnitude of the file drawer problem that can result if findings are not carefully replicated before publication.) While my description of experimental P data analysis methods is harsh, I think they can again be understood as partly forgivable in the light of other features of psychological research style. Experimental P's often have access to large subject pools at low cost (since subjects are often not paid at all), so free subject labor can substitute for expensive experimenter human capital in designing efficient statistical tests. And in experimental P, the experimenter writes a script (perhaps a short vignette, like a joke) or lets a drama unfold that illustrates an informal claim about human nature. Readers give the experimenter the benefit of the doubt (trusting her results and not needing to see raw data), much as theatergoers trust the playwright. In this view, the file drawer effect results from refusing to stage bad plays.

Deception: The use of deception is another important stylistic difference between E's and P's: E's hate it, and P's often don't mind. For the sake of discussion, define deception as actively claiming something to be true, when you know it to be false. For example, some experimenters have implicitly promised to pay subjects money in a way that (saliently) depended on choices, then told them at the end they would

earn a flat amount. In other cases a confederate subject, playing a role necessary to create a social illusion or treatment, is implicitly introduced as a fellow subject, experimenter, former subject, etc.

Experimental E's hate deception because they feel it is often unnecessary, and more importantly, harms the credibility of all experimenters. Since American Psychological Association rules require experimenters to inform a subject about any deception in a post-experiment debriefing, veteran subjects always know they have been lied to before. Some studies also indicate that subjects who are told about a deception in a debriefing do not alway s believe the debriefing either. (Should they?) An example will help illustrate some features of this disagreement about deception.

Many experimenters have studied "ultimatum" games in which one person offers a division of a sum X to a responder. Then the responder either takes it, or rejects it and leaves both with nothing (see Güth et al, 1982; Camerer & Thaler, 1995). Suppose you were interested in whether a sense of "entitlement" affected the kinds of divisions players offered and accepted. Hoffman et al (1994) studied this by having pairs of subjects play a simple game of skill. Those who won got to make the offer in a subsequent ultimatum game; those who lost could accept or reject the winner's offer. They found that winners offered less (keeping more for themselves) and losers were willing to accept less. Winning created entitlement.

Or did it? There is a subtle problem with this design: It perfectly correlates game-playing ability and entitlement generated by winning. Their results are consistent with the hypothesis that skilled game-players always demand more, unskilled players always accept less, and game-playing simply sorted players by their bargaining aggressiveness (or timidity) rather than created entitlement per se. For a psychologist, the natural way to break this two-variable confound is to play a game, then randomize feedback on whether players won or lost. That way, half the skilled game-players land in the low-entitlement group (they are told, deceptively, they lost); and half the unskilled players land in the high-entitlement group. Deception is required in this alternative design; it is hard to see how to break the skill-entitlement confound otherwise. One can see how experimental P's might criticize the Hoffman et al design for allowing a confound, and if one is eager enough to truly separate skill and entitlement, deception may be scientifically useful.

This example illustrates the scientific benefit of deception. But it is important to take account of the possible costs of deception and weigh costs and benefits. Two costs loom largest: First, repeated deception probably harms overall experimenter credibility, but the extent of harm and whose experiments are jeopardized is not well understood. Second, since deception is generally used to create an unnatural social situation-- such as a non-relation between skill and entitlement-- one must ask: If the situation being created is sufficiently unnatural to require deception, will subjects belie\e the deception? If subjects entertain the hypothesis that they are being deceived, then the most necessary deceptions-- to produce the most unnatural situations-- are least likely to be believed. This point can be illustrated by further ultimatum experiments.

Polzer, Neale & Glenn (1993) (PNG) used an ultimatum game design that did break the confound between skill and entitlement. After their subjects did a "word find" task, they were given deceptive feedback about actual performance, in order to sort half the high-skilled players into a low-entitlement condition (by telling them they performed more poorly than the subject they were paired with). PNG reported that entitled ("justified") players offered a modest $.32 less than nonentitled players (out of $10 being divided). More interestingly, the responder subjects who were nonentitled (were told they had low performance) demanded $.41 more than entitled players did. A natural possibility is that the deceptive feedback was not believed by some of the players who were told they were less skilled. Evidence for this possibility is the fact that the surprising increase in demands of nonentitled players is larger among groups of (self-selected) friends, who came to the experiment together and, knowing more about each other, were more likely to doubt the false feedback, than among groups of strangers.

3 An Example: Fairness in Surveys and Markets

An example will help illustrate some of the differences in experimental E and P styles summarized in Table 2.

3.1 Surveys of Fairness

Many social scientists have been interested in perceptions of fairness and their impact on social life. Kahneman, Knetsch & Thaler (KKT) (1986) set out to describe some of the rules people use to judge whether economic transactions are fair. They decided to use surveys in which short vignettes were described, and respondents could answer whether a transaction was fair or not. For example, subjects were told:

> A hardware store has been selling snow shovels for $15. The morning after a large snowstorm, the store raises the price to $20. Please rate this action as:
>
> Completely Fair   Acceptable Unfair   Very Unfair

Of 107 respondents, 82% said the store's price increase was unfair or very unfair. From dozens of questions like this, they induced several basic propositions about perceptions of fairness.

Their method illustrates some classic features of experimental P. The survey data are used to construct theory, not test it. The natural context makes the question more engaging than a drier, abstract version ("Is it fair to mark up prices after shortages?") would be, and perhaps easier to comprehend as well. The subjects were randomly chosen and telephoned, because KKT thought that average folks were obviously the people whose behavior they wanted to model, and students might be unfamiliar with many of the contexts they asked questions about (e.g., employers raising or cutting

wages). The data are answers to questions, rather than choices with financial consequences. No repetition is necessary (though many different questions were asked) since nothing much can be learned from asking the same question twice. The data analysis is simple (and in this case, need not be complicated). No deception is involved.

From an experimental E view, these data tell us nothing about "real" behavior with substantial consequences for respondents. But the data are clearly useful for theory construction if one presumes only that (i) subjects are intrinsically motivated to answer accurately (i.e., to report what they really would feel about fairness if they rushed in from the snow and picked up a shovel marked $15), or have no reason not to; and (ii) their perceptions of fairness have something to do with their willingness to forego buying a shovel they think is priced unfairly. A critic who thinks the answers are worthless is implicitly assuming that either (i) or (ii) are false, which reflects a judgment about human behavior that itself seems empirically false to psychologists who have experience with survey design.

3.2 Fairness in Markets

Inspired by the KKT results, Kachelmeier & Limberg & Schadewald (KLS 1991) designed a traditional E experiment to see whether fairness effects would affect prices and quantities in a market. Their clever design starts with the proposition, derived by KKT from their surveys, that a price increase is fair if it is accompanied by an increase in marginal costs, and is unfair otherwise. (For example, raising the snowshovel price to $15 after the snowstorm-induced demand shock is unfair-- according to respondents to the question above-- but subjects also said raising the retail price of lettuce is fair after a transportation mixup created a shortage and raised the wholesale price.)

Their design began with several periods of posted-bid trading in which each of several buyers posted a bid at which they would buy a unit of an unspecified commodity from a seller. (This unusual design enabled direct observation of how buyers reacted to a visible cost change, the closest analogy to the KKT surveys.) Than partway through, a tax on seller revenues was imposed. The tax effectively shifts the marginal cost curve (since the price at which a seller must sell to earn zero after-tax profit is now higher).

In a control condition, the buyers knew nothing about the tax; the fairness theory predicts they may object to price increases in this case. In the cost-disclosure condition, buyers were told about the revenue tax and it was explained why this might raise prices. Fairness theory predicts they will treat price increase as more fair in this case, and prices will rise more rapidly. In a third profit-disclosure condition, buyers were told the effect of the tax on the sellers' share of total surplus (at various possible prices). By design, the revenue tax also raised the sellers' share of total surplus at every price, so fairness theory predicts that buyers might resist paying higher prices in this case.

As the survey results implied, behavior in the three disclosure conditions was different. When the cost effect was disclosed, prices rose more rapidly to the new equilibrium price (buyers appeared to forgive price increases, or treated them as more fair). When the unfair profit effect was disclosed as well, price increases were slower. The no-disclosure control condition prices fell in between. Deng et al (1994) replicated the KLS findings using a more traditional posted-offer institution, and found similar convergence patterns but little difference in equilibrium prices across the information treatments.

The ingenious KLS design takes the KKT results seriously but plays by experimental E rules. The experiment tests theory, in an abstract context (making the influence of fairness all the more remarkable and plain-- it stems purely from attitudes about divisions of money, not about shovels or lettuce). Subjects make choices (bids and sales) with financial consequences, over groundhog-day repeated trials to see if fairness effects "wear off" (they do, to some extent). The time series of data are reported, so readers can judge how quickly equilibration occurs, and tests for differences in conditions are sophisticated (and conservative). No deception is involved.

The KKT-KLS interplay also shows how much can be learned by cross-fertilization of the E and P approaches. The KLS design would never have come about if many P-style survey questions did not reveal the texture of perceptions of fairness, and provide a clear hypothesis testable in later work. At the same time, survey questions are not well-designed, as the experimental market is, to test how much people will pay to punish unfair behavior, and how their behavior changes with repetition. So the two methods are productive complements in producing a fuller understanding of fairness in economics.

4. Conclusions and Caveats

4.1 Which Approach is Better?

My own tastes find something of special value in each of the E and P approaches that could be usefully imported into the other.

The modelling tradition in E, and guidance from theory, imposes useful discipline on economics experiments. Designing an experiment with a theory in mind forces you to be crystal clear about why all the pieces of the design are there. (It also clarifies theory, by forcing a theorist to be clear about at least one concrete domain in which the theory applies.) Designing to test theory has enabled us to create extraordinarily efficient experiments in domains where multiple theories compete. For example, Schotter, Weigelt & Wilson (1994) and Banks, Camerer, Porter (1994) each constructed a series of games to test several different equilibrium concepts. Designs of such efficiency are rare in psychology, at least partly because

specific theories which predict point estimates or different directional responses to variables are less common.

Experimental E's are also extremely good about reporting raw data (many articles include raw data and instructions, to make reanalysis or replication by readers easy).

On the other hand, many precepts in the E approach are restrictive. Ignoring almost all data but choices wastes a valuable opportunity to learn something more from a group of subjects who are often eager to explain their thinking processes and inferences. (Whether their thoughts are useful or not is difficult to answer, but it is surely less difficult to answer if we collect such data!) Also, many types of apparently non-choice behavior are choices and should be given equal or comparable status. For example, the time it takes a subject to respond in making a choice is itself a choice (of how much time to spend answering the question); theories of decision cost might make predictions about this time-choice.[4]

Insisting on paying subjects according to performance may miss the opportunity to learn something by varying incentives (see Smith & Walker, 1993), impose special burdens on poorly-funded investigators, and complicate some experiments unnecessarily.

In addition, the distaste for "rich", naturally-labelled contexts has inhibited the scope of problems people have studied (particularly those lying most squarely at the crossroads of psychology and economics); though happily, I think this stricture has weakened substantially in recent years.

4.2 Some Caveats

I end with some caveats. Obviously, there are many experimental E's and P's who do not fit well into the Table 2 columns, or fit in the "wrong" one. Mathematical psychologists are few in number but respect formalism and contribute substantially to it. Cognitive psychologists often study abstract tasks and offer modest incentives (perhaps to alleviate boredom). Some psychologists, like Amnon Rapoport, Ward Edwards, and Robyn Dawes, play by experimental economics rules for the same reasons E's do-- to force subjects to take tasks seriously, to make incentives (and hence performance) salient, to study the effect of learning over repeated trials, etc.

And of course, some groups of experimental E's are more curious about psychological regularities, and less inclined to express findings in narrow formalisms. Prominent among such eclectic E's are members of the "German school".

Which brings me to Reinhard Selten, who founded and inspired the German school along with Heinz Sauermann. I wrote this essay for a book honoring the

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[4] There is a substantial literature in psychology on response times (for example, they are often used as a primary dependent variable in theories of memory). Among decision theorists, John Hey, Nat Wilcox, and others have found them useful to study.

remarkable Reinhard Selten because his own work combines the very best of both the E and P approaches. But how?

Selten does it by having an open mind, reading and thinking voraciously, and by taking a "dualist" position. The dualist position is that formal theories of rational behavior are important for answering sharply-posed analytical questions, and possibly as normative bases for ideal living, but it is silly to think such theories are the best possible account of complicated behavior of actual humans. Instead, rational theories and behavioral (descriptive) theories must flow along separately.

Consequently, Selten and others in the German school mix and match conventions of the experimental E and P approaches as they see fit. Selten's experimental papers reflect the P's characteristic attention to details of subjects' decision rules and individual differences. For example, Selten developed the "strategy method", a survey/choice hybrid in which subjects report what they would choose (and then are forced to) for each of several possible realizations of a random variable, like a private value in an auction, or a randomly-determined Harsanyi "type" in an incomplete-information game. For most purposes, the strategy method produces much richer data and enables within-subject tests which are much more statistically efficient than single-choice methods.

To achieve mastery of a single mathematical approach or experimental style in a lifetime is difficult enough. To master two in a lifetime hardly seems possible, except that Reinhard Selten has done it.

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Reciprocity: The Behavioral Foundations of Socio-Economic Games

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Abstract. Growing experimental evidence and field research results on small group behavior suggest that the assumption of self interest must be expanded to allow more general forms of commitment in the form of reciprocity, defined as a response to another person's action that forgoes immediate benefits or incurs immediate costs. Negative and positive reciprocity have emerged as solutions to two important socio-economic activities: distribution and investment. The development of social institutions can be viewed as attempts to encourage reciprocity. Evolutionary psychologists hypothesize that the widespread development of reciprocity in human society results from the evolution of specialize mental algorithms for developing and punishing cheaters who behave noncooperatively in social exchange. We build on this work, and extend it as an organizing principle to examine subject behavior in ultimatum, dictator, and more general extensive form bargaining games.

Keywords. Experimental Games, Reciprocity, Evolutionary Psychology

1. Introduction

Why do people cooperate? In many 'commons'-type environments economists theorize that individual incentives to 'free ride' will interfere with attempts to increase social efficiency. While there exist numerous examples of noncooperative behavior, particularly under private information, the puzzle for economists is the extensive existence of small group cooperative behavior. Anthropological and archeological evidence suggests that sharing behavior is ubiquitous in primitive cultures that lack modern economic institutions for storing and redistributing wealth (see Cosmides and Tooby, 1987, 1989; Isaac, 1978; Kaplin and Hill, 1985; Lee and De Vore, 1968; Tooby and De Vore, 1987; and Trivers, 1971). Extensive field research by Ostrom et. al. at Indiana University have identified many contemporary and historical cases of decentralized cooperative solutions to commons problems (see Ostrom, Gardner, and Walker, 1994).

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Experimental evidence on small group behavior suggests that the assumption of self interest must be expanded to allow more general forms of commitment in the form of reciprocity (see Axelrod, 1984). *Reciprocity* is defined as a response to another person's action that forgoes immediate benefits or incurs immediate costs. In addition we will maintain the perspective that the use of reciprocity can be either a conscious, or an unconscious attempt to achieve greater individual fitness. Reciprocity can be divided into *negative reciprocity*, a form of commitment, such that individuals are willing to punish inappropriate behavior, and *positive reciprocity*, a form of obligation such that individuals are willing to reward appropriate behavior.

From a rational choice perspective, subjects who deviate from the subgame perfect strategies must believe this will increase their expected return; but why would they believe this? One approach is to look at the role of reciprocity in a sequence of games; either self-enforcing equilibria or reputational equilibria (see for example Kreps, 1990). But the literature on folk theorems for repeated games tell us the following: while equilibria with efficient social exchange can emerge in repeated games, they represent only a small subset of the possible equilibria that may occur. While the theory of repeated games is useful for explaining how reciprocity may be supported by noncooperative behavior, it does not explain the origin of this behavior.

One promising approach to explain the existence of reciprocity is to derive this behavior as an evolutionary stable strategy, as for example, in the work by Robert Frank (1988) on the strategic role of the emotions. The evolutionary approach is currently being studied in economics (see Güth and Kliemt, 1994) evolutionary psychology (Cosmides and Tooby 1992, herein denoted CT), and evolutionary biology (Smith, 1989).

The thesis of this paper is that in addition to extending the gains from exchange to nonmarket settings, negative and positive reciprocity have emerged as solutions to two important socio-economic activities: distribution and investment. Furthermore, the emergence of some social institutions can then be explained as attempts to encourage reciprocity. In developing this thesis we note that opportunities for reciprocity, together with the possibility of bounded rationality (Simon, 1979 and Selten, 1989) and the endogenous development of common expectations, suggest many potential, and often subtle, interactions between players' expectations, preferences, messages, and environmental factors. These complexities make it difficult to propose positive theories without experimental testing. One experimental approach, developed by evolutionary psychologists (CT, 1992), has been to study possible specialized mental algorithms for social exchange.

In the next sections we outline the approach developed in evolutionary psychology and relate the outcomes of that research to results in experimental economics on cooperation in bargaining games.

2. Experimental Detection of Specialized Mental Algorithms for Social Exchange

Neural science has developed a large body of knowledge on the specialized architecture of the brain. For example, we know that the brain has a dedicated architecture for the input and processing of visual sense data. The idea that the brain is specialized has led to the development of computational theory by evolutionary biologists to explain the development of brain architecture as the solution to particular adaptive problems. An example is the work by Marr (1982) on the adaptive problem of visual perception. Also see Wallman (1994) on the evidence that what the eye sees affects how it grows. Thus myopia and hyperopia, while having a genetic basis, are also influenced by growth factors affected by reading (the eye focuses on the page) and outdoor living (the eye focuses on infinity). Similarly, the language ability of the human brain is linked to the development of the human vocal tract. Up to the age of three months the human infant's larynx engages the nasal passage, making it impossible to breathe and drink simultaneously; it is also impossible to form words. This throat structure is similar to the structure of all other adult nonhuman mammals who, likewise, cannot speak. After the age of three months the larynx descends deep enough into the throat to allow the tongue to move back and forth, producing the vowel sounds emitted by adult human beings. This adaptation illustrates the biological basis of human language behavior, which in turn is linked inseparably with human social behavior.

While sensory input may depend on specialized mental algorithms, it has long been thought that the analysis of this information involves general logic-driven mental algorithms. However, experimental evidence questions the validity of this conjecture. For example Wason (1966) developed a task to study how efficient people are at detecting violations of material implication. The logic of falsifying a statement of the form "if P, then Q," requires people to understand that it is violated only under the condition that P is true and Q is false. In the Wason task, four cards each contain an antecedent on one side and a consequent on the other. A subject is shown one side of each of four cards, displaying a true antecedent (P), a false antecedent (not P), a true consequent (Q), and a false consequent (not Q).

Subjects are asked to indicate only those card(s) that definitely need to be turned over in order to see if any cases violate the rule. The correct answer is to select the cards showing P (to see if there is a not-Q on the other side) and not-Q (to see if there is a P on the other side). Here is an example of such a task. A subject is informed that part of your new clerical job at the local high school is to make sure that student documents have been processed correctly. Your job is to make sure the documents conform to the following alphanumeric rule:

If a person has a 'D' rating, then his document must be marked code '3'.

(If P[D] then Q(3))*

You suspect the secretary you replaced did not categorize the students' documents correctly. The cards below have information about the documents of four people who are enrolled at this high school. Each card represents one person. One side of a card tells a person's letter rating and the other side of the card tells that person's number code. Indicate only those card(s) you definitely need to turn over to see if the documents of any of these people violate this rule.

| D |	| F |	| 3 |	| 7 |
(P)	(not-P)	(Q)	(not-Q)*

*Parenthetical entries were not included in actual instructions.

The correct responses are to indicate the cards showing the letter D (P) and the numeral 7 (not-Q). The results of this experiment show that less than 25% of college students choose both of these cards correctly.

Now consider an example that concerns a law which states that "If a person is drinking beer, then he must be over 20 years old." The cards read: drinking beer, 16 years old, not drinking beer, 25 years old. The correct response is to choose the card "drinking beer" and the card "16 years old." In this experiment about 75% of college students get it right. There are many differences between this example and the previous one, but note that one difference is that the second rule involves a social contract.

Another interpretation of the difference in results between the example of the students' documents problem and the underage drinking problem might be that the underage drinking problem refers to a more familiar situation. In that case we might say that there are content-effects; the familiar content is responsible for the improved rate of positive responses. However, in a survey of this literature (Cosmides, 1985) suggests that "Robust and replicable content effects were found only for rules that related terms that are recognizable as benefits and cost/requirements in the format of a standard social contract ..." (CT, 1992, p. 183). Sixteen out of sixteen experiments using social contracts showed large content effects. For non-social contract rules, of 19 experiments 14 produced no effect, 2 produced a weak effect and 3 produced a substantial effect. The bottom line from several such studies was "Familiarity ... cannot account for the pattern of reasoning elicited by social contract problems" (CT, 1992, p. 187).

In all of the above cases, the correct adaptive response is identical to the correct logical response. Can more crucial experiments be designed to separate this confounding effect? The evolutionary psychology literature has produced two distinct groups of experiments that confront this issue: (i) experiments that switch the P and Q in the standard social exchange task thereby breaking the identity between cheating detection and the correct logical response; (ii) experiments that change the subjects' perspective from own to other, thereby changing the interpretation of what constitutes cheating. For example "If an employee gets a pension, then the employee must have worked for the firm for at least 10 years."

What constitutes cheating is different from the perspective of the employee than from that of the employer. But the correct Popperian response is independent of perspective. The experiments show that subjects' answers switch with perspective, thereby supporting the adaptive hypothesis.

3. Constituent Games For Social Exchange

If humans have specialized mental algorithms that assist them in achieving cooperative outcomes in social exchange environments, then factors that facilitate the operation of these natural mechanisms should increase both cooperative behavior and cooperative outcomes. For example, cooperative behavior should increase if individuals can observe and monitor one anothers behaviors, even if there are no direct mechanisms for enforcement. If it is possible for agents to directly punish cheating by other agents, cooperative behavior should increase even further. Similarly, if agents can communicate with one another, they can frame a group decision as a social exchange problem and activate natural inclinations to cooperate. If, in the absence of direct communication, agents can signal other agents of their intentions, they can create shared expectations of social exchange and cooperative behavior.

If reciprocity has evolved in response to particular adaptive problems, then this should have implications for behavior in basic constituent games for social exchange: the ultimatum (dictator) game and the investment game. Ultimatum (dictator) games have been used to study the effect of negative reciprocity on bilateral bargaining. From these studies we learn that negative reciprocity moderates selfish behavior through norms which place limits on noncooperative behavior. This moderation has a direct consequence for terms of trade and ultimately the distribution of goods and services. Investment games have been used to study the effects of positive reciprocity on investment decisions. From these studies we have learned that positive reciprocity can increase investment opportunities, thus having a direct consequence for economic growth.

3.1. The Ultimatum Game

In an ultimatum game, player 1 offers \$X to player 2 from a total of \$M. (The amount M has varied in experiments: \$5, \$10 and \$100). If player 2 accepts the offer, then player 1 is paid \$(M-X) and player 2 receives \$X; if player 2 rejects the offer, each gets \$0. In the dictator game player 1 makes an offer that player 2 must accept.

If we assume subjects come to an ultimatum or dictator game with no preconceived norms of behavior in such an environment, that all subjects prefer more money to less, and that all subjects have common knowledge of these characteristics of all other subjects, then the noncooperative equilibrium of the ultimatum game is for player 1 to offer player 2 the smallest unit of account, and for

player 2 to accept this offer. However, player 2 *can* punish player 1 for "cheating" on an implied social norm of sharing across time, learned in social exchange experience, by rejecting player 1's offer. In the absence of common knowledge, the possibility of punishment may change player 1's equilibrium strategy. In the dictator game, noncooperative game theory predicts that player 1's should keep all the money, i.e. send nothing to player 2's.

Brewer and Crano (1994), a recent social psychology textbook, identifies three important norms of social exchange that may apply in ultimatum games. The norm of equality implies that gains should be shared equally in the absence of any objective differences between individuals that would suggest some other sharing rule. The norm of equity implies that individuals who contribute more to a social exchange should gain a larger share of the returns. The norm of reciprocity implies that if one individual offers a share to another individual, the second individual reciprocates as soon as possible.

The ultimatum game was first studied by Güth, Schmittberger, and Schwarze (1982, herein GSS) and has now been replicated a large number of times. Figure 1A

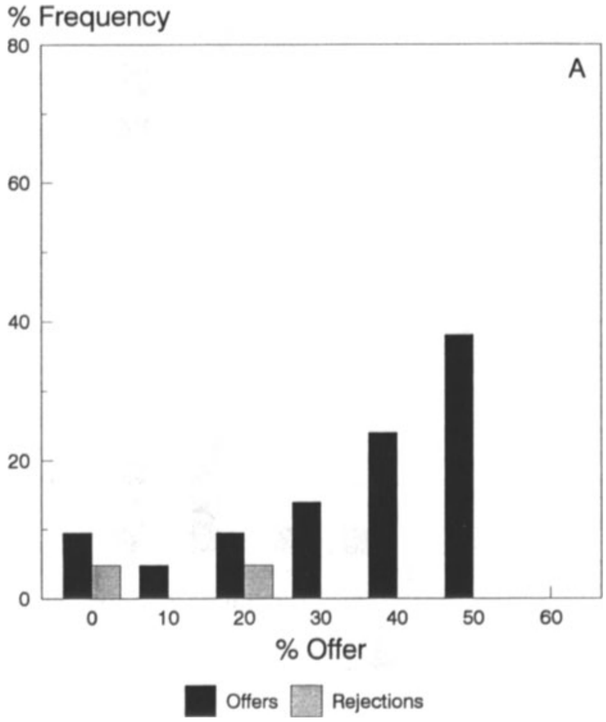

Figure 1A. Percentage frequency of percentage of pie offered to player 2 in ultimatum game experiments. (A) GSS, variable pie ultimatum, inexperienced subjects, N = 21. Offers are rounded to the nearest 10%.

graphs the percentage frequency of percentage pie offered to player 2 (rounded to the nearest 10%) in the GSS ultimatum games with inexperienced subjects (M is variable and the number of pairs is N = 21). This figure is typical of ultimatum games in that subjects often make proposals which offer receivers much more than the smallest $ unit of account. The modal proposal is often 50%. Furthermore player 2s will often reject asymmetric proposals even though these proposals offer them as much as 40% of M.In a direct test of the importance of negative reciprocity in explaining proposals in ultimatum games, Forsythe, Horowitz, Savin and Sefton (1994, herein FHSS) compare ultimatum game results to dictator game results. Figure 1B shows the FHSS replication of the ultimatum game using M = $10 (N = 24). Comparing Figure 1A with 1B we observe that the FHSS replication has led to even stronger sharing. Hoffman, McCabe, Smith (1994a) suggest that this is due to the strengthening of norms of reciprocity found in the FHSS instructions and procedures. FHSS also find that removing the threat of punishment, as in the

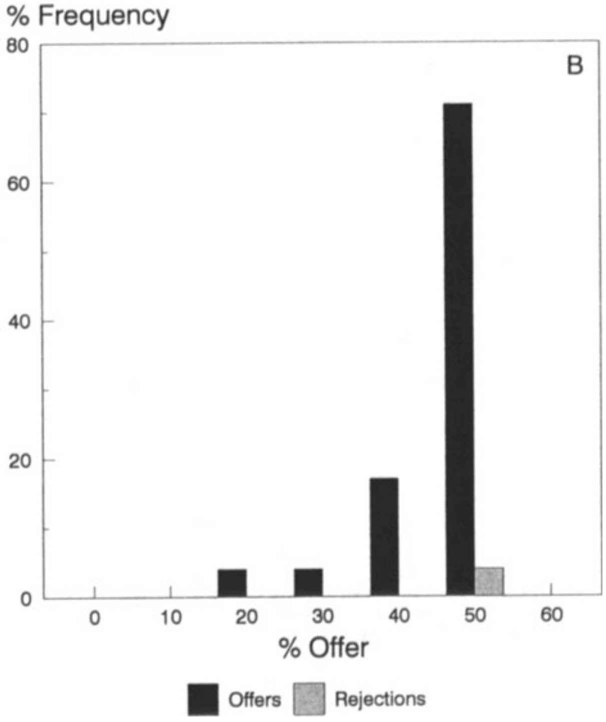

Figure 1B. Percentage frequency of percentage of pie offered to player 2 in ultimatum game experiments. (B) FHSS, $10 ultimatum, inexperienced subjects, N = 24. Offers are rounded to the nearest 10%.

dictator game, reduces the amounts given to receivers, but not by as much as noncooperative game theory predicts. This is seen by comparing Figure 2A with Figure 1B. Hoffman, McCabe, Shachat, and Smith (1994, herein HMSS) design a version of the dictator experiment, which provides dictators with complete anonymity through *double-blind* controls.

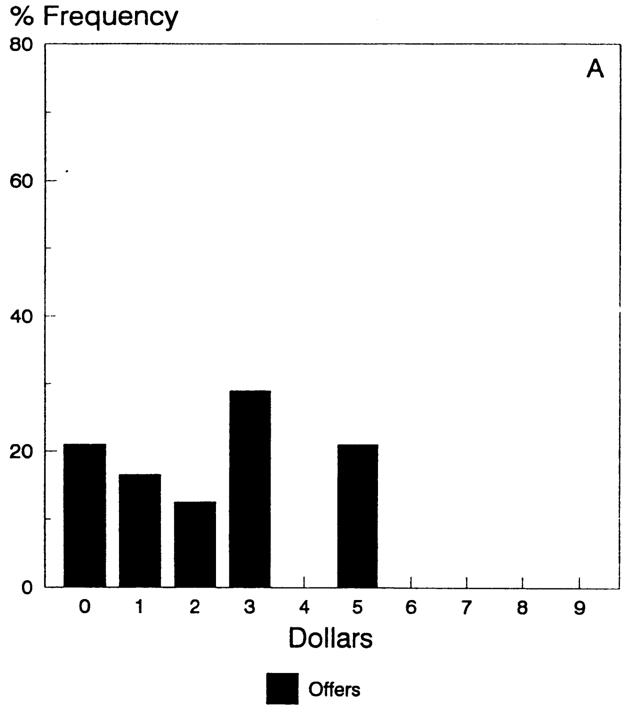

Figure 2A. Comparison of percentage frequency of offers to player 2's in Dictator Experiments. (A) FHSS, Dictator, Divide $10 experiment, N = 24.

The distinguishing characteristic of the double-blind design is that dictators' decisions cannot be identified by the experimenter, any other person who might observe the experiment or study the results, or the dictators' counterparts. Dictators and counterparts are recruited to separate rooms. Each dictator randomly draws an opaque envelope from a box. Each envelope contains twenty pieces of paper. Twelve contain ten $1 bills and ten white slips of paper. Two contain twenty white slips of paper. Each dictator takes the envelope to a private place removes ten pieces of paper, seals the envelope, and leaves the room without identification. When all dictators have finished, the box is delivered to the room where the counterparts are located. Each counterpart randomly selects an envelope. A subject monitor opens it

and records the number of $1 bills. The HMSS results are strikingly different from the dictator results summarized in FHSS, in which subjects were observed by the experimenter (compare Figures 2A and 2B). In the double-blind dictator experiments, 64% of players take all the money.

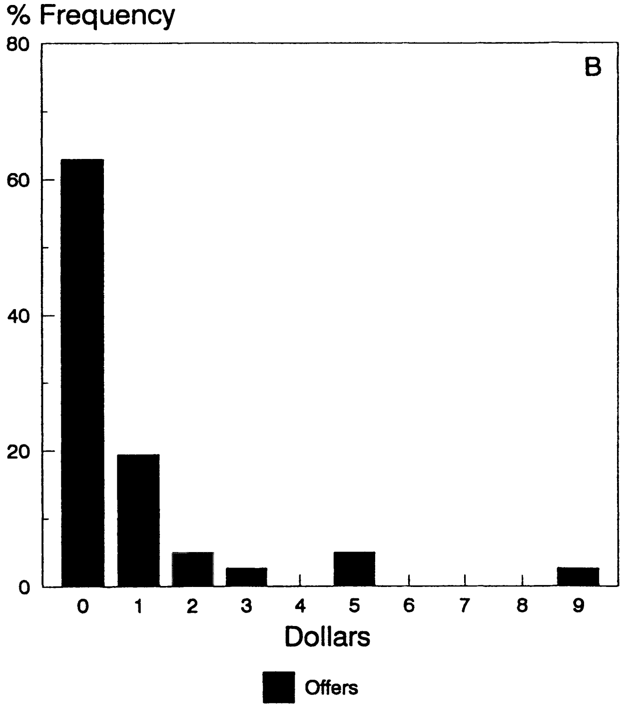

Figure 2B. Comparison of percentage frequency of offers to player 2's in Dictator Experiments. (B) HMSS, Dictator, Double-Blind 1 experiment, N = 36. An Epps-Singelton test statistic of 23.6 is significant at the .001 level.

Hoffman, McCabe and Smith (1994b) then vary each of the elements of the double-blind dictator experiment in ways intended to reduce the degree of "social distance" between the experimenter and others who might see the data, and the subject dictators while preserving complete anonymity between the dictator and their counterpart. Comparing the results on social distance to the authors' replication of FHSS's dictator experiments, they find that the experimental results form a predicted ordered set of distributions. As the social distance between the subject and others increases, the Player 1s offers to Player 2s decreases. These results demonstrate quite

strongly the power of observability in invoking innate social norms of reciprocity, even where direct punishment is not possible.

Recognizing that the threat of punishment might create expectations that change player 1's behavior, HMSS also consider experimental treatments explicitly designed to manipulate subject expectations about the social acceptability of proposals in ultimatum games. FHSS make no role distinction between the two individuals "provisionally allocated" $10, and they are told to "divide" the money. Not surprisingly, perhaps, in such an environment, more Player 1s fear deviations from equal division will be punished as "cheating" on the implied social exchange. HMSS replicate the FHSS results in a slightly different experimental environment. The key elements they maintain are: (1) players 1 and 2 are randomly assigned to those positions; and (2) the task is described as proposing a "division" of $10 "provisionally allocated to each pair." They refer to this treatment as random/divide $10.

HMSS then explore two treatment changes, in a 2x2 experimental design. First, without changing the reduced form of the game, HMSS describe it as a market in which the "seller" (player 1) chooses a price (division of $10) and the "buyer" (player 2) indicates whether he or she will buy or not buy (accept or not accept). From the perspective of social exchange, a seller might equitably earn a higher return than a buyer. Treatments with buyers and sellers are referred to as exchange treatments. Second, they make player 1s earn the right to be a seller by scoring higher on a general knowledge quiz. Winners are then told they have "earned the right" to be sellers. Going back to social exchange, equity theory predicts that individuals who have earned the right to a higher return will be socially justified in receiving that higher return. Treatments using a general knowledge quiz to assign property rights in the position of seller or player 1 are referred to as contest treatments.

In a situation in which it is equitable for player 1 to receive a larger compensation than player 2 (i.e., contest/exchange) player 1 offers significantly less to player 2, and player 2 accepts with the same probability (no significant difference in the rejection frequencies) (compare Figures 3A and 1B). These results suggest that the change from (random/divide) to (contest/exchange) alters the shared expectations of the two players regarding the social exchange norm operating to determine an appropriate sharing rule. Moreover, Hoffman, McCabe, and Smith (1994a) find no significant reduction in offers when the amount to be divided is increased from $10 to $100 in these two treatments (compare Figures 3A and 3B). This result suggests that attitudes toward socially acceptable proposals are very strong; as the stakes are increased, so is the opportunity cost of proposing an improper division and getting punished. Finally, the difference between (random/divide) and (contest/exchange) carries over to a non-double-blind dictator experiment as well. Thus, the change in expectations takes place even when there is no threat of punishment from player 2.

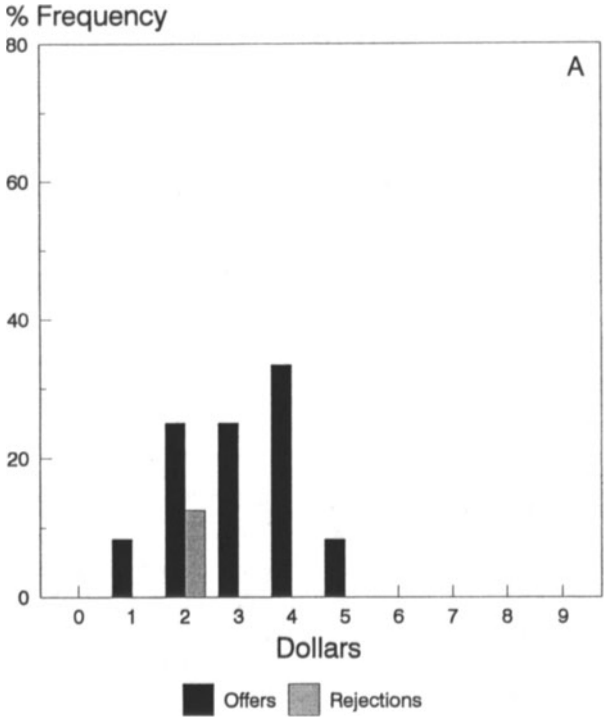

Figure 3A. Comparison of percentage frequency of offers to player 2s in (A) HMSS $10 contest/exchange ultimatum game, N = 24.

3.2. The Investment Game

The *investment game* allows us to isolate the effect of positive reciprocity on investment behavior. Player 1 is given $M ($10) as a show up fee. In stage one, player 1 in room A decides how much of the $M (denoted $S) to send to an anonymous counterpart called player 2 in room B. It is common information that whatever amount is sent will be multiplied by some factor (f = 3) by the time it reaches player 2. In stage two, player 2 then decides how much of the money (f x $S) to return to player 1. Note that stage two is a dictator game where the non-cooperative prediction is for player 2 to return nothing. But this in turn implies that player 1 should send nothing. Trust occurs in the investment game whenever a positive amount is sent in stage one. Positive reciprocity occurs when subject 2 sends back an amount greater than the amount sent.

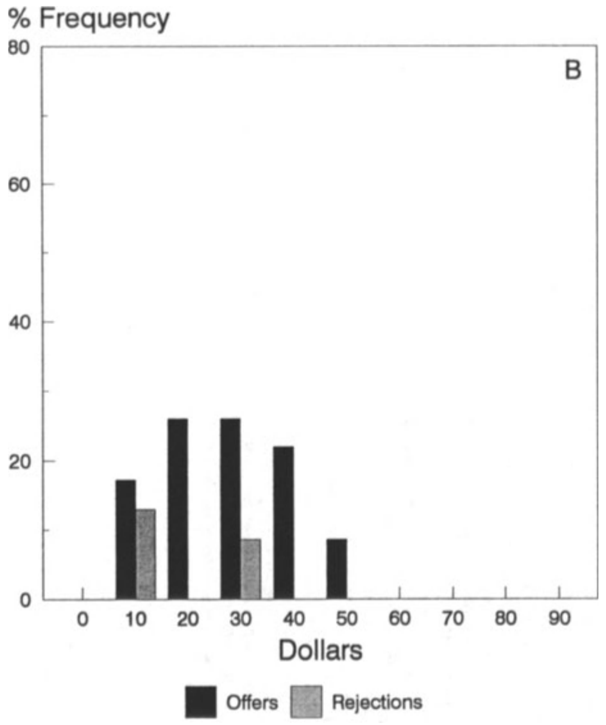

Figure 3B. Comparison of percentage frequency of offers to player 2s in (B) HMSa, $100 contest/exchange ultimatum game, N = 23. An Epps-Singleton statistic of 1.37 leads to the conclusion that offer distributions are the same in the $10 and $100 treatments. However, the increase in rejection rates in the $100 ultimatum games is significant at the .001 level.

In implementing the investment game, first studied by Berg, Dickhaut and McCabe (1995, hereinafter BDM), BDM modify the double-blind procedures used by HMSS in order to control for the possibility of repeat game effects and thus protect any observed results from being attributed to reputation, collusion, or the threat of punishment. Figure 4A graphs the amount sent from room A (shown as open circles; data has been sorted on amount sent), the tripled amount which reached room B (shown as grey bars), and the amount returned to room A (shown as solid circles). BDM observe a large (although variable) degree of trust and a significant (compared to double blind dictator games) degree of positive reciprocity (compare Figure 4A to 2B).

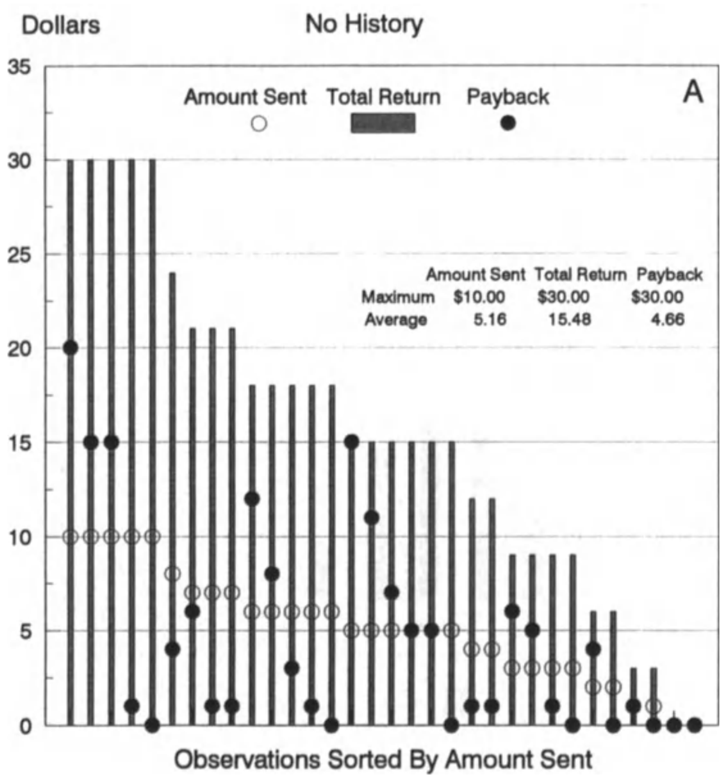

Figure 4A. Comparison of amounts sent by player 1's ($S), total return to the pair (3 x $S), and payback decisions ($ amount returned to player 1's by player 2's), in the investment game: (A) BDM, Treatment 1, No History, M = $10, f = 3, N = 32.

There are many instances within organizations where public information reflects an organization's social history. Since social history provides common information about the use of trust within an organization such a history may reinforce individuals' predisposition towards trust. Thus, Coleman (1990) argues that norms are more likely to be internalized when an individual clearly identifies with a particular group. The process of having an individual identify with a group is termed socialization. In the initial BDM experiment subjects were all University of Minnesota undergraduates. BDM run a second treatment with subjects from the same pool, where subjects were provided a social history about the behavior of subjects in the first treatment. By providing social history in a double-blind,-one-

shot setting, BDM are able to focus on the internalization of social norms, as opposed to other potential mechanisms for reciprocity such as reputation building.

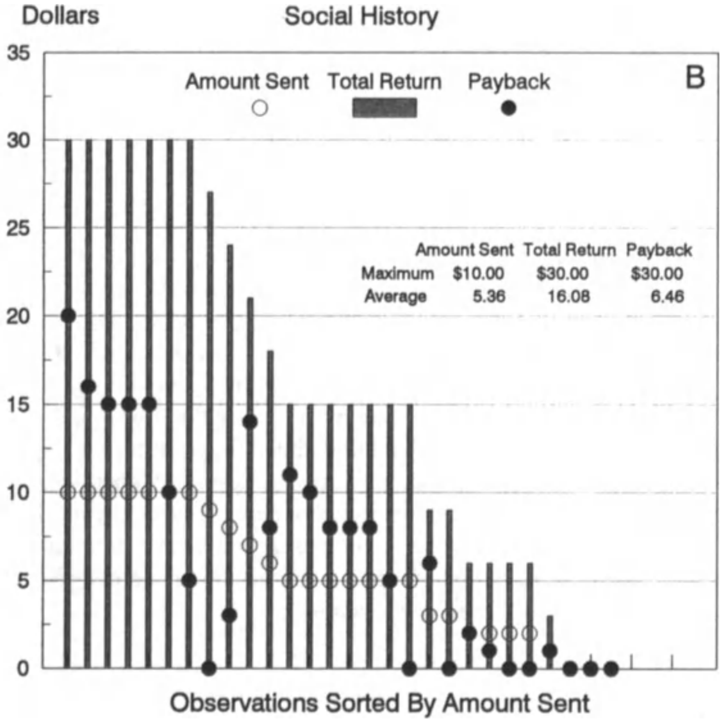

Figure 4B. Comparison of amounts sent by player 1's ($S), total return to the pair (3 x $S), and payback decisions ($ amount returned to player 1's by player 2's), in the investment game: (B) BDM, Treatment 2, Social History, M = $10, f = 3, N = 28.

Comparing Figure 4B to Figure 4A we observe an increase in both trust and reciprocity. BDM calculate Spearman's rank correlation coefficient, r_s between the paired room A and room B decisions. Since absolute amounts sent and amounts returned will bias this statistic upwards, i.e., low amounts sent preclude some high returns, we compare amount sent to percentaged returned. An $r_s = .01$ suggests no correlation between amounts sent and payback decisions in Figure 4A. An $r_s = .34$ was found to show a significant (at the .06 level using resampling methods) increase in correlation between amounts sent and payback decisions in Figure 4B where social history is provided.

In settings characterized by repeat interactions people can build a reputation for positive reciprocity. To study this behavior we will extend the basic investment setting studied in BDM to two periods. Subjects will be assigned partners at the beginning of the experiment and will keep the same partner for both periods. We will extend the double blind controls to this setting to allow direct comparisons of the data with both the no history and social history treatments in BDM. Pilots for this design have been run by Dickhaut, Hubbard, and McCabe (1995). Preliminary results suggest an increase in reciprocity in period one followed by a significant drop in reciprocity in period two.

4. Discussion

New modeling efforts have incorporated reciprocity into the utility assumptions of game theory. Bolton (1991), incorporates a loss to be 'unfairly' treated into subject's utility functions in order to explain a willingness to reject unacceptably low offers in ultimatum games. But his model is contradicted by results in the dictator game. Rabin (1993) more generally captures reciprocity by incorporating a 'kindness' function into subjects' utility in such a way as to capture the following behavior: as one's counterpart increases his or her 'kindness', the utility maximizing response is to be kinder in return. Putting norms directly into subjects utility functions is simply a shortcut for modeling a predisposition towards reciprocity; i.e. expectations are represented as utilities. An alternative might look at individuals learning strategies. How do individuals form initial expectations towards reciprocity? How do individuals confirm or modify these expectations?

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Adaptation of Aspiration Levels - Theory and Experiment -

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1. Introduction

Stimulated by the work of *Herbert A. Simon*, *Reinhard Selten* has concerned himself with *aspiration levels* and *bounded rationality* since the beginning of his academic career.[2] His joint paper with *Heinz Sauermann* "*Anspruchsanpassungstheorie der Unternehmung*" was published already 1962.[3]

After presenting the most important components of this theory I will show how it has influenced my own work. Thirty years ago the two authors suggested to me to investigate also macroeconomic relations experimentally.[4] For this purpose a sufficiently complex macroeconomic model had to be constructed which could be used as an experimental environment for negotiations with macroeconomic impact. For this model *Reinhard Selten* proposed the name KRESKO. I will show in which way the model is governed by multilateral adaptation processes of aspiration levels. I will then discuss bargaining theories controlled by aspiration levels which were developed exploratively based on these experiments. Finally I will touch on some open research questions.

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[1] Translation of a paper presented at a colloquium in honor of *Reinhard Selten* in Bonn, January 27th, 1995. I hope that *Reinhard Selten* and myself will be able to continue and to complete the work on aspiration levels and bounded ratoionality started 30 years ago.

[2] Cf. *R. Selten*, In search of better understanding of economic behavior, in: *A. Heertje* (Ed.), The Makers of Modern Economics, New York et al. 1994, pp. 115-139; *H. A. Simon*, Theories of Decision Making in Economics and Behavioral Science, The American Economic Review, Volume 49 (1959), pp. 253-283; *H. A. Simon*, New Developments in the Theory of the Firm, The American Economic Review, Volume 52 (1962), Papers and Proceedings, pp. 1-15.

[3] *H. Sauermann* and *R. Selten*, Anspruchsanpassungstheorie der Unternehmung, Zeitschrift für die Gesamte Staatswissenschaft, Volume 118 (1962), pp. 577-597.

[4] *H. Sauermann*, Preface of the Editor to: *R. Tietz*, Ein anspruchsanpassungsorientiertes Wachstums- und Konjunkturmodell (KRESKO), Tübingen 1973, p. V.

W. Albers et al. (eds.), *Understanding Strategic Interaction*
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2. The Unilateral Adaptation Theory of Aspiration Levels of the Firm

The term aspiration level comes from experimental psychology where it was first used by *Lewin* et al.[5]. *Sauermann and Selten* define the *aspiration level* as the *bundle of all aspirations that must be fulfilled in order to satisfy the individual.*[6] An aspiration level is considered realistic if the aspirations can be expected to be reached.[7] Only realistic aspiration levels are of importance in a theory of bounded rationality. An aspiration level is adapted upwards when possibilities of action are seen which allow a better solution. An aspiration level which no longer seems to be attainable has to be adapted downwards.

The *theory of adaptation of aspiration levels*, presented by *Sauermann and Selten* for the situation of the firm, is a general theory of decision behavior in unilateral decision situations.

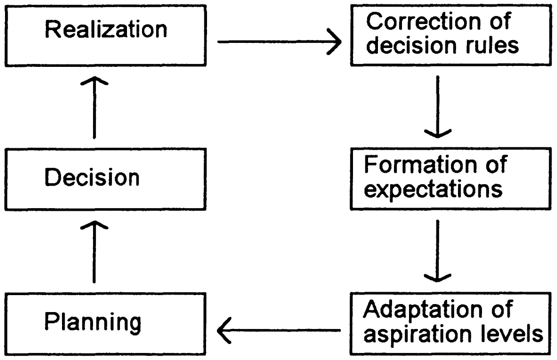

Figure 1: Flow Chart of the Decision Process[8]

As shown in *Figure 1*, this theory can be seen as a dynamic process consisting of 6 overlapping phases with feedback: Correction of decision rules, formation of expectations, adaptation of aspiration levels, planning, decision, and finally, realization.

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[5] *K. Lewin, R. Dembo, L. Festinger,* and *P. P. Sears*, Level of Aspiration, in: *J. McV. Hunt* (Eds.), Personality and the Behavior Disorders, New York 1944, pp. 333-378.

[6] *H. Sauermann* and *R. Selten*, l.c., p. 579.

[7] Ibid.

[8] Figures 1 to 6 are taken from *Sauermann* and *Selten*, l.c.

The two most important components of the theory are the scheme of adaptation of aspirations and the scheme of influences. The *scheme of adaptation of aspirations* is defined as the grid of discrete aspiration values in the space of the goal

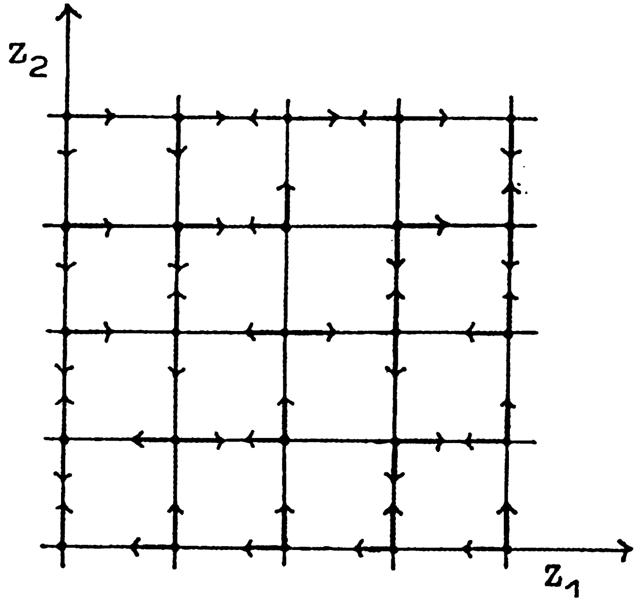

Figure 2: The Scheme of Adaptation of Aspirations

variables. Each point of the grid represents a *potential aspiration level.* For each such point an *order of importance* determines which of the respective goal variables is to be adapted upwards first.

For an adaptation downwards a *sacrifice variable* is defined in which a sacrifice, a reduction of an aspiration, has to be made. For two goal variables, for instance rentability and market share, the aspiration scheme could have the form shown in *Figure 2.* At each grid point the arrows indicate the upward and downward directions of adaptations.

Figure 3 shows an upward adaptation from an earlier aspiration level (circle) to a new one (square) in accordance with the scheme of adaptation of aspirations of *Figure 2.* The *realization area* as *expected* by the decision maker is regarded here as given and is indicated by hatching. The old aspiration level was not Pareto-optimal, thus inducing an upward adaptation. The last adaptation step switches to the second most important variable Z_2, since an increase in the most important variable Z_1 would lead out of the realization area and therefore is not allowed.

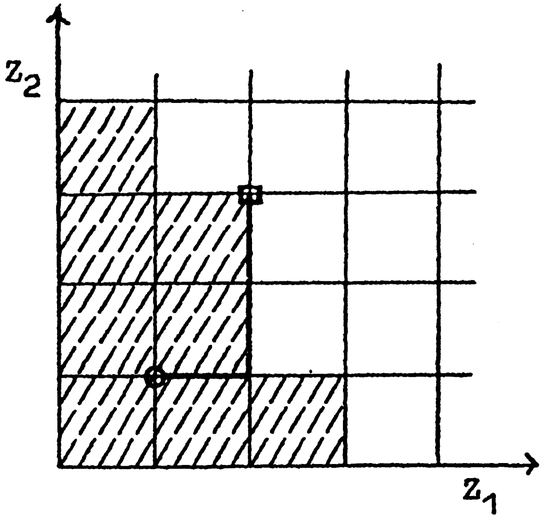

Figure 3: Upward Adaptation of Aspirations

In contrast, *Figure 4* shows in the first step a downward adaptation since the last aspiration level lies outside the realization area relevant here. The subsequent upward adaptation in the last step has to use the second most important variable, here Z_1, until a Pareto-optimal point is reached as an actual aspiration level.

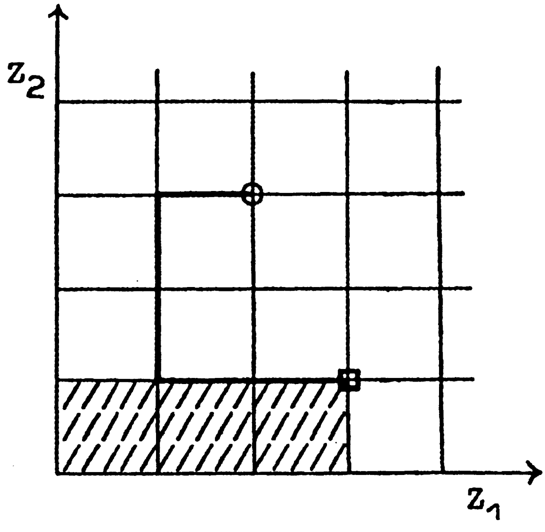

Figure 4: Downward Adaptation of Aspirations with Subsequent Upward Adaptation

As the theory has been formulated rather generally, *Sauermann and Selten* have presented two more concrete models, the routine model and the planning model. The *routine model* disregards the first phase of *Figure 1*, the correction of decision rules. The search for new possibilities of action is thus omitted. The formation of expectations is restricted to changes of the scheme of influence.

	Z_1	Z_2
A_1	o	—
A_2	+	—
A_3	o	+

Figure 5: The Scheme of Influence

The *scheme of influence* (*Figure 5*) represents the decision maker's subjective ideas of causal influences only in a qualitative way. In a matrix a positive influence of a possible action (A_1, A_2, or A_3) on a goal variable (Z_1 or Z_2) is represented by '+', a negative one by '-', and missing influence by 'o'. A *possible action* is a plan in which instrumental variables are changed. Possible actions can also be the objects of a search process governed by planning rules. The scheme of influence is modified by means of expectation-forming rules according to the decision maker's experience.

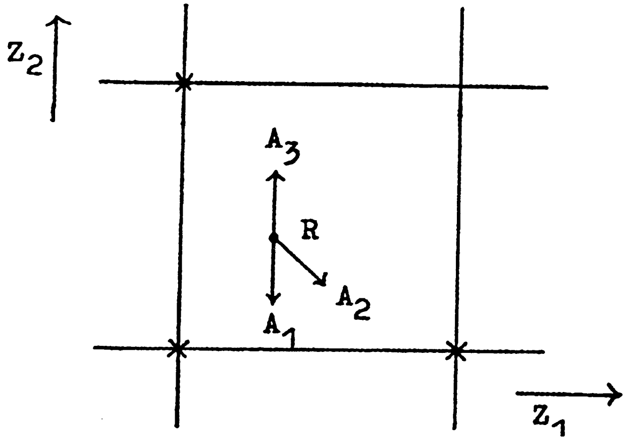

Figure 6: Construction of the Expected Realization Area

As an example, *Figure 6* shows how the *expected realization area* is derived from the scheme of influence in combination with the last realization of the goal variable R. The scheme of influence from *Figure 5* as transferred to the scheme of aspiration adaptation is represented by arrows indicating the direction of reaction. Potential aspiration levels that are still attainable are indicated by asterisks.

In the *planning model* the rules of forming expectations are corrected in accordance with the realization. The phases of the routine model are transposed to the planning level. Calculation by which the effects of possible actions are computed takes the place of the realization. The scheme of influence is changed in accordance with this calculation. The determination of the realization area in the routine model is replaced by the derivation of the *calculation area* at the planning level.

The work of *Sauermann and Selten* in the form of the models sketched here is not to be seen only as a descriptive approach to boundedly rational behavior. In its general form the theory is at the same time a program for research which in some aspects is still awaiting concretization. It poses many questions which should be clarified by empirical, experimental, and theoretical investigations: Which rules prevail or are expedient in the formation of expectations? How is the order of importance derived? How many potential aspiration levels are useful? How are the potential aspiration levels formed and modified? In which way is the adaptation of aspiration levels effected in bilateral or multilateral situations? Which normative or, in a weaker form, semi-normative properties should be required for a dynamic theory of boundedly rational decision behavior controlled by aspiration levels?

The idea of aspiration-guided decision behavior has also influenced explanatory approaches of *Selten* to the behavior in *oligopoly experiments*.[9] There, new aspirations are formed in each period. As modelled in a simulation study,[10] the aspirations are not directly concerned with the goal variables; on the contrary, the adaptation is accomplished between various criteria which deliver aspirations in a more natural way. For instance, the principle of sustaining the net profit is replaced by the weaker principle of sustaining the same gross profit as before, if the aspiration of sustaining the net profit appears to be unrealizable.

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[9] *R. Selten*, Investitionsverhalten im Oligopolexperiment, in: *H. Sauermann* (Ed.), Contributions to Experimental Economics (Volume 1), Tübingen 1967, pp. 60-102; *R. Selten.*, Ein Oligopolexperiment mit Preisvariation und Investition, ibid., pp. 103-135; *R. Selten*, Die Strategiemethode zur Erforschung des eingeschränkt rationalen Verhaltens im Rahmen eines Oligopolexperimentes, ibid., pp. 136-168.

[10] *R. Tietz*, Simulation eingeschränkt rationaler Investitionsstrategien in einer dynamischen Oligopolsituation, in: *H. Sauermann* (Ed.), Contributions to Experimental Economics (Volume 1), Tübingen 1967, pp. 169-225. Cf. also: *R. Selten* and *R. Tietz*, Zum Selbstverständnis der experimentellen Wirtschaftsforschung im Umkreis von Heinz Sauermann, Zeitschrift für die gesamte Staatswissenschaft, Volume 136 (1980), pp. 12-27.

3. Multilateral Processes of Adaptation of Aspirations as a Macroeconomic Concept of Coordination of Decisions

The fascination arising from the dynamic approach of adaptation of aspiration levels led me to construct a complex *macroeconomic model* which could be used as an experimental environment for bargaining experiments with macroeconomic relevance. In accordance with principles of *bounded rationality* this model is solved in each period by markets and by mutual processes of adaptation of aspiration levels.[11]

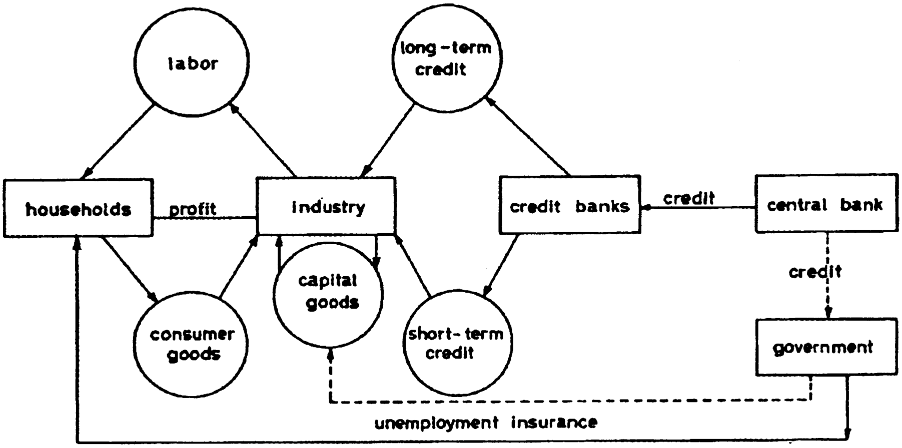

Figure 7: The Monetary Circulation of the KRESKO Economy[12]

This model of a closed national economy includes four sectors: households, industry, credit banks, and the central bank, which are interpreted as representative decision units (*Figure 7*). The government is represented only rudimentarily. The sectors are connected with each other by markets for labor, consumer goods, capital goods, short-term credit, and long-term credit. The production is organized by a putty-clay production function with endogenous technical progress. By means of *dimensional analysis* principles of construction were derived in order to

[11] *R. Tietz*, Ein anspruchsanpassungsorientiertes Wachstums- und Konjunkturmodell (KRESKO), Tübingen 1973.

[12] Figure 7 is taken from *R. Tietz*, The Macroeconomic Experimental Game KRESKO, in *H. Sauermann* (Ed.), Contributions to Experimental Economics, Volume 3, Tübingen 1972, pp. 267-288, esp. p. 269.

analyze the relations between about 300 variables and to ensure the existence of *equilibrium paths* for this *growth and business cycle system.*

Here, we will restrict detailed consideration to the *individual decision and coordination process,* which is ruled by aspiration levels in a way similar to the theory of *Sauermann and Selten.* In each period the aspiration levels were formed by means of expectation and planning variables (*Figure 8*[13]).

The univariate *formation of expectation* in field E1 uses in the first step only information on the past of the regarded variable (I1) by means of an adaptive exponential smoothing procedure, which is controlled by the prediction quality. The multivariate formation of expectations in E2 uses in addition information on the assumed causal structure from I2. *Planning variables* are formed in P3 using expectations on the behavior of other decision makers (E3). If these planning variables also have control and regulation functions they also assume the character of *aspiration variables* (A3). On the basis of these plans preliminary decisions are made in D4 which are tentatively coordinated in M5 with preliminary decisions of other decision makers (D4'), e.g. in markets. If necessary, additional expectations, plans, and decisions are made for other variables (fields E6, P7, A7, and D8). If the aspiration level formed by the aspirations defined so far can be reached (W9), the decision process for this topic is finished. Failure to reach an aspiration level triggers a hierarchically organized *process of adaptation of aspirations.*

Besides the aspirations formed in the fields A3 and A7 which are concerned with economic variables, there are aspirations that limit the number of adaptations of aspirations and govern the selection of variables to be adapted. In the KRESKO model these aspirations are represented by *counting-variables* in hierarchical order. Thus, field W10 examines whether the "strong" adaptations over K13 and Z13 have been performed less than v10* times. If this is the case a similar test for the variable v11 controls the "weaker" (or inner) adaptation via K12 and Z12. In K12 smaller revisions are made only for the variables formed in E6, P7, and A7. Based on these corrected data, new decisions are made in D8. After the limit v11* for the counting-variable for weak adaptations is reached, a strong (or outer) adaptation occurs via K13. This leads to corrections of expectations from E2 and of plans and aspirations from P3 and A3. Thereby new decisions have to be made in D4 which again necessitate coordination on the market (M5). While the weak adaptations are made only internally by the individual, the strong adaptations also require revisions by other decision makers.

If the limit for the maximum of strong adaptations would be violated in W10 by an additional strong adaptation, after some final corrections in K14 and D14, other decision makers also have to make adaptations in D14'. The revised deci-

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[13] Figures 8 and 9 are taken from *R. Tietz* 1973, l.c., pp. 105, 112, resp. 465.

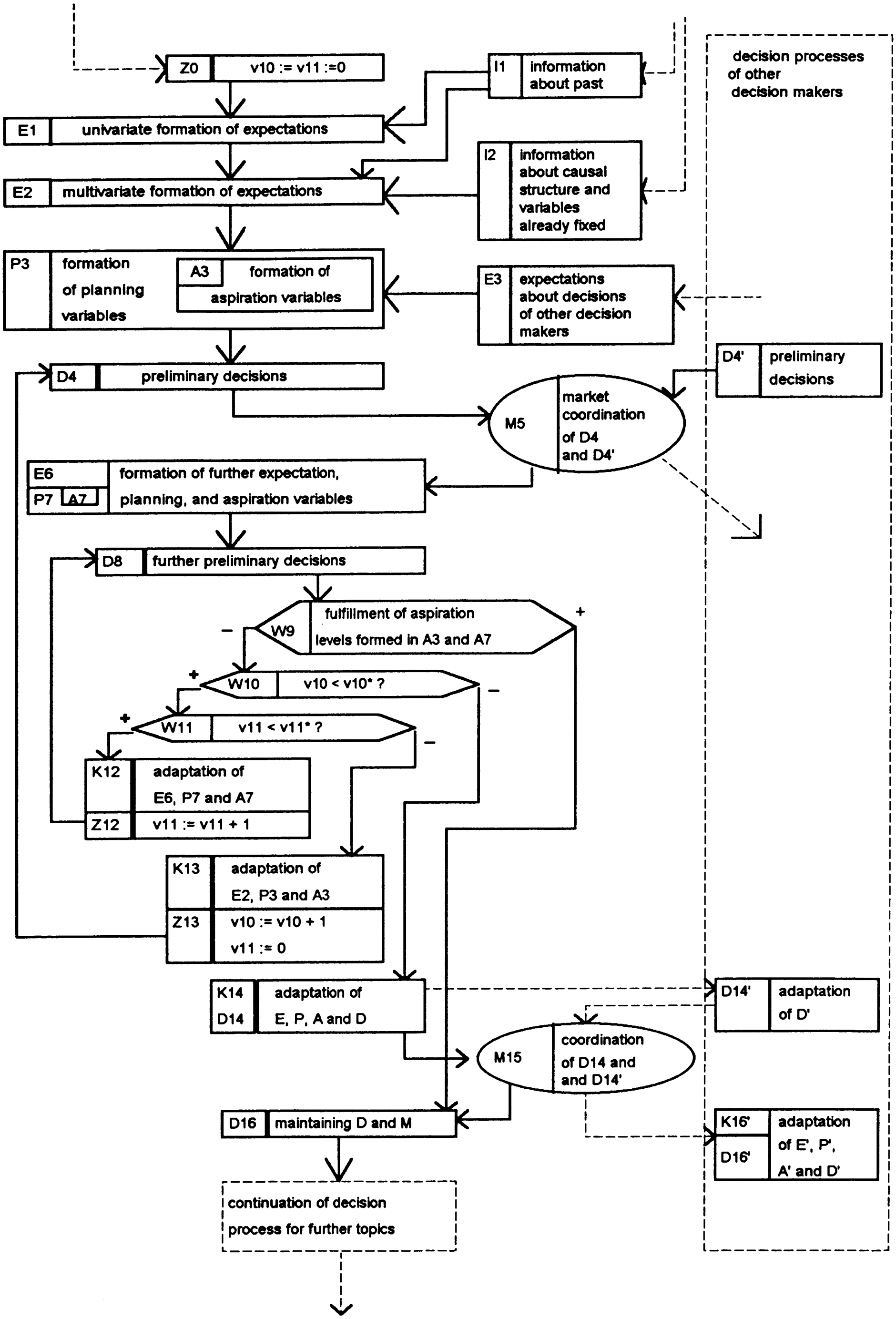


Figure 8: The Individual Decision Process

sions of both sides are again coordinated in M15. Thereafter the burden of adaptation switches over to other decision makers in K16' and D16'.

The *macroeconomic process of coordination* between the sectors regarded as representative decision units corresponds in its main features to the individual process. (Therefore, it may be sufficient to reproduce *Figure 9* only in German.) For instance, *Figure 9* shows that, because of violated liquidity aspirations of the firms (Liquiditätsansprüche der Unternehmen) in field W1.13, a weak adaptation occurs in K1.17. In this case the firms adapt their liquidity aspirations in the form of time deposits downwards, and their demand for short-term credit (Nachfrage nach kurzfristigen Krediten), upwards. Subsequently the market for short-term credit (Geldmarkt) M5.1 is again passed through. On the other hand, the strong adaptation in K1.19 concerns the planning of production (Produktionsplanung) with feedback via Z1.20 and L111 to P1.3, where decisions on production, investment, and employment (Produktion, Investition, Beschäftigung) again have to be made.

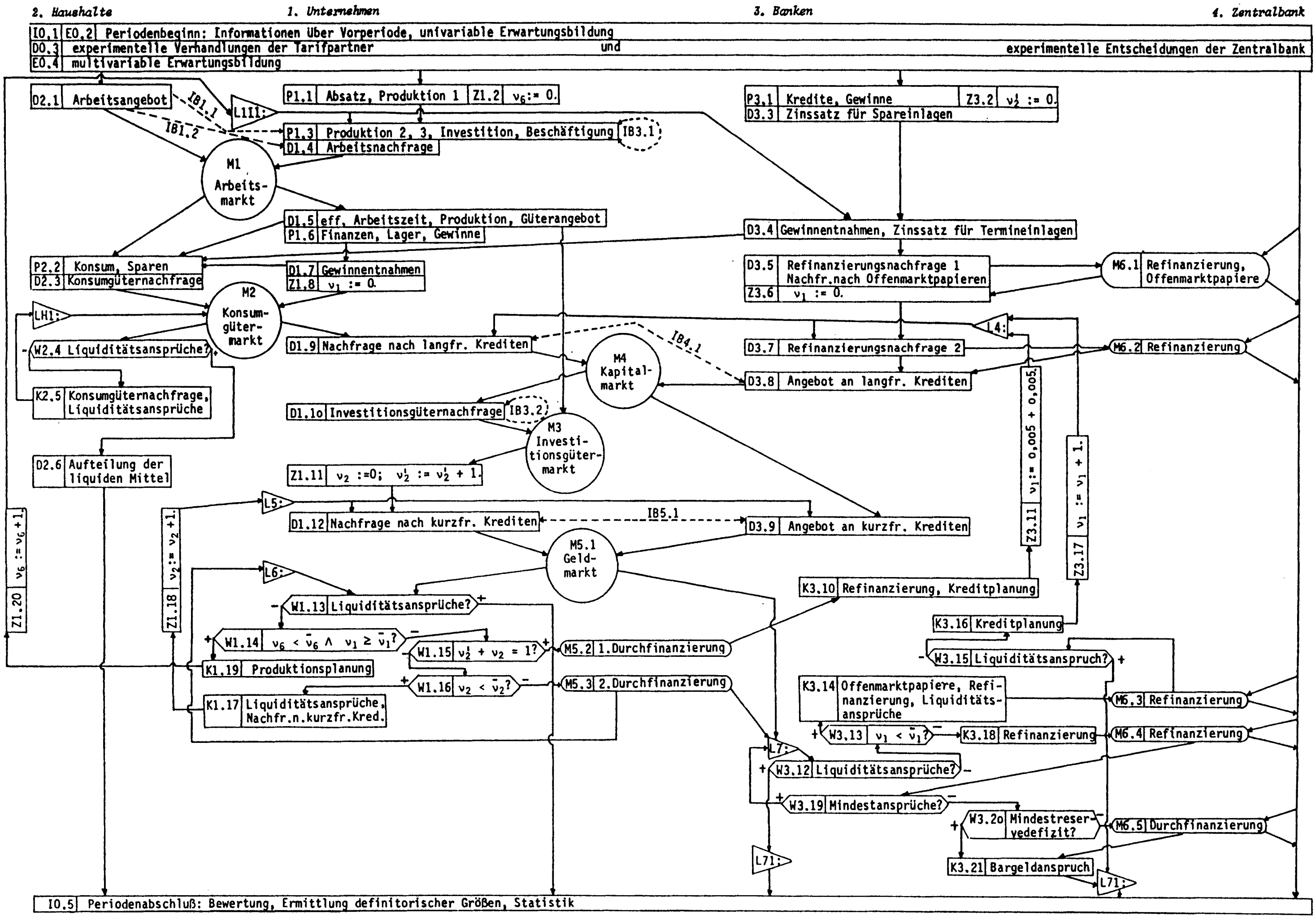
Finally the adaptation process shifts via M5.2 or M5.3 to the sector of the credit banks. By the so-called *"through financing"* (Durchfinanzierung) the reduced liquidity aspirations of the firms are preliminarily fulfilled. In K3.10, for example, the banks revise their plans for refinancing and credit (Refinanzierung, Kreditplanung).

The - in a sense - hierarchical order of the sectors and the concept of "through financing" guarantee a solution of the system: The central bank, via the through financing in M6.5, in the end has to fulfill the liquidity aspirations of the banks that were greatly reduced several times.

In the given form, KRESKO is one possible model of a multilateral macroeconomic process of adaptation of aspirations. One may criticize the relatively rigid hierarchical structure between and within the sectors. Possibly, it favors the tendency of the model to exhibit depressions rather than inflations. The question could also be asked whether the formation of aspiration levels has to take place on an equilibrium path in the same way as in a disequilibrium, meaning whether the process shall always start with the highest aspiration level. We will return to these questions after discussing the experimental results which have been obtained later.

4. Bilateral Theories of Bargaining Based on the Adaptation of Aspiration Levels

The great importance of qualitatively distinguished aspiration levels, derived in a seemingly natural way, was especially evident in bargaining experiments


2. Haushalte
1. Unternehmen
3. Banken
4. Zentralbank
I0.1 E0.2 Periodenbeginn: Informationen über Vorperiode, univariable Erwartungsbildung
D0.3 experimentelle Verhandlungen der Tarifpartner und experimentelle Entscheidungen der Zentralbank
E0.4 multivariable Erwartungsbildung
D2.1 Arbeitsangebot
IB1.1
IB1.2
L1I1:
P1.1 Absatz, Produktion 1
Z1.2 v6 := 0.
P1.3 Produktion 2, 3, Investition, Beschäftigung
IB3.1
D1.4 Arbeitsnachfrage
P3.1 Kredite, Gewinne
Z3.2 v'2 := 0.
D3.3 Zinssatz für Spareinlagen
M1 Arbeitsmarkt
D1.5 eff. Arbeitszeit, Produktion, Güterangebot
P1.6 Finanzen, Lager, Gewinne
D3.4 Gewinnentnahmen, Zinssatz für Termineinlagen
P2.2 Konsum, Sparen
D2.3 Konsumgüternachfrage
D1.7 Gewinnentnahmen
Z1.8 v1 := 0.
D3.5 Refinanzierungsnachfrage 1
Nachfr. nach Offenmarktpapieren
Z3.6 v1 := 0.
M6.1 Refinanzierung, Offenmarktpapiere
M2 Konsumgütermarkt
LH1:
L4:
W2.4 Liquiditätsansprüche?
D1.9 Nachfrage nach langfr. Krediten
IB4.1
D3.7 Refinanzierungsnachfrage 2
M6.2 Refinanzierung
M4 Kapitalmarkt
D3.8 Angebot an langfr. Krediten
K2.5 Konsumgüternachfrage, Liquiditätsansprüche
D1.10 Investitionsgüternachfrage
IB3.2
M3 Investitionsgütermarkt
Z3.11 v1 := 0,005 + 0,005
Z3.17 v1 := v1 + 1.
D2.6 Aufteilung der liquiden Mittel
Z1.11 v2 := 0; v'2 := v'2 + 1.
L5:
D1.12 Nachfrage nach kurzfr. Krediten
IB5.1
D3.9 Angebot an kurzfr. Krediten
Z1.20 v6 := v6 + 1.
Z1.18 v2 := v2 + 1.
M5.1 Geldmarkt
L6:
K3.10 Refinanzierung, Kreditplanung
W1.13 Liquiditätsansprüche?
K3.16 Kreditplanung
W1.14 v6 < v̄6 ∧ v1 ≥ v̄1?
W1.15 v'2 + v2 = 1?
M5.2 1. Durchfinanzierung
W3.15 Liquiditätsanspruch?
K1.19 Produktionsplanung
W1.16 v2 < v̄2?
M5.3 2. Durchfinanzierung
K3.14 Offenmarktpapiere, Refinanzierung, Liquiditätsansprüche
M6.3 Refinanzierung
K1.17 Liquiditätsansprüche, Nachfr. n. kurzfr. Kred.
W3.13 v1 < v̄1?
K3.18 Refinanzierung
M6.4 Refinanzierung
L7:
W3.12 Liquiditätsansprüche?
W3.19 Mindestansprüche?
W3.20 Mindestreservedefizit?
M6.5 Durchfinanzierung
L71:
K3.21 Bargeldanspruch
L71:
I0.5 Periodenabschluß: Bewertung, Ermittlung definitorischer Größen, Statistik

designed to explain bargaining process and bargaining result by means of processes of adaptation of aspiration levels. Such potential aspiration levels may be recorded with the so-called *planning-report method.* By means of questionnaires the test persons are induced to concern themselves intensively with the forthcoming decision situation.

In the experiments concerning *labor market negotiations* between the parties of a wage agreement, for instance, questions regarding expectations about the development of the general economic situation were asked. Subsequent questions were concerned with the three bargaining variables: standard working hours, increase of the standard wage rate, and the period of notice, with the first of these questions regarding their order of importance. Next, subjects were asked for their *expectations* about plans (or better, aspiration levels) of the bargaining partner, that is to say about his first demand $\hat{D}$ and the respective conflict limit $\hat{L}$.

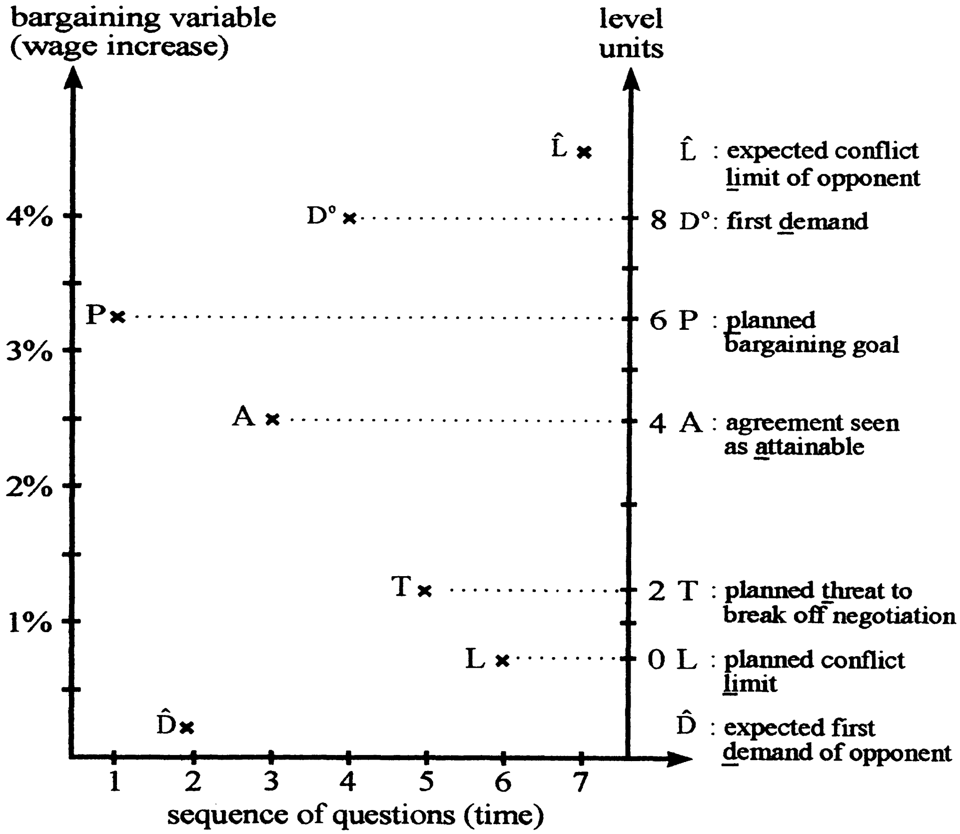


Figure 10: Planning Report and Aspiration Levels (Labor Union)

Figure 10 shows the aspiration values for one bargaining variable (left ordinate) in the respective order of preference and in the chronological order of questions (abscissa). Questions about expectations ($\hat{D}$ and $\hat{L}$) are followed by the question about the true aspiration level, namely the planned or target bargaining result P. The next lower potential aspiration level is derived from the question about the value regarded as attainable A. In a certain sense it represents the expected willingness-to-yield of the bargaining partner and is perhaps derived from an estimate of the opponent's P-value. The lowest aspiration level is the planned conflict limit L at which the bargaining is broken off. From these values more tactical variables can be derived, such as the planned threat to break off negotiations T - mostly situated between A and L - and finally one's own first demand D^o, which is the actual aspiration level in the beginning of the negotiation process.[14] *Marlies Klemisch-Ahlert* could show experimentally that the aspiration levels discussed here are of great importance before and during bargaining, even in situations in which no written planning report is asked for.[15]

Besides the original five aspiration levels described above, the midpoints of adjacent levels can become potential aspiration levels also. Values in the ten areas outside and between these nine aspiration levels are seen as respective equivalents. This results in a 19-level rating scale in *aspiration* or *level units* (right ordinate).

A comparison of the aspiration grids of both bargaining partners yields, e.g. by means of the *static aspiration balancing principle*, an agreement value or range at which both partners reach, as far as possible, equally high aspiration levels. The basically two-stage *"planning-difference theory"*, first developed with *Hans-Juergen Weber*, determines the degree of toughness of a negotiation according to the differences between the mutual aspiration levels P.[16] Then it is determined which partner has to concede first; we call this partner player 1. In hard bargaining situations the partner with the greater *concession reserve*, the difference between the first demand D^o and the level regarded as attainable A, expresses the greater willingness to concede and therefore is predicted to concede first. In more relaxed situations in which concessions below the value P are not necessary for an agreement, the so-called *"tacit concession"* gives a better explanation. It is the difference between the expected and the actual first demand of the opponent. The partner with the smaller tacit concession has to concede first, since the other one

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[14] Both of these values (T and D^o) also have the character of potential aspiration levels, but they may be varied more easily by tactical considerations.

[15] *M. Klemisch-Ahlert*, Bargaining in Economic and Ethical Environments - An experimental study and normative solution concepts, Berlin-Heidelberg-New York-London-Paris-Tokyo 1995, p. 20.

[16] *R. Tietz* and *H.-J. Weber*, On the Nature of the Bargaining Process in the KRESKO-Game, in: *H. Sauermann* (Ed.), Contributions to Experimental Economics, Volume 3, Tübingen 1972, pp. 305-334.

has implicitly made the greater concession in advance (as related to the planning phase). In that case the agreement occurs at the P-value of player 2, the player who does not concede first; in hard bargaining situations player 1 reaches his A-value unless the respective value of player 2 is better for him.[17] We also call attention to the case in which the two planned aspiration values P coincide. This situation can be interpreted as equilibrium, since according to both theories discussed above, agreement likewise coincides with the P-values.

Finally, by means of a third theory, the *dynamic aspiration-balance theory*, all individual steps of the bargaining process are represented. Here the important *aspiration-securing principle* is added. It compares the aspiration levels secured by the opponent's actual bid. Besides the compatibility test of the conflict limits, the aspiration-securing principle is the first so-called *decision filter* for the first concession (cf. *Figure 11*). The partner with the higher secured aspiration level has an *aspiration-securing advantage* measured in level units and therefore is predicted to concede first (I = 1, resp. J = 1).

Subsequently, the same filters as in the planning-difference theory, the concession reserves and the tacit concessions, are compared. If a filter does not lead to a decision by virtue of the equality of the compared variables, the next filter is tested. For completeness, the three filters are followed by a random filter which selects the first concession maker with equal probability.

In addition, the statements of the filters determine the *relative strength* of player 1. He is regarded as strong if a postpositioned filter contradicts the deciding filter; his strength in this case is rated with the value 3 (F0 = 3). If player 1 is selected only by the third filter, he has the moderate strength of 2 (F0 = 2). If the decision occurs by the first or second filter without contradictions, player 1 is regarded as weak, with a strength value of 1 (F0 = 1). This rating of strength influences the extent of the concessions of both players in the subsequent stage of the process of bargaining, which we will not discuss here in detail.[18]

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[17] *R. Tietz* and *H.-J. Weber*, Decision Behavior in Multivariable Negotiations, in: *H. Sauermann* (Ed.), Bargaining Behavior, Contributions to Experimental Economics, Volume 7, Tübingen 1978, pp. 60-87, esp. p. 66.

[18] Cf. e.g., *R. Tietz*, Der Anspruchsausgleich in experimentellen 2-Personen-Verhandlungen mit verbaler Kommunikation, in: *H. Brandstätter* and *H. Schuler* (Eds.): Entscheidungsprozesse in Gruppen, Beiheft 2 der Zeitschrift für Sozialpsychologie, Bern-Stuttgart-Wien 1976, pp. 123-141; reprinted in: *H. W. Crott* and *G. F. Müller* (Eds.): Wirtschafts- und Sozialpsychologie, Hamburg 1978, pp. 140-159; *R. Tietz*, Anspruchsanpassung und Verhandlungsverhalten, Frankfurter Arbeiten zur experimentellen Wirtschaftsforschung, Discussion Paper No. A 25, Frankfurt 1987.

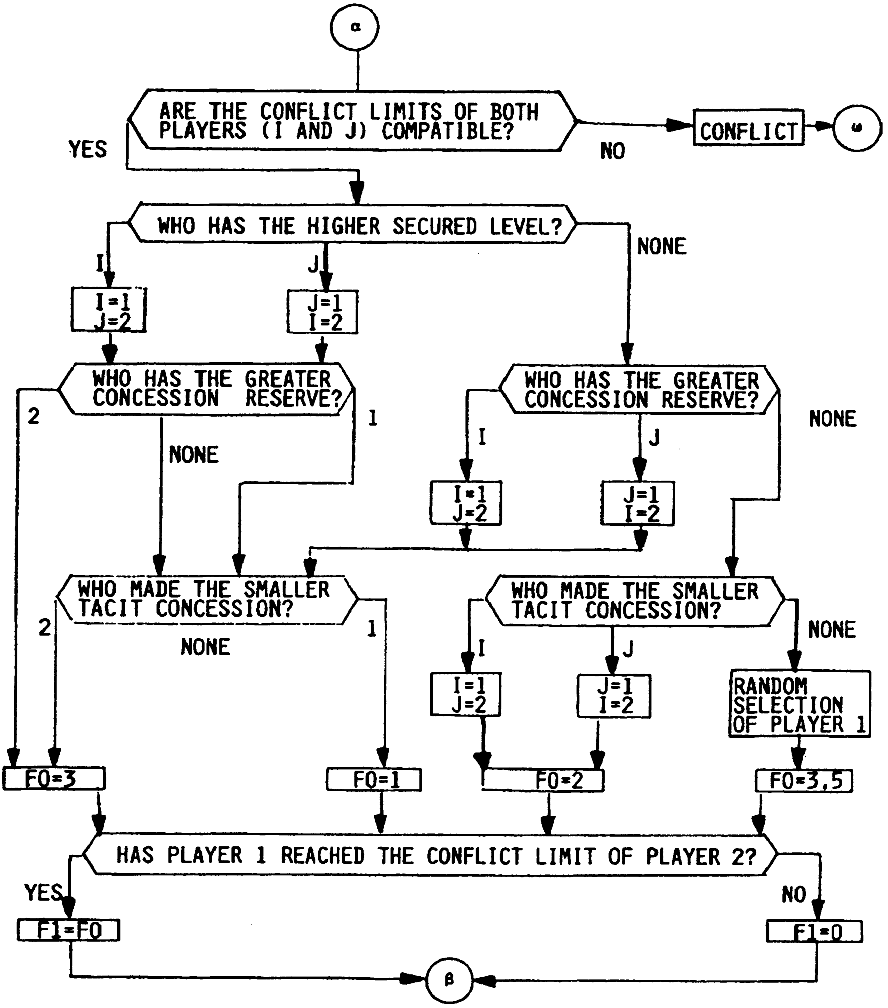

Figure 11: Who Makes the First Concession?

It is important to note that under certain conditions a weaker player 1 has to make comparatively greater concessions, whereas his partner can limit himself to smaller ones. In this manner the strength of the player contributes to an adverse effect of excessive demands. The aspiration-securing principle works in a similar way by modifying the principle of alternating concessions. It prevents concessions which clearly would reinforce an aspiration-securing disadvantage.

As could be shown by simulation studies, the dynamic aspiration-balance theory under *semi-normative aspects* has the desirable property of a *concave payoff*

function over the tactical variables first demand and conflict threat.[19] Therefore, aggressiveness pays only up to a certain degree. This means that the best replies lie in a moderate range, which also favors moderate equilibria. The theory possesses a certain *inner stability against itself.* Escalations are avoided. The theory can be used semi-normatively for recommendations. This characteristic is not shown in other, empirically comparable, bargaining theories, e.g. the one predicting an agreement at the middpoint between the first offers. *Reinhard Selten* has always demanded that a good bargaining theory should have this inner stability as a prerequisite, even under bounded rationality.

The dynamic aspiration-balance theory has, in a way similar to the two simpler theories, delivered good predictions for various experiments with a total of over 300 negotiations. Essential components of this theory were also used by *Jürgen Crößmann* in order to represent combined quantity and price negotiations.[20] The market experiment he used was originally developed by *Reinhard Selten* and represents a negotiation market with eligible partners.[21] The phenomenon of *demand inertia* observed by *Crößman* was previously detected by *Selten.*[22] It can also be characterized as *relation equilibrium* and was later explained by the principles of fulfillment and balancing of aspiration levels.[23]

Various concepts have effects on the *formation of aspirations*, meaning the adaptation of aspirations between negotiations.[24] Besides the *prominence principle* and changing conditions of the *experimental environment,* the *success* measured in aspiration levels explains only about half of the cases: *the aspiration level*

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[19] Cf. *R. Tietz, W. Daus, J. Lautsch,* and *P. Lotz,* Semi-Normative Properties of Bounded Rational Bargaining Theories, in: *R. Tietz, W. Albers,* and *R. Selten* (Eds.), Bounded Rational Behavior in Experimental Games and Markets, Berlin-Heidelberg-New York-London-Paris-Tokyo 1988, pp. 142-159; *R. Tietz,* Semi-normative Theories Based on Bounded Rationality, Journal of Economic Psychology, Volume 13 (1992), pp. 297-314.

[20] *H. J. Crößmann,* Entscheidungsverhalten auf unvollkommenen Märkten, Frankfurt 1982; *H. J. Crößmann* and *R. Tietz,* Market Behavior Based on Aspiration Levels, in. *R. Tietz* (Ed.), Aspiration Levels in Bargaining and Economic Decision Making, Berlin-Heidelberg-New York-London-Paris-Tokyo 1983, pp. 171-185.

[21] *R. Selten,* Ein Marktexperiment, in: *H. Sauermann* (Ed.), Contributions to Experimental Economics, Volume 2, Tübingen 1970, pp. 33-98.

[22] Ibid., pp. 70-72.

[23] *R. Tietz,* 1992, l.c.

[24] *H.-J. Weber,* Theory of Adaptation of Aspiration Levels in a Bilateral Decision Setting, Zeitschrift für die gesamte Staatswissenschaft, Volume 132, 1976, pp. 582-591; *H.-J. Weber,* Zur Theorie der Anspruchsanpassung in repetitiven Entscheidungssituationen, Zeitschrift für experimentelle und angewandte Psychologie, Volume 24, 1977, pp. 649-670; *R. Tietz, H.-J. Weber, U. Vidmajer,* and *C. Wentzel:* On Aspiration-Forming Behavior in Repetetive Negotiations, in: *H. Sauermann* (Ed.), Bargaining Behavior, Contributions to Experimental Economics, Volume 7, Tübingen 1978, pp. 88-102.

is raised after success, and lowered after failure.[25] But our investigations have also shown that in repetitive negotiations contrary behavior may be in the interest of cooperation and may favor relation equilibria. In this modification it is expedient for the more successful player to show more willingness to yield by *expanding his aspiration grid*, whereas the less successful player *compresses his aspiration grid*, thereby expressing less willingness to concede.

In *negotiations with two or more variables* the position of a variable in the order of importance influences the degree of differentiation of the respective aspiration grid and thereby also the number of concessions in this variable.[26] Likewise in these negotiations, the predictions computed by the above mentioned univariate theories show relatively high hit rates at least in one variable. The most successful theory here is the dynamic aspiration-balance theory since deviations lie outside a meaningful compensation area in only 15 % of the cases; this means that disadvantages for a player are cumulated across all variables in only a few cases.[27]

5. Final Remarks

I tried to show how the theory of adaptation of aspiration levels by *Sauermann and Selten* could be further developed and extended to bilateral and multilateral problems. The research program postulated by this theory is still not completed. On the basis of the realized experiments we have a certain knowledge of the boundedly rational decision behavior guided by aspiration levels. Some questions still await detailed investigation: for example, under which conditions in multivariate bargaining do the partners continuously switch between individual variables, and when are concessions made in bundles?[28]

Likewise the macroeconomic approach may be improved in the light of experimental results:[29] The aspiration-adaptation processes complementing the coordination by markets, such as between industry and credit banks, could be modeled explicitly as *bilateral negotiations* in accordance with the *dynamic aspiration-*

[25] Cf. e.g. *F. Hoppe*, Erfolg und Mißerfolg, Psychologische Forschung, Volume 14 (1931), pp. 1-62; *K. Lewin et al.* l.c.; *H. Heckhausen*, Motivanalyse der Anspruchssetzung, Psychologische Forschung, Volume 25 (1955), pp. 118-154.

[26] *R. Tietz* and *H-J. Weber*, 1978, l.c., esp., p. 76 f.

[27] Ibid. p. 82.

[28] Other questions are: How is the order of importance adapted? Is it possible to predict the forming of adaptations more precisely?

[29] Since the various bargaining experiments were performed after the completion of the macroeconomic model, we could not take into consideration the experimental results. One may ask, which features should be reflected - beside an international version - in a revision of the KRESKO-model, especially for educational purposes.

balance theory. This might perhaps make it possible to do without the rigid hierarchy of sectors.[30] Depending on the constellation of the aspiration grids, sometimes the industry, sometimes the banks would have to concede first. Perhaps, this change would reduce the tendency of the model to yield depression and increase the tendency for inflation under a loose monetary policy.

Of course, the *five aspiration levels* should then be formed explicitly in a more natural way. It is not sufficient to start from a high level which is adapted downwards by small proportional reductions. Some considerations in this direction: The desired and *planned bargaining result P* is the level that will, according to the bargaining theories, be reached by both partners if their plans coincide. This fulfillment of plans characterizes a state of equilibrium. On an equilibrium path, at which all expectations are fulfilled and therefore are extrapolated, the aspiration level P must be derived directly from the expectations. In an equilibrium, the aspiration level D^o, usually positioned higher, should converge to P and coincide with it at least in the long run. The difference between D^o and P, the *tactical reserve*, could depend, e.g., on the uncertainty about the economic situation, which may be derived from the difference between the short-run and the long-run trend estimations of the univariate expectation-forming procedure. Likewise, more natural points of reference could be found for the other three aspiration levels.[31] This modified concept of forming aspirations would presumably lead to more realistic behavior of the model.

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[30] The hierarchy of sectors is reflected especially in the pragmatic concept of counting variables.

[31] The "information bridges" often positioned before the bilateral relations (cf. "IB 5.1" in Fig. 9) could be used for forming the third aspiration level A, the value regarded as attainable. The conflict limit L could be derived from minimal requirements. For the conflict threat T the experience from earlier negotiations with the same partner could be used.

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A Model of Boundedly Rational Experienced Bargaining in Characteristic Function Games

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Abstract. The approach models the negotiations process in characteristic function games. The aim is to give a model of the behavior of idealized experienced players. They do not make mistakes, but they have all properties of a boundedly rational player that do not disappear with experience. The negotiation process and negotiation motives of such ideal players are modeled, with the idea to characterize the 'limit' of subjects' behavior who developed growing experience with a given game. – The approach models the phenomena prominence, structure of proposals, dominance, loyalty, attributed (revealed) demands, stepwise coalition formation, and foresight, and presents a solution concept for the negotiation process. Possible modifications and refinements are given. – A final section contains several examples, and comments on relations to Bargaining Set, Demand Equilibrium, and Equal Share Analysis.

1. The Model

The concept is given by a set of definitions, which model the negotiation process of experienced boundedly rational players. Basic elements of the approach are proposals, i.e. coalition structures, and corresponding payoff arrangements (1.1). Negotiation processes are sequences of proposals, each of which dominates the preceding one. The exactness of perception of numerical payoffs is restricted by a smallest perceived money unit. Accordingly, only such proposals are considered which can be generated on this level of exactness (1.2). Dominance is modeled in the usual way, the minimal improvement in a normal dominance is one smallest money unit. Loyalty is the phenomenon that 'similar' players spontaneously prefer to coalesce with one another, and do not leave a coalition with each other as easily as nonsimilar players. According to this phenomenon the usual definiton of dominance is modified (1.3). Every payoff, a given player received in an earlier stage of the negotiation process, generates the assumption that this player will thereafter enter a coalition only if she gets at least the same amount as before (attributed demands). These demands depend on the coalition structure (1.4). By the condition of attributed demands possible dominances are essentially reduced, all negotiation processes end after a finite number of steps. This permits rollback analysis of possible future dominances. Strategies on the space of possible negotiation chains are defined. Stable strategies are specified as

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solutions in a way which models the foresight of the players (1.5). An additional condition is given concerning the preference for agreements with high loyalty index (1.6) Negotiations on the strategy level are modeled in (1.7).

The model is based on the observations of more than 1000 experimental games. It addresses the (special) negotiation situation, where the formation of a coalition has to be announced to the experimenter, and becomes a final result when it has not been replaced by a new announcement within a certain time (usually 20-30 minutes). This procedure excludes ultimative elements, and forces agreements to reflect the power structure of the game, it has been introduced by SELTEN,SCHUSTER (1964). The model addresses that type of experience that is generated, when subjects play the same game repeatedly for several rounds with changing partners, and communication is allowed.

A problem of the development of the concept from the observed data was that experienced players do not show processes. They apply foresight, and immediately select stable states. Experiments with unexperienced players produce observable processes, but at least every second performed dominance is a mistake, which experienced players would not make, since they would apply foresight, and stop the process. Nevertheless, the observable processes admit insight into general patterns, motives, and the structure of actions. The results of games with experienced players permit to decide, whether the predictions of the model concerning the observable part of the negotiation process are correct.

As far as we know, there are no experimental results, that contradict the basic points of the model, although there are experimental results in which deviations can be observed for unexperienced subjects. Nevertheless, we expect that future experimental results will make it reasonable to refine the model in one and the other detail. Suggestions for such refinements are given in some of the remarks.

1.0 Basic Notations

A game is (N,v) is a set of players $N=\{1,...,n\}$, and a function v, which assigns a value v(C) to every coalition $C \subset N$, where $v(\emptyset)=0$, and $v(\{i\}=0$ for all $i \in N$. P(N) is the set of all subsets of N. Subsets of P(N) are denoted as **A**,**C**,**L**,**P**,.. For given **P**, $A \in \mathbf{P}$, $B \in \mathbf{P}$: $A \subset B$ denotes that A is a strict subset of B. $A \subset_1 B$ denotes that $A \subset B$ and there is no $C \in \mathbf{P}$ such that $A \subset C \subset B$. $\mathbf{P}_{\subset B} := \{A \in \mathbf{P}: A \subset B\}$, and $\mathbf{P}_{\subset_1 B} := \{A \in \mathbf{P}: A \subset_1 B\}$. #A is the number of elements of A. R is the set of real numbers. R^N is the real N-space. For $x \in R^N$: $x(S) := \Sigma(x(i): i \in S)$.

1.1 Proposals

Definition: $\mathbf{P} \subseteq P(N)$ is a **coalition structure**, iff (1) $\{i\} \in \mathbf{P}$ for all $i \in N$, and (2) $A,B \in \mathbf{P} \Rightarrow A \cap B = \emptyset$, or $A \subseteq B$, or $A \supseteq B$.

Notation: For p: P(N) $\rightarrow$ R^N, A$\subseteq$C$\subseteq$N let
pC:= p(C) ($\in R^N$) (i.e. the image of C under p),
$p^+C:=p^+(C):= \Sigma($ pA: A$\in$P(N), A$\subseteq$C) ($\in R^N$),
pC(A):= $\Sigma($ pC(i): i$\in$A),
$p_{tot}:= \Sigma($ pC: C$\subseteq$N).
For a characteristic function v: P(N) $\rightarrow$ R^N, and a given coalition structure **P**, C$\in$**P** let v'(C):= v(C) - $\Sigma($ v(A): A$\in \mathbf{P}_{C|C})$.

Definition: Let **P** a coalition structure, and p: P(N) $\rightarrow$ R^N. (**P**,p) is a **proposal**, iff for all C$\subseteq$N: (1) pC(C)=v'(C) for all C$\in$**P**, and (2) pC(i)=0 if i$\notin$C or C$\notin$**P**.

Coalition structures are not defined as partitions (as for instance in the bargaining set), but reflect the whole structure of possible subcoalitions as they can be formed in a process of stepwise coalition formation. – Proposals distribute for every formed coalition C its marginal value v'(C) (obtained by forming C) among the members of C. A proposal contains the information of all prior agreements of the process of coalition formation for all coalitions that have been formed, and not been broken afterwards.

1.2 Prominence

Definitions: The **prominent numbers** are PRO:= $\{a\cdot 10^i$: a$\in\{1,2,5\}$, i integer$\}$. – Σ (a(p)·p: p$\in$PRO) is called a **presentation** of a real number x, iff (1) x= Σ(a(p)·p: p$\in$PRO), and (2) all coefficients a(p) are +1,–1, or 0. – The **exactness of a presentation** is the smallest prominent number p of the presentation with coefficient a(p) unequal zero. – The **exactness of a number** x is the crudest exactness over all presentations of x. – A **number has prominence** d, iff its exactness is $\geq$ d.
A **proposal (P**,p) **has prominence** d iff for all D$\in$ **P**$\cup$\{N\} there is a coalition structure **Q**$\supseteq$**P** (i.e. **Q** finer than **P**), such that for all C$\in \mathbf{Q}_{C|D}$ the partition $\mathbf{Q}_{C|C}$ of C fulfills
(1) ('numerical prominence') pD(A) (or p^+D(A)) has prominence d for all but one A$\in \mathbf{Q}_{C|C}$, or
(2) ('equality') pD(A)=pD(B) for all A,B$\in$ $\mathbf{Q}_{C|C}$ (or $p^+D(A)=p^+D(B)$ for all A,B$\in \mathbf{Q}_{C|C}$).

The definition of prominent numbers and presentations follows ALBERS, ALBERS (1984), see also ALBERS (1996). Every (real) number can be presented as a sum of prominent numbers with coefficients 0,–1,+1 (where no prominent number is used more than once), for instance 16=20–5+1. This presentation need not be unique, for instance 3= 5–2=2+1. The exactness of a number is the smallest prominent number necessary for its presentation. If several presentations are possible, then that presentation defines the exactness, for which the exactness is maximal. Examples of presentations (exactnesses in brackets) are: 1=1(1), 2=2(2), 3=5–2(2), 4=5–1(1), 5=5(5),

6=5+1(1), 7=5+2(2), 8=10-2(2), 9=10-1(1), 10=10(10), 11=10+1(1), 12= 10+2(2), 13=10+5-2(2), 14=10+5-1(1), 15=10+5(5), 16=20-5+1(1).

A subject who 'operates with exactness 5' considers numbers as 5,10,15,.. i.e. the numbers with prominence 5 (exactness $\geq$5). To generalize the notation of prominence to proposals, three ways of generating a numerical payoff p(A) (or $p^+(A)$) for a coalition A are specified: (1a) to select p(A) as a prominent number, or (1b) to compute a remainder $v(C) - \Sigma(p(A)\colon A \in \mathbf{P}_{C|C})$ (where all p(A) are generated according to (1a)), or (2) to devide a joint payoff equally among the coalitions of a partition $\mathbf{P}_{C|C}$.

Assumption: In the following it is assumed that the prominence with which a game is played, is given and greater zero, and that all proposals have this prominence.

Remarks: 1. To avoid to be misunderstood: The prominence is not implemented by the experimenter, for instance by saying that only multiples of certain amounts are permitted as payoffs. The prominence structure of proposals is spontaneously selected by the subjects, similar as in the response on the question 'how many inhabitants has Cairo', which is usually given as one number with prominence 500.000. — 2. On the linearity of perception: The theory of prominence is developed as a theory of perception, and subconscious construction of numerical responses in individual decision situations. The result is that perception of differences has a logarithmic character: the relative, and not the absolute differences of numbers are perceived. For distributional situations as here, the problem is that a high outcome of one player (generating a low sensitivity for the payoffs of this player) is related to a low payoff of another player (generating a high sensitivity for her payoffs). Since usually all players attend the payoffs of all others, it seems reasonable to assume constant sensitivity over the whole range of reasonable alternatives. — 3. There is not yet a rule which predicts the prominence of a game. But experimental results indicate that the prominence is determined by the game, and the conditions of communication. As a rule of thumb we use: (a) for face to face communication the prominence is as fine as possible, but not finer than 5% of the maximal payoff, (b) for communication via terminals experienced players reach a prominence of 1, when the total payoff is 100.

A remark concerning the prediction of prominence: The exactness of a numerical response depends on the exactness of the stimulus. In certain situations, practitioners characterize the exactness of a signal by its 'range of reasonable alternatives', characterized by its extremes 'worst case', and 'best case'. (As a rule of thumb we use that worst case and best case are the cuts of the 10% tales, for measurable distributions.) The prominence can be predicted from the range of reasonable alternatives as follows:

Prominence Selection Rule: For a numerical stimulus with a given range of reasonable alternatives the prominence d of the response is se-

lected in a way that there is a set X of at least 3, and at most 5 numbers such that (1) x has prominence d (for all $x \in X$), (2) $y - x \geq d$ (for all $x < y \in X$), (3) X is maximal with (1),(2).

In the bargaining situations considered here, the extension of the range of reasonable alternatives seems mainly to be given by the inpreciseness of the predictions of reactions of others. Part of the inpreciseness is induced by the problem to predict loyalty (see below). It seems that for games with loyalty under certain conditions the range of reasonable alternatives coincides with the range of outcomes predicted by Equal Share Analysis, or Equal Division Bounds (SELTEN 1972,1982).

1.3 Loyalty, and Dominance

In the following it is assumed that the smallest money unit, d, also defines the minimal incentive under which a player (or a jointly acting group of players) changes a coalition. The incentives, under which a coalition is changed, essentially depend on the question, if a player belongs to, forms, or leaves a loyal group.

Definition: Let (**P**,p) a proposal, $C \in \mathbf{P} \cup \{N\}$, and $\mathbf{L} \subset \mathbf{P}_{\subset 1C})$. **L** is a **loyal group** of (**P**,p) below C, iff (1) ('symmetry') $pE(L)=pE(M)$ (or $p^+E(L)=p^+E(M)$) for all $E \in \mathbf{P}_{\supseteq C}$, $L,M \in \mathbf{L}$, and (2) ('maximality') **L** is a maximal subset of $\mathbf{P}_{\subset 1C})$ that fulfills (1).

Notation: For a given proposal (**P**,p) and $i \in N$ let $\mathbf{LL}_i(\mathbf{P},p)$ the union of all loyal groups that contain i.

Predominance rules which parts of an old proposal are distroyed, and which are kept, when a new coalition is formed. Dominance in addition defines which incentives are necessary that the players agree to perform the dominance.

Definition: A proposal (**Q**,q) **predominates** a proposal (**P**,p) via a coalition D, iff
(1) $\mathbf{Q} = \{A \in \mathbf{P}: A \subset D\} \cup \{A \in \mathbf{P}: A \cap D = \emptyset\} \cup \{D\}$, and
(2) $qA = pA$ for all $A \in \mathbf{P} \cap \mathbf{Q}$.

Definition: Let (**Q**,q) predominate (**P**,p) via D, and for every $i \in D$ let $\mathbf{L}_i$ the unique loyal group of (**Q**,q) below D that contains i. (**Q**,q) **dominates** (**P**,p) via D [notation: $(\mathbf{P},p) \rightarrow D\ (\mathbf{Q},q)$], iff for all $i \in D$: (1) $q_{tot}(i) \geq p_{tot}(i)$, and (2):
(a) ('deletion of a loyal group') if there is $\mathbf{L} \in \mathbf{LL}_i(\mathbf{P},p)$ such that $\#\mathbf{L} > 1$, and $D \cap (\cup \mathbf{L}) \subset \cup \mathbf{L}$, then
$q_{tot}(\cup \mathbf{L}_i) - p_{tot}(\cup \mathbf{L}_i) \geq d + l \cdot d$, or $q_{tot}(\cup \mathbf{L}_i)/p_{tot}(\cup \mathbf{L}_i) > 5/3$ (where l denotes the degree of loyalty), (b) ('extension of loyal groups') otherwise, if $\mathbf{LL}_i(\mathbf{P},p) \subset \mathbf{LL}_i(\mathbf{Q},q)$ then $q_{tot}(i) - p_{tot}(i) \geq 0$,
(c) ('neither deletion nor extension') otherwise
$q_{tot}(\cup \mathbf{L}_i) - p_{tot}(\cup \mathbf{L}_i) \geq d$, or $q_{tot}(\cup \mathbf{L}_i)/p_{tot}(\cup \mathbf{L}_i) \geq 5/3$.

Ad (a): The degree of loyalty, l, depends on the conditions of communication. Under free communication, the degree of loyalty is l=1 (sometimes l=2). When subjects communicate via terminals, then there is usually no loyalty (l=0).

A loyal group **L** is a maximal set of subcoalitions of a larger coalition, where each subcoalition receives the same payoff. The larger coalition may be formed recently, or in some former dominance of the process of coalition formation. The implicit assumption of the model is, that the symmetry among players (or groups of players), given by the fact that they receive equal payoffs, creates a feeling of solidarity which causes them not to leave a coalition with one another, except for substantial additional payoffs. (Compared to former definitions of a loyal group, we here omitted the condition that the game must be symmetric with respect to the coalitions of **L**.)

A new coalition is formed by a dominance (**P**,p) →D (**Q**,q). Coalitions of the preceding proposal are broken, if they intersect D. Old coalitions which are subsets of D, or do not intersect D, are kept, their payoff agreements remain unchanged (see the definition of a predominance). Thereby a proposal keeps track of those coalitions and agreements which have not been broken in later dominances.

Every member of a new coalition D is assumed to agree to form D only, when she thereby improves her payoff by at least d. If she is in a coalition L of a loyal group, then it is sufficient that the sum of the improvements of the members of L add up to at least d (condition (c)). If in (**Q**,q) a new loyal group is generated, or an old loyal group is extended, and this group includes i, then it is sufficient, that the payoff of i does not decrease (condition (b)). A player leaves a loyal group only, if she improves her payoff by an extra amount l·d, which may be interpreted as the amount necessary to corrupt the player to betray her partners of the old loyal group. This amount has to be paid to every player of the old loyal group, who enters D. In addition, she needs the 'regular' incentive d. to change the coalition, as in the normal case (condition (a)). For low payoffs, the additive improvement d can be replaced by a relative improvement of 5/3. This relative improvement is also sufficient, to compensate the bad feeling connected with leaving partners of a loyal group.

Remarks: 1. The described phenomenon of loyalty has been verified in several experiments (see for instance esamples 2.5-2.7). A detailed experimental analysis on loyalty in Apex Games is given in HAVENITH (1991). — 2. Experimental results indicate that the amount of loyalty, l·d, and the prominence of bargaining are of about the same size. It seems that, in situations with positive loyalty, the amount of loyalty can only be roughly predicted. According to the diffuseness of the prediction of loyalty, all predictions of future moves become unprecise. However, future moves can be analyzed in a nonprobabilistic way, if the prominence of analysis is made sufficiently crude. Accordingly, the prominence should be at least as large as the amount of

loyalty, l·d. — 3. In the approach here, loyalty is generated by symmetry of the players with respect to payoffs. A question is, whether players can feel loyalty even in situations, where their payoffs are not quite equal, or when their payoffs are given by some other rule of fairness. — 4. The model predicts that players switch into a socially desired coalition even, if they thereby do not increase their payoff. This observation has been made in games with free communication (face to face, and via terminals). We are not sure, if the same kind of pattern establishes, when communication is restricted to very formal pieces of information. — 6. That a relative improvement of payoff greater than 5/3 is evaluated as essential, seems to be related to the structure of 'spontaneous numbers'. The spontaneous numbers are $\{\epsilon \cdot a \cdot 10^i: \epsilon \in \{-1,+1\}, a \in \{1,1.5,2,3,5,7\}, i \text{ integer}\}$. Under 'ordinary' conditions, they seem to be the finest perceived numerical stimuli (see ALBERS 1996). x=5/3 is the minimal number with the property that any two spontaneous numbers a>b are necessarily more than one step apart, if a/b>x.

1.4 Negotiation Chains, and Attributed Demands

Definition: A **negotiation chain** is a sequence of proposals $(\mathbf{P}_0,p_0)$ $\rightarrow D_1$ ... $\rightarrow D_r$ $(\mathbf{P}_r,p_r)$ of which each dominates the preceding one, and which starts with $\mathbf{P}_0=\{\{1\},...,\{n\}\}$, $p_0\equiv 0$.

Notation: Two situations $[\rightarrow D\ (\mathbf{P},p),A]$, $A\in\mathbf{P}_{\subseteq D}$, and $[\rightarrow D'\ (\mathbf{P}',p'),A']$, $A'\in\mathbf{P}'_{\subseteq D'}$ are **comparable**, iff there is a permutation π of the set of players, such that (1) $v(\pi S)=v(S)$ for all $S\subset N$, and which (2) maps $[D,(\mathbf{P},p),A]$ to $[D',(\mathbf{P}',p'),A']$, and (3) maps the loyal groups of $(\mathbf{P},p)$ below D to the loyal groups of $(\mathbf{P}',p')$ below D'. (Note that p and p' are only used to get the loyal groups.)

Notation: Every dominance .. $\rightarrow D$ $(\mathbf{P},p)$ defines an **attributed demand** $\alpha([D,(\mathbf{P},p),A]= q_{tot}D(A)$ for every coalition $A\in\mathbf{P}_{\subset D}$.

Definition: A negotiation chain fulfills the **attributed demands condition**, iff the attributed demands of comparable situations are not less for later than for earlier proposals.

A negotiation chain describes the complete history of a process of coalition formation. In every step of the process one coalition is formed according to the conditions of dominance. The respective last proposal of a negotiation chain gives a protocol of all agreements that have not been cancelled by later dominances.

It is assumed that players, who have to decide whether to perform a next dominance use the demands others made during the preceding part of the process, to predict, whether these others will be content with certain payoffs in a new coalition, or whether they will leave the coalition later to improve their payoff. It seems to be reasonable to assume that they will be content with a given amount x(A), if they did not leave 'comparable' previous coalitions, where they received the same amount x(A) (or less than

that). It seems also reasonable to assume that they will leave a coalition, in which they get x(A), in a future dominance, if they left another coalition in a comparable situation, where they received the same payoff x(A) (or more). Moreover, players assume that a coplayer's demand is the same, in symmetric situations (generated by a permutation of the set of players). For a group A getting a share from the joint payoff of a coalition D a situation is defined to be comparable, if the maximal coalitions outside D, the maximal coalitions inside D, that do not cut A, and the corresponding payoffs, are the same, and if the loyal groups below the coalition D formed in the dominance, are identical.

Remarks: 1. An interesting question is, who generates these demands. It seems that in experimental situations it is rather the others who deduce reasonable demands from the behavior, than that oneself has the demand. It is possible, that the central issue of the learning process happening in subjects playing a given game (repeatedly) is not that much the logic of the procedure, but to learn the adequate demands of the players in different situations. — 2. It even happens, that subjects transfer ideas of adequate demands from one game (they played repeatedly) to a similar but different next one. — 3. Attributed demands, as modeled here, do not permit that subjects reduce their demand during the bargaining process. Accordingly, the concept can only model situations, where the adequate demands have already been learned. (It may be remarked that subjects apply principles of attributed demands also during the learning phase, but in a modified way: they forgive a too high demand, as long as the demand setting player did not actively change the coalition to improve her payoff.) — 4. It is not clear, whether attributed demands can also be transferred (by permutation of players) to situations that are less symmetric. Conditions of 'tolerated' asymmetry have to be explored. They may depend on the number of players. — 5. There is some experimental evidence for the attributed demand condition. As an example we mention spatial games which were arranged by a strongly formalized nonverbal procedure. Coalitions which violated the attributed demand condition were in more than 90% of the cases thereafter broken by the player whose attributed demand was not satisfied. This happened only in the games of the first round. In games of later rounds, coalitions were only entered, when the attributed demands of all members were fulfilled.

A remark concerning the linearity of negotiation chains: An interesting question is, if negotiation processes do really follow a linear structure as modeled by a sequence of dominations. Another scenario might be for instance, that players build up a tree-shaped structure of alternatives, where they negotiate through subtrees, redecide early moves in the tree, etc. Experimental results support the following model: Proposals have the character to suggest 1. coalitions, or 2. payoffs, or 3. payoffs in coalitions. Define a proposal to be complete, if it gives a coalition and a payoff distribution within the coalition. And define a complete proposal to be essential, if it is dominated by

the very next proposal. The following statements can be made: (1) Every complete proposal dominates the last essential proposal. (2) The essential proposals define a chain of which each dominates the preceding one. Experimental results support that (1) is correct in more than 95% of the complete proposals, and that (2) is correct in more than 90% of the essential proposals. This strongly suggests, that the assumption of a mainly linear structure of negotiation chains is reasonable. (But the approach here permits to model different structures as well.)

1.5 Strategies and Solution Concept

An important consequence which follows from prominence structure and attributed demands condition, is that all negotiation chains have finite length (which can be estimated by a universal upper bound). This permits roll back analysis of future moves, and keeps the complexity of strategic analysis within certain bounds.

Definition: For every game (N,v) the corresponding **negotiation game** is defined as a generalized extensive game, with a tree-shaped ordered set of states, and a payoff function, which assigns a payoff to every maximal state: The **nonmaximal states** are the negotiation chains that fulfill the attributed demand condition. They are ordered in a natural way by 'inclusion': $X<Y$ iff there is $r \in N$ such that X consists of the first r dominances of Y. ($X<_1Y$ denotes that there is no state Z such that $X<Z<Y$.) – For every nonmaximal state X (= negotiation chain), with a last proposal (**P**,p), there is a corresponding **maximal state** $X^*>_1X$ defined as $X^*:= (X \rightarrow (\mathbf{P},p))$. The **payoff** in X^* is defined by p_{tot}. – The unique minimal state is is $(\mathbf{P}_0,p_0):= (\{\{1\},...,\{n\}\}$.

Definition: A **strategy** S_i of a Player i is a set of states, such that for every nonmaximal state X there is exactly one state $Y>_1X$ in S_i (i.e. S_i assigns to every nonmaximal state X a coalition D and a proposal (**P**,p), and thereby a next state $Y=(X \rightarrow D\ (\mathbf{P},p))$). – If for a given nonmaximal state X there is a unique state $Y>_1X$, $Y=(X\rightarrow D\ (\mathbf{P},p))$, such that Y is contained in the strategies S_i of all $i \in D$, then the **move** $X \rightarrow Y$ is performed. If there is no such Y (including the case that Y is not unique), then the move $X \rightarrow X^*$ is performed. – Every n-tuple $S= (S_1,...,S_n)$ of strategies defines a **path** path(S), i.e. a unique sequence of moves through the game tree starting in the unique minimal state, and ending in a maximal state. The payoff of this maximal state defines the **payoff** of S. – The payoff of the restriction of an n-tuple of strategies to a '**subgame after** X' is the payoff in the endpoint of the unique path path($S_{>X}$ starting in X, and defined by the strategy.

Definition: An n-tuple of strategies is **stable**, if for no state X there is a coalition $C \subset N$ which can increase the payoff of all of its members in the subgame above X by changing their strategies in their decisions

Y>$_i$X in such a way, that the sequence (old follower after X) →C (new follower after X) fulfills the conditions of dominance. — A proposal (**P**,p) is **stable**, iff there is a stable strategy such that its path ends with a state X*, of which (**P**,p) is the last proposal.

The solution concept works on the tree-shaped ordered set of possible histories, which define the states (= negotiation chains). Thereby every subdecision of a strategy in every given state depends on the special history in which the state is reached. Strategies are stable if there is no sufficient motive to change them, when they are once established (by a social norm or joint agreement). Examples of stable proposals are (80,40,–,–,–), (–,30,30,30,30), and (24,24,24,24,24) in the Apex Game with payoff 120 (see example 2.5). If Player 2 decides between these alternatives by her payoff, she will prefer (80,40,–,–,–). The condition of maximal loyalty (see 1.6) selects (24,24,24,24,24). There are also reasons to select (–,30,30,30,30).

Remarks: 1. We suggest, that every characteristic function game with finitely many players has at least one stable strategy, and that on the path of a stable strategy no coalition is 'broken'. — 2. The approach does not permit that two disjoint coalitions are formed in the same move. This makes sense, since thereby the players can only support a dominance into a state they know in advance. However, there are also games, for which it makes sense to support a dominance independently from decision of others to form a coalition at the same time. In the model here, two coalitions can only be formed in two steps.

The concept as an instrument to decide about stability: As long as subjects are still learning, players do not assume that every payoff agreed upon in an essential proposal really settles an unreducable demand. The 3-person game (n=3, v(1,2)=v(1,3)=100, v(S)=0 otherwise) may serve as an example. Here the following chain of arguments may happen within an experimental game (with 5 as smallest money unit): (50,50,–) → (55,–,45) → (60,40,–) → ... → (90,10,–) → (95,–,5) (and thereafter → (100,0,–) → (–,0,0) and no further dominance possible). There is no reason for Player 1, not to follow this chain up to (95,–,5). However, the solution concept stops the chain after (50,50,–) → (55,–,45) with the argument, that Player 2 cannot enter (60,40,–), since he revealed a demand of 50 before. Neither the total sequence, nor the chain (50,50,–) → (55,–,45) is stable. Stable states consist of only one proposal (except for ($\mathbf{P}_o$,p_o)), namely (95,5,–), or (95,–,5). – The example shows that the solution concept is an instrument to decide about stability, or to predict negotiation chains of experienced players, but not a tool that describes the learning process of unexperienced players.

1.6 Preference of States with Higher Loyalty-Index

Notation: Let (**P**,p) →D (**Q**,q), and i∈D. The **loyalty-index** of this dominance for a player i∈D is zero, if there is a loyal group of (**P**,p) which

is not loyal group of (**Q**,q). Otherwise the loyalty index of the dominance is given by the number of sets of the unique loyal group of (**Q**,q) below D that contains i.

Definition: A stable strategy S is **loyalty-maximal**, iff it is loyalty-maximal in all states X; it is loyalty-maximal in a state X, iff (1) it is loyalty-maximal in all states Y>X, and (2) for every stable strategy T which is loyalty-maximal for all Y>X, holds:
if (**P**,p) is the maximal element of X, (**P**,p) →D (**Q**,q) the move after X selected by S, (**P**,p) →D (**Q**',q') the move after X selected by T, then either (a) there is a player i∈D∩D' for whom the loyalty index of (**P**,p) →D (**Q**,q) is greater than the loyalty index of (**P**,p) →D (**Q**',q'), or (b) the loyalty indices of the two dominances are equal for all i∈D∩D'.

It is assumed that - whenever coalitions unite - subjects prefer arrangements, where the uniting coalitions share equally, as long as these alternatives generate solution strategies. Moreover, in every state X they prefer such a stable strategy which brings them (or their group) into a loyal group with a maximal number of coalitions. This principle selects among the stable strategies. The idea is that - as long as there are no different forces - the precommitment of the subjects is, to select a coalition with maximal loyalty index, and that they deviate from this only, when the obtained state is not stable.

Remark: Up to now there have been only very few experiments, that address the condition of maximal loyalty. The approach here is guided by experimental observations (in characteristic function and spatial games) which strongly indicate that - if possible - players like to enter states with high loyalty index. Moreover, it seems to be true that higher payoffs cannot compensate a lower loyalty index, as long as both alternatives are stable. (See for instance the Apex Game with payoff 120 in example 2.5.)

1.7 Negotiations on the Strategy-Level

On a higher level of reflection, players negotiate about possible strategies in the negotiation game. Accordingly, an n-tuple of strategies may not only be changed in one move above one state X by one coalition, but for several moves, by possibly different coalitions.

Notation: For any state X, and any strategy S let ($\mathbf{P}_X$,p_X) the last proposal of X, and ($\mathbf{P}_{S>X}$,$p_{S>X}$) the last proposal of path($S_{>X}$).

Definition: An n-tuple of strategies T=(T_1,...,T_n) **dominates** an n-tuple of strategies S=(S_1,...,S_n), iff for every state X with $\{Z\} = T_{>1X} \neq \{Y\} = S_{>1X}$, Z≠X*, and the coalition D with X →D Z holds $(p_{T>X})_{tot}(A) \geq (p_{S>X})_{tot}(A)$ for all A∈$(\mathbf{P}_Z)_{CD}$.

Notation: Every dominance of states X →D Y with Y≠X defines attributed demands via the corresponding dominance ($\mathbf{P}_X$,p_X →D ($\mathbf{P}_Y$,p_Y) (see section 1.4). — Every n-tuple of strategies S defines for every state X a

follower Y, and thereby an attributed demands via the corresponding dominance $X \to_D Y$, $\{Y\}=S_{>1X}$ (if $Y \neq X^*$). — Every dominance of n-tuples of strategies $S \to T$ defines attributed demands for every state X with $\{Z\}=T_{>1X} \neq \{Y\}=S_{>1X}$, $Z \neq X^*$, and the coalition D which enacts the dominance $X \to_D Z$, namely $\alpha[D,(\mathbf{P}_Z,p_Z),A]= (p_{T>X})_{tot}(A)$ for all $A \in (\mathbf{P}_Z)_{CD}$.

Definition: A sequence $S^1,...,S^r$ of n-tuples of strategies (of which each dominates the preceding one) fulfills the **attributed demands condition**, iff (1) every strategy S^k fulfills the attributed demands condition, and (2) the attributed demands of S^k are not less than the attributed demands of the comparable dominances of all S^l, $l<k$.

We restrict the considerations to sequences of dominating strategies, that fulfill the attributed demands condition. In the same way, as for the sequences of dominating proposals (in section 1.5), we now consider sequences of strategies as negotiation chains, define such sequences as states (of higher order), obtain moves, and strategies (of higher order). Undominated strategies aredefined as stable. – For a given game (N,v), the obtained game is called the negotiation game on the strategy level, or shortly the **strategy game** of (N,v).

When a player changes her strategy above a state X, she needs not meet her attributed demands immediately in the first move after X, but in the last of move of the path of the new strategy after X. This implies a kind of foresight on the strategy level. Nevertheless every dominance on every path($S_{>X}$) defines attributed demands.

In the new approach, too high demands of dominated strategies of the bargaining history can be punished. Thereby a kind of force is implemented that drives the game to a solution. (In the approach of section 1.5, strategies were simply replaced by new ones without generating attributed demands, the 'force of attributed demands' was not implemented on the strategy level.)

Remarks: 1. The analysis via the strategy game seems necessary for games, in which nested coalitions are formed. — 2. Solutions of the strategy game seem to select such sequences of strategies which have length 1, i.e. contain only one strategy. Moreover, we suppose that the path of such a strategy is a sequence of proposals, in which no coalition is broken. — 3. The approach implicitly models the blocking power of coalitions, and does not need to replace the characteristic function of the game by a power function (see MASCHLER 1963, or THRALL,LUCAS 1963).

2. Solutions of Selected Games

In the following, solutions of some selected games are presented. The corresponding main arguments for the stability of the results are sketched by selected negotiation chains, which present the essential ideas of the corresponding proofs. It may be remarked, that for most of these games, the

negotiation chains of the essential ideas can be joined to one large sequence. This makes it reasonable, that subjects can learn the reasoning in limited time.

2.1 The 3-person game with quotas 80,40,40: The Loyalty Paradox

Game: N={1,2,3}, v(1,2)=v(1,3)=120, v(2,3)=80, v(S)=0 otherwise, exactness 10, loyalty 10. – Stable: (70,50,–), (70,–,50), (–,40,40). – Arguments for the stability of (70,50,–),(–,40,40), instability of (80,40,–): ('—' denotes that, using foresight, this player will not perform the dominance. '———' denotes that no further dominance is possible):

70 50 --	70 50 --	80 40 --	-- 40 40
--	--	-- 40 40	--
80 -- 40	-- 60 20	--------	60 60 --
-- 40 40	70 -- 50		70 -- 50
--------	--------		--------

Selecting between the stable alternatives (70,50,–),(70,–,50),(–,40,40), Players 2 and 3 prefer (70,50,–), (70,–,50) by payoff, while (–,40,40) is loyalty maximal.

Modification: If in the same game the loyalty is 0, then (–,40,40) is no longer stable, since (-,40,40) → (70,50,-), and (70,50,-) is stable.

Remarks: The modified example shows the 'loyalty paradox', namely that by the incentives of loyalty, (70,50,–) becomes stable, while (–,40,40) is outruled. Experimental data support this phenomenon (see ALBERS,CROTT,MURNIGHAN 1985).

2.2 The 3-Person Game with Quotas 80,80,40: Loyalty Paradox, and Results Outside the Range Predicted by Equal Share Analysis

Game: N={1,2,3}, v(1,2)=160, v(1,3)=v(2,3)=120, v(S)=0 otherwise, exactness 10, loyalty 10. – Stable: (90,–,30), (–,90,30), (80,80,–). – Arguments for the stability of (70,50,–), (–,40,40), instability of (80,–,40):

90 -- 30	90 -- 30	80 -- 40	80 80 --
--	--	80 80 --	--
100 60 --	-- 80 40	--------	100 -- 20
-- 90 30	80 80 --		-- 90 30
--------	--------		--------

Modification: If the same game is played with exactness 1, loyalty 0, then (81,–,39), (–,81,39) are stable. – Arguments for the stability of (81,39,–), instability of (80,–,40), (80,80,–):

```
81 -- 39   81 -- 39   80 -- 40   80 80 --
--               --   80 80 --   81 -- 39
82 78 --   -- 80 40   --------         --
-- 81 39   80 80 --              -- 80 40
--------   --------              80 80 --
                                 --------
```

Remarks: The example shows, that the outcome of a game can be outside the range predicted by equal share analysis. – The predictions of the modification are supported by experiments via terminals with experienced players.

2.3 The 3-Person Game with Quotas 100,0,0: An Example of Blocking

Game: N={1,2,3}, v(1,2)=v(1,3)=100, v(S)=0 otherwise, exactness 5, loyalty arbitrary. – Stable: (95, 5,–), (95,–, 5). – Arguments for the stability of (95, 5,–), instability of (100, 0,–):

```
 95  5 --   100  0 --
 --          --  0  0
100 --  0   --------
 --  0  0
--------
```

Modification: If v(N)=100, then coalition 2,3 gets blocking power. Stable are (75,25,–), (75,–,25), and, in addition, (50,25,25) on the strategy level. – Arguments for the stability of (75,25,–), (50,25,25) and instability of (80,20,–), (70,30,–):

```
75 25 --   80 20 --   70 30 --   (--  0  0
--         (--  0  0  75 -- 25   (50 25 25
80 -- 20   (50 25 25  --------         --
(--  0  0  --------              65 35 --
(50 25 25                        70 -- 30
--------                         --------
```

(50,25,25) is no solution, of the negotiation game, since (–,0,0) is dominated by (75,25,–). It is a solution of the strategy game, since (–,0,0) $\rightarrow$ (50,25,25) can only be dominated by giving Player 1 less than 75, what is dominated thereafter.

Remarks: The example demonstrates the dominance of strategies. – The preference of (75,25,–) and (75,–,25) in the modified game is supported by experiments of MURNIGHAN,ROTH (1977).

2.4 A Simple Symmetric 4-Person Game: Blocking, and Instability of (40,40,40,–)

Game: N=(1,2,3,4), v(S)=120 if #S=3, v(S)=0 otherwise, exactness 10, loyalty 0. – Stable (up to symmetry): (55,55,10,–), (57,57,5,–). – Arguments for

the stability of (55,55,10,–), instability of (60,60,0,–), (40,40,40,–) (using the maximal loyalty condition of 1.6):

```
55 55 10 --    55 55 10 --    60 60  0 --    40 40 40 --
-- --          --             (-- --  0  0   55 55 -- 10
60 60 --  0    70 -- 25 25    (10 -- 55 55   (stable,s.a.)
(-- --  0  0   -- 40 40 40           -- --
(10 -- 55 55   -----------    --  0 60 60
       -- --                  ( 0  0 -- --
--  0 60 60                   (40 40 -- 40
( 0  0 -- --                  -----------
(40 40 40 --
-----------
```

Modification: If the same game is played with v(N)=120 (instead of v(N)=0), then the 2-person coalitions get blocking power. – Stable (up to symmetry): (45,45,30,–). – Arguments for the stability of (45,45,30,–), instability of (40,40,40,–):

```
45 45 30 --    45 45 30 --    40 40 40 --
-- --          --             45 45 -- 30
50 50 -- 20    60 -- 30 30    (stable,s.a.)
-- --  0  0    -- 40 40 40
30 30 30 30    -----------
-----------
```

Remarks: The example shows that the solutions can be essentially different from the predictions of the demand equilibrium and equal share analysis (both predict (40,40,40,–), ..., (–,40,40,40)). – The predictions concerning the modified game are supported by experimental results.

2.5 The 5-Person Apex-Games with Payoffs 100, and 120: Relevance of the Numerical Payoff

Game: N=(1,2,3,4,5), v(S)=100 if (1∈S and #S>1) or if S=(2,3,4,5), v(S)=0 otherwise, exactness 5, loyalty 5. – Stable: (65,35,–,–,–), ..., (65,–,–,–,35). – Arguments for the stability of (65,35,–,–,–), instability of (70,30,–,–,–), (60,40,–,–,–), (–,25,25,25,25), (20,20,20,20,20):

```
65 35 -- -- --    65 35 -- -- --    70 30 -- -- --    -- 25 25 25 25
--                --                -- 35 22 22 22    65 35 -- -- --
70 -- 30 -- --    -- 40 20 20 20    --------------    (stable,s.a.)
-- 22 35 22 22    65 -- 35 -- --
--------------    --                60 40 -- -- --
                  70 -- -- 30 --    65 -- 35 -- --    20 20 20 20 20
                  -- 22 22 35 22    (stable,s.a.)     65 35 -- -- --
                  --------------                      (stable,s.a.)
```

Modification: If the same game is played with payoff 120 (instead of 100), the subjects spontaneously select exactness 10, loyalty 10. – Stable are (up to symmetry) (80,40,–,–,–), (–,30,30,30,30), (–,40,27,27,27), (24,24,24,24,24).

– Solutions are of type (80,40,–,–,–). – The main arguments for the stability of (80,40,–,–,–), (–,30,30,30,30), (–,40,27,27,27), and the instability of (–,30,30,30,30) are:

```
80 40 -- -- --      80 40 -- -- --      90 30 -- -- --      -- 30 30 30 30
--                  --                  -- 40 27 27 27      --
90 -- 30 -- --      -- 50 23 23 23      --------------      70 50 -- -- --
-- 27 40 27 27      80 -- 40 -- --                          80 -- 40 -- --
--------------      --                  70 50 -- -- --      (stable,s.a.)
                    90 -- -- 30 --      80 -- 40 -- --
                    -- 27 27 40 27      (stable,s.a.)
                    --------------

-- 40 27 27 27      -- 40 27 27 27      -- 50 23 23 23      24 24 24 24 24
   --                  --               80 -- 40 -- --      --
70 -- 50 -- --      70 50 -- -- --      (stable,s.a.)       70 50 -- -- --
80 -- -- 40 --      80 -- 40 -- --                          80 -- 40 -- --
(stable,s.a.)       (stable,s.a.)                           (stable,s.a.)
```

Remarks: The example shows, that the result of a game can essentially depend on the numerical structure generated by payoff, prominence, and loyalty. - Experiments showed surprising differences in the results of the two games (the first game was played with payoff 1000, prominence of 100 and loyalty 100, this is structurally equal to the game with payoff 100, prominence 10 and loyalty 10):

type of result	(x,y,-,-,-)	(-,y,z,z,z)	(-,a,a,a,a)	(b,b,b,b,b)
v(N)=1000	19 (13)	3 (2)	2 (-)	- (-)
v(N)=120	3 (-)	1 (-)	1 (-)	18 (13)

Table 2.5: Frequencies of final results over all rounds (rounds 3-5)

The Apex Game with payoff 100 has been played on the Jerusalem Conference (1964) by game theoreticians. The result was (65,35) in all but one cases. The deviating result (62.5,37.5) was proposed by a player who proportionally transformed his experimental result (30,10) from the game with payoff 40.

2.6 Stability of (–,40,20,20,20) after (70,30,–,–,–) in the Apex Game

A crucial point of the new approach is the instability of (70,30,–,–,–). The approach here says that (70,30,–,–,–) $\rightarrow$ (–,40,20,20,20), and that thereafter no further dominance is possible, by loyalty of 3,4,5, and by the fact that Player 1 revealed a demand of 70. The Bargaining Set approach says that the argument is countered by (70,30,–,–,–) $\rightarrow$ (–,40,20,20,20) $\rightarrow$ (70,–,30,–,–).

Therefore we ran an experiment with confederates in the position of Player 2, who had the following orders: step 1: immediately contact Player 1, and form a coalition with payoffs (70,30,–,–,–), step 2: leave this coalition, and enter (–,40,20,20,20), step 3: be quiet, and wait for a 20 minutes time. The

prediction of our concept is, that (–,40,20,20,20) remains stable. – The result was as predicted in 12 of 12 cases.

2.7 Stability of (70,30,–,–,–) in the Apex Game, if (2,3,4,5) Can only Split Equally

The result above can be reformulated as '(70,30,–,–,–) is not stable for the Apex Game with payoff 100'. The crucial point for the instability is, that it is dominated by the uneven split (–,40,20,20,20). But what happens, if only even splits are permitted for coalition (2,3,4,5)? The prediction is the result (70,30,–,–,–). (Since (75,25,–,–,–) → (–,25,25,25,25) → (end). (65,35,–,–,–) is unstable, since [65,35,–,–,–) → (70,–,30,–,–) (=stable as before).) – This prediction was the outcome of all 5 experimental games of this type.

3. Relations to Other Solution Concepts

3.1 Bargaining Set

The Bargaining set was proposed by AUMANN,MASCHLER (1964). There are several points in which the bargaining set deviates from the concept here, and in all points experimental evidence clearly supports our approach.

1. In our approach the coalition structures are not only partitions. Stepwise nested coalitions are permitted. The bargaining set models only the first step of coalition formation.

2. Different from the Bargaining set, the approach here does not only consider argument-counterargument chains of length three, but of arbitrary length. By the model here the stability is not decided by the existence of a counterargument for any argument. In addition the stability properties of the counterargument are checked. – The Apex Game may serve as an example. Here the bargaining set argues that (60,40,–,–,–) is stable. The complete analysis of the approach here shows, however, that (60,40,–,–,–) is not stable:

```
60 40 -- -- --
65 -- 35 -- --
      --
-- 40 40 10 10
65 -- -- 35 --   *)
--
70 -- -- -- 30
-- 22 22 22 35
--------------
```

```
*)  not (70,--,--,30,--), since thereafter (--,22,22,35,22), and no
    further dominance possible
```

In a similar way, the negotiation chain related to the loyalty paradox 2.1 confirms, that counterarguments do not count, if they do not have sufficient stability properties: (–,40,40) → (70,50,–) –/→ (80,–,40) → (–,40,40) (end).

3. The implicit assumption of the bargaining set is, that every player has a demand which applies for every coalition he enters. This is not supported by experimental evidence. Contrary to that, demands are functions of the coalition structure. For instance in the Apex Game, the observed counterargument after (60,40,–,–,–) → (65,–,35,–,–) is not (–,40,40,10,10), but (–,20,40,20,20). When the role of the selected partner of Player 1 has shifted to Player 3, and after Players 3,4,5 formed a block, the attributed demand of Player 2 can change (and does change) essentially. Experimental results show, that the result of type (–,20,40,20,20) can become stable (see 2.6), while proposals of type (40,40,10,10) were never observed (not even mentioned in any argument of any player).

4. Social preferences for certain coalitions are not modeled. For example, experiments show that in the Apex Game the proposal (75,25,–,–,–) is dominated by (–,25,25,25,25) (with equal payoff for Player 2). This type of dominance is not permitted in the Bargaining Set. (With a modified definition of dominance, the Bargaining Set would capture this phenomenon.)

3.2 Demand Equilibrium

The demand-equilibrium has been defined by ALBERS (1974), see also ALBERS (1979,1981), or TURBAY (1979), BENNETT (1980). The approach generates a generalization of the main simple solution of VON NEUMANN and MORGENSTERN (1944). It is related to the competitive bargaining set of HOROWITZ (1973).

In this concept demands again only depend on the person, and not on the coalition structure. Again only the respective next step of coalition formation is considered.

The concept consideres demand-profiles $d \in R^N$, and uses the notations $coa(d) := \{S \subseteq N: d(S) \leq v(S)\}$. $coa(i,d) := \{S \in coa(d): i \in S\}$.

Definition: A demand profile $d \in R^N$ is a **demand-equilibrium**, iff

(1) $d(S) \geq v(S)$ for all $S \subseteq N$ ('no slack'),

(2) $coa(i,d) \neq \emptyset$ for all $i \in N$ ('coalition forming ability'), and

(3) $coa(i,d) \subset\neq coa(j,d)$ for no pair $i,j \in N$ ('no player depends on another')

To obtain a better description of the process of coalition formation, one would have to introduce demands as functions of coalition structures. But even then the 'no slack' condition (condition (1)) would not fit with observed behavior in cases, where players form a certain kind of block. An example is given by the block of Players 1,2 in the outcome (45,45,30,–) of the 4-person game 2.4. The interesting feature of this kind of block is, that it is stabilized by the fact that, whenever a Player i of the block leaves the other(s) by a dominance (here for instance to (50,–,35,35)), then she can be replaced by one of the other players of the block, (here via (–,45,37,37)), where the replacing player receives as much as before. Accordingly, there may be slack, as long as the

block players are together and get the same payoffs. - It seems that this kind of block gets increasing importance with increasing number of players (see also ALBERS 1996).

3.3 Equal Share Analysis, and Stages of Learning

There is empirical evidence that unexperienced players start their first investigations in a new game with equal share considerations, and select that coalition as a first candidate, which has the highest equal share value. They revise their demands according to the respective strength tested in the negotiation process. For instance, they reduce their demands, when they are not feasible (there is no coalition that can fulfill their demands), or when they depend on other players (see condition (3) of the definition of demand equilibrium in 3.2).

This process of learning the structure of the game is described for spatial games in ALBERS (1984), ALBERS,BRUNWINKEL (1983). The first two steps of reflection of this learning process correspond to the predictions of Equal Share Analysis (SELTEN 1972), and Equal Division Bounds (SELTEN 1982). (For a detailed definition of steps of reflection see ALBERS 1984.) Experienced players, as they are modeled here, will usually not remain in these early stages of reflection, so that more detailed or even deviating predictions are possible (see for instance 2.2). (Experimental results indicate that a procedure where subjects play a series of different games, the game changes in every round, and communication is not permitted, reduces the learned level of reflection, and tends to support a behavior as predicted by Equal Share Analysis.)

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Theory, Field, and Laboratory: The Continuing Dialogue

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Abstract. Reinhard Selten has proposed a four-step schema for policy analysis, consisting of (1) focus on a section of reality, (2) formal modelling of that reality, (3) comparison of outcomes under different institutional arrangements, and (4) pursuit of empirical research based on those comparisons. In this paper, we apply Selten's schema to common-pool resources and the problems that they face. We show how the last decade of research has conformed to Selten's pattern. In particular, we stress the role field studies and controlled laboratory experiments have played in building behavioral theories for these resources - a role we expect to see expand in the future.

In *Guidance, Control, and Evaluation in the Public Sector*, Reinhard Selten concludes his analysis of institutional utilitarianism with the following observations:

> Generally, a section of reality like the provision of health care is shaped by a multitude of legal and organizational facts which are constraints on individual behavior and creates incentives and disincentives. Thereby, individual behavior is limited and guided by a system of rules. The use of the words institutional arrangement refers to a rule system of this kind.
>
> It is important to compare the social desirability of different institutional arrangements which can be proposed for the same section of social reality. . . . In order to do this, one has to construct formal models of the institutional arrangements. . . . Hypothetical exercises in the framework of abstract models without immediate empirical application enhance the understanding of reality by a clarification of conceptual issues. Moreover, the results of purely theoretical investigations may inspire the improvement of empirical research techniques (1986: 258-59).

Selten's advice on how to conduct research that contributes to policy analysis can be summarized as follows: (1) focus on one section of reality at a time, (2) develop formal models of institutional arrangements and incentives for that reality, (3) compare the social desirability of outcomes from alternative institutional arrangements, and (4) pursue empirical research based on these comparisons.

The research program that we have pursued over the past decade is strongly

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influenced by Selten's recommendations.[1] We have attempted to engage in a dialogue between theory and two kinds of empirical research - field studies and laboratory experiments. This research has been driven by an effort to predict and explain complex empirical phenomena. Here we describe the highlights of that decade-long dialogue.

1 A Focus on Common-Pool Resources

The section of reality that we have chosen as our focus is a class of empirical and analytical problems related to the joint use of common-pool resources (CPRs). CPRs are natural or man-made resources in which (1) exclusion is nontrivial (but not necessarily impossible) and (2) yield is subtractable (Gardner, Ostrom, and Walker, 1990). Problems related to these resources have frequently been treated as if they were pure public goods, since exclusion is difficult in both domains. Unlike pure public goods, however, the subtractability of the units in a CPR makes them subject to congestion and even destruction. Examples of CPRs abound in the natural world, including inshore and ocean fisheries, grazing lands, forests, lakes, and the stratosphere. Examples of man-made CPRs include irrigation systems, parking lots, and computer networks. Families, firms, and legislatures are social arrangements that create pools of resources that can also suffer from similar problems of overuse.

Individuals jointly using a CPR are thought of as facing an inevitable social dilemma, referred to as the tragedy of the commons (Hardin, 1968). In this tragic scenario, individual users ignore the externalities they impose on others, leading to suboptimal outcomes. There are several ways out of such dilemmas, all of which are themselves problematic. For instance, it may be possible to create a set of institutional arrangements to transform the incentives of a CPR problem; this transformation can create a second-order public good problem, which is again a social dilemma. Monitoring rules of a transformed situation so that users conform to them is likely to become a third-order public good problem, yet another social dilemma. Since the users of the resource appear to be stuck in a social dilemma no matter what they do, analysts tend to propose reform imposed from the outside. Thus, one has sweeping calls for privatization or central government control of CPRs, as well as calls for institutional arrangements that do not fit neatly into either category of state or market solution (Ostrom, 1990).

The impetus for focusing on CPRs was Ostrom's participation in an interdisciplinary panel of the National Academy of Sciences (NAS) of the United States to examine various forms of common-property arrangements (National

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[1] Ostrom had the privilege of participating with Selten in the ZiF Research Group that produced *Guidance, Control, and Evaluation in the Public Sector* (Kaufmann, Majone, and Ostrom, 1986). Both Gardner and Ostrom had the privilege of participating in Selten's ZiF Research Group on Game Theory and the Behavioral Sciences, 1987-88. All three of us had the privilege of participating in the Bonn Summer Workshop on Experimental Economics and Bounded Rationality, organized by Selten in 1991. All these experiences have had a profound effect on the research discussed here. The authors wish to acknowledge the support of the National Science Foundation (grant SBR-9319835).

Research Council, 1986). The case studies brought together by NAS revealed that the users of many CPRs had escaped from the tragedy of the commons. These users, after suffering from overuse of the resource, recognized the problem and the need for institutional reform. They crafted institutional arrangements of their own to cope with the problem. Further, they monitored their own agreements and sanctioned those who did not conform. In this way, they overcame all three orders of social dilemma. Tragic outcomes were not inexorable. At the same time, these success stories were not the only ones observed. Sufficient evidence existed of major failures, including species extinctions, to pose a genuine intellectual puzzle. How and why did some participants in difficult social dilemmas find a way out, while others did not? With this question inspired by field studies, the dialogue between theory and field begins.

2 Formal Models of CPR Situations

One way to approach the above question is to model a CPR situation as a game. Suppose the game has only inefficient equilibrium outcomes, so that even the most optimistic equilibrium selection theory will predict a suboptimal outcome. Then one can treat a restructuring of the rules of the game as a way for players to achieve better outcomes through better equilibria. This approach we call the technique of rules and games (Gardner and Ostrom, 1991; Ostrom, Gardner, and Walker, 1994), and it applies with considerable success to CPRs.

The first question one must answer in any game modeling attempt is what game to use. Hardin had presented a story that could easily be translated into a Prisoners' Dilemma (PD) as the basis of his gloomy predictions, and the PD has been used extensively as a game model of CPRs (Dawes, 1973; Stevenson, 1991). Having a large number of case studies forced us to recognize the prevalence of many different game structures. These structures included assignment and provision games (see Gardner and Ostrom, 1991), monitoring games (Weissing and Ostrom, 1991, 1993), all coexisting within games involving appropriation externalities (see Ostrom, Gardner, and Walker, 1994: ch. 3 for details).

Whether any of these games lead to a social dilemma depends on particular parameter values. Consider the number of users, n. Harvesting from a fishery for one's own consumption ($n = 1$) has a very different equilibrium outcome from harvesting from the same fishery to sell in competition with other fishers in a market ($n > 1$). For values of n below some critical value n^*, optimal outcomes of some sort are achievable. For large values of n ($n > n^*$), however, outcomes are increasingly inefficient (see Gardner, 1995: ch. 4). The resource may be physically the same, but changes in the parameter in question change the predicted equilibrium.

Based on the above two considerations, we decided not to rely on the PD as our primary model. Instead, our initial base appropriation model involved n-players in a one-shot game where the return that any one player obtained from harvesting resource units depended on the level of harvesting undertaken by others. Since most natural resource systems are characterized by nonlinear relationships, we used a quadratic production function that meant that the initial units appropriated are more valuable than units appropriated later. If appropriators withdrew a sufficiently large number of resource units, however, the outcome was counterproductive. The maximum outcome occurred when appropriators harvested some, but not all, of the resource

units available. A rational player in this game, without communication and external enforcement of agreements, would be led at equilibrium to harvest more resource units than was optimal. The aggregate payoff to the players at any equilibrium for this game was substantially less than the maximum aggregate payoff, on the order of 40%. If this game adequately reflected the game that appropriators in field settings were playing, how did they obtain outcomes better than those available at this equilibrium?

3 Comparing the Social Desirability of Institutional Arrangements

In field settings, as opposed to the above noncooperative game model, players tend to play an appropriation game more than once, thus creating the possibility of repeated game effects. However, as long as the one-shot game is finitely repeated, its subgame perfect equilibria (Selten, 1975) will continue to be inefficient (the equilibrium efficiency is only 40% in the example above). (Of course, if players delude themselves into thinking they are playing an infinitely repeated game, better subgame perfect equilibria are available.) It would not appear that mere repetition of a game will lead to an easy escape from a dilemma outcome. Repetition of the dilemma outcome is what subgame perfect equilibrium predicts.

Again, players in field settings have the opportunity to communicate with one another. The importance of individuals communicating with one another in settings where mutual promises are not enforced by an external authority has been heavily discounted in game-theoretic analysis (see Harsanyi and Selten, 1988). Talk is cheap. Without external enforcement, individuals do not have an incentive to keep promises that are out of equilibrium. Thus, even with communication, subgame perfect equilibrium outcomes of the repeated game would still be inefficient (see Ostrom, Walker, and Gardner 1992 for details).

Field studies, however, show that many self-organized groups have developed and agreed upon their own rules, monitor each other's performance, and engage in graduated sanctions for those instances where individuals do not keep their promises (see literature reviewed in Ostrom, 1994 and Ostrom, Gardner, and Walker, 1994). Thus, the institutions of communication, monitoring, and sanctioning appear to improve observed outcomes in such groups. These institutions transform the game in a way that the model does not capture.

Exactly how these institutions accomplish this transformation, however, is impossible to determine from field studies alone. There are several reasons why. First, in contrast to a solved game model, scholars can rarely obtain quantitative data about the potential benefits that could be achieved if participants cooperate at an optimal level or about the level of inefficiency yielded when they act independently. Second, it is difficult to determine how much improvement has been achieved as opposed to the same setting without particular institutions in place. Third, without using expensive time series designs, studies only include those institutional arrangements that survive and the proportion of similar cases that did not survive remains unknown. Fourth, many variables differ from one case to the next. A large number of cases is required to gain statistical control of the relative importance of

diverse variables. Some of our own students have been able to conduct studies of a relatively large number of field cases so as to engage in multivariate statistical analysis (see Tang, 1992; Schlager, 1990; Lam, 1994). To complement such studies, the techniques of laboratory investigation play an important role.

4 Pursuit of Model-Based Empirical Research

The designer of an experiment knows exactly what the achievable optimum is for each experimental set of conditions, and how far from optimum any equilibrium is. In an experiment, one can measure the effect of communication, for instance, by how close an experimental group that agrees to some joint strategy sticks to that agreement and achieves a good outcome. Groups that fail to achieve an agreement and continue to make independent and inefficient decisions continue to be a part of the data produced by experiments. Thus, the proportion of groups achieving a joint agreement under diverse conditions and the level of conformance to agreements are known. Finally, a major advantage of the experimental method is better control of relevant variables. Laboratory settings are sparse in contrast to natural settings, but that is part of their advantage.

Experimental studies have generated data that is highly consistent with the cases describing smaller and more homogeneous field settings (see Ostrom and Walker, forthcoming). Individuals (typically, students in groups of 8) using CPRs and acting independently tend to overharvest, congest, and/or destroy these resources. When given an opportunity to communicate on a face-to-face basis about the situation they are in, however, these same individuals tend to agree upon joint strategies and carry these out. In repeated settings, individuals use communication as a mode of sanctioning others (Ostrom, Walker, and Gardner, 1992).[2] Moreover, when given an option to impose sanctions on themselves in a repeated setting, individuals who agree upon a joint strategy and on sanctions achieve close to 100% efficiency. Often, they do not even need to use the sanctioning capability they have devised for themselves (ibid.).

Findings from experiments where no communication is allowed are consistent with the outcome predicted by subgame perfect equilibrium of the finitely repeated game. Findings from experiments where communication is allowed are not consistent with that same theory. The finding that "mere talk" can produce an agreement that is enforced by the participants themselves needs to be explained theoretically and not simply with *ad hoc* adjustments.

The problem of explaining behavior observed in the laboratory has long inspired Reinhard Selten's theory of bounded rationality (see Selten, 1978 and 1991, for example). A variant of a suggestion by Selten and his coworkers has proved crucial to our own work (see Selten, Mitzkewitz, and Uhlich, 1988). Suppose that in a game

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[2] In an interesting variation of our communication experiments, Rocco and Warglien (1995) replaced face-to-face communication with communication via email. Subjects tended to come closer to optimum in their agreements, but had greater difficulty in conforming to agreements they had made.

with communication an agreement has been reached. Consider a player in that game who has just observed a deviation from the agreement. We call the response of such a player *measured* if, when the deviation observed is small, the player's own subsequent deviation (if any) is small. In this theory, a deviation is small if it lies between the agreement and the equilibrium for that player. A measured response is a heuristic for dealing with deviation from equilibrium, and as such is fundamentally different from the threats and punishments involved in subgame perfect equilibrium.

We have developed a data-based theory of measured response to explain how individuals who adapt this heuristic can agree upon reasonably efficient joint strategies and sustain those agreements. What is crucial here is not just the opportunity to communicate but also the extent to which deviations remain small. As long as no more than 10% of the deviations are large, then an agreement is sustained and efficiencies of at least 80% (up from 40%) are observed. On the other hand, if more than 10% of deviations are large, then agreements break down and play reverts to that predicted by subgame perfection. Using measured responses, players can achieve high levels of efficiency without external enforcers (see Ostrom, Gardner, and Walker, 1994: ch. 9, for details).

5 Where the Dialogue Goes from Here

The study of CPRs being used by a small number of relatively homogeneous participants has produced a progressive research program over the past decade. While theory produced a set of predictions that cooperation was unlikely in CPR situations, field research produced some contrary evidence. Carefully designed experiments produced evidence that was consistent with received theory under one set of institutional conditions, and with field studies, under another set.

In addition, these advances have spurred the building of new theories besides that of measured response. Bester and Güth (1994) have recently demonstrated that individual preferences including a concern for the payoffs received by others (called "altruism" by Bester and Güth) are evolutionarily stable under specific conditions. In another paper, Güth and Kliemt (1994) have shown that having internal costs associated with breaking promises is also evolutionarily stable under similar conditions (see also Crawford and Ostrom, 1995). In both cases, the identity and "type" of all players are known (or can be known at low cost) to other players before playing a round of a game. This kind of low cost information about the trustworthiness and other-regardingness of participants is more likely to be available in small, homogeneous communities that have been studied in the field and in the lab.

Typically in science, research starts on the simpler systems and only later moves to more complex systems. A simple CPR setting exists when the incentives are highly salient and clear to the participants, who are themselves relatively homogeneous and few in number. Relaxing any of these features results in complexity. The most complex CPRs are global in nature: the open seas, the atmosphere, outer space. Nevertheless, we are confident that the dialogue that we have started will continue to prove useful in the face of additional complexity.

6 A Continuation of the Dialogue into the Future

One way of moving toward explanations of behavior in larger and more complex CPRs is to take incremental steps in our theory, field, and laboratory studies in that direction. In the field, we are now studying local institutions as they affect the behavior of appropriators harvesting from forests in Bolivia, India, Nepal, and Uganda (see Gibson, McKean, and Ostrom, forthcoming). This is a logical next step for field studies, since appropriation from forests produces concentrated local impacts as well as affecting larger systems at a landscape and global level. In the laboratory, we are conducting two series of experiments that take small incremental steps. In one of these, subjects face asymmetric payoff matrices. In the other, subjects participate in a repeated base CPR game and also vote (using simple majority rule, majority rule with symmetric proposals, and unanimity) on allocation rules for the later rounds of the game they are playing (Walker et al., 1995). These new empirical thrusts challenge us to extend our theoretical understanding of the issues surrounding multiple, simultaneous scales, asymmetries among players, and the choice of game structure by those involved.

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Naive Strategies in Competitive Games[1]

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Abstract. We investigate the behavior of players in a two-person competitive game. One player "hides" a treasure in one of four locations, and the other player "seeks" the treasure in one of these locations. The seeker wins if her choice matches the hider's choice; the hider wins if it does not. According to the classical game-theoretic analysis, both players should choose each item with probability of .25 . In contrast, we found that both hiders and seekers tended to avoid the endpoints. This bias produces a positive correlation between the players' choices, giving the seeker a considerable advantage over the hider.
J.E.L Classification numbers: C7,C9.

1. Introduction

In this article we investigate the behavior of players in several two-person competitive (i.e., zero-sum) games. In each game, one player "hides" a treasure in one of four locations, and the other player "seeks" the treasure in one of these locations. The seeker wins if her choice matches the hider's choice; the hider wins if it does not. Because the game has only two possible outcomes, and each player would rather win than lose, a rational player should maximize the probability of winning.

This game has a unique Nash equilibrium with mixed strategies: Each player selects the four alternatives with equal probability (i.e. .25). The proof of this

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proposition reveals the strategic considerations induced by this game. In equilibrium, player 2 (the seeker) does not choose any alternative unless she believes that it is (one of) the most likely alternatives to be chosen by player 1. Similarly, in equilibrium, player 1 (the hider) does not choose any alternative unless he believes that it is among the alternatives that are least likely to be chosen by player 2. Thus in equilibrium the probability of any alternative that may be chosen by player 2 cannot exceed .25 . Consequently, player 2 (and analogously player 1) selects each of the alternatives with equal probability, and player 2 wins with probability .25 .

The following study compares the solution prescribed by game theory to the choices made by subjects. In particular, we explore the effects of three factors : the spatial position of the alternatives (endpoint versus middle point), the relative distinctiveness of the alternatives (focal versus non-focal items), and the role of the player (hider versus seeker). Although these factors are not considered in the formal game theoretical analysis, there are reasons to believe that they might influence the players' choice.

2. Method

The subjects in the present study were undergraduate students at Stanford University who had taken an introductory psychology course, and were required to participate in a few experiments. All subjects were presented with the same form, consisting of six sets with four items each, as shown in Figure 1.

The six sets of items, each of which corresponds to a different game, were constructed as follows. Three of the sets consist of pictorial items (sets 1,3 and 5), and the other three of verbal items (letters or words). Each set includes one distinctive (focal) item. We constructed three types of focal items: neutral (sets 1 and 4); positive, on the background of negative items (sets 5 and 6); and negative, on the background of positive items (sets 2 and 3). Each type consists of one verbal and one pictorial set. We also distinguish between the items in the two ends, called endpoints, and the two items in the middle.

Each participant in the competitive games was assigned to play either the role of the hider or the role of the seeker in all six games. The players were informed that they will be randomly paired with a person playing the other role. They were told that the hider's task is "to hide the treasure behind one of the four items", whereas the seeker's role is "to guess the location of the treasure". If the seeker guesses correctly - she wins $10 and the hider receives nothing; if the seeker does not guess correctly - she receives nothing and the hider receives $10. The participants were told that "both players are informed of the rules of the game". They were promised that five pairs of players will be selected at random and will be paid according to the outcome of the game.

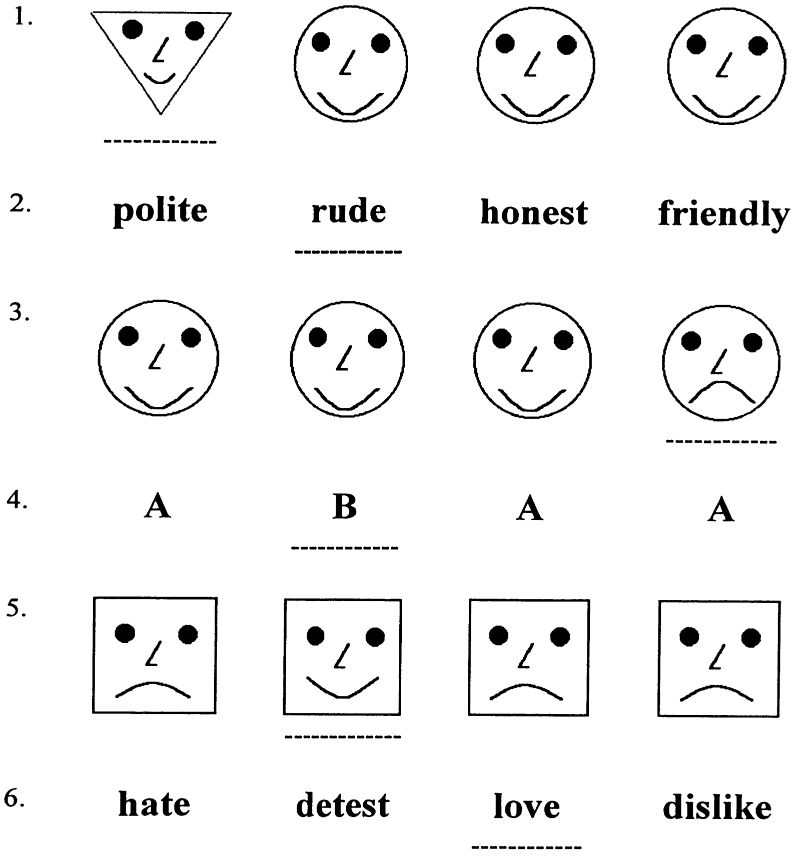

Figure 1. The six sets of items used in all games. The focal item is marked in each set.

We also investigated two non-competitive two-person games based on the same sets of items. In the coordination game, both players receive $10 if they select the same item, and nothing if they select different items. In the discoordination game, both players receive $10 if they select different items, and nothing if they select the same item. Two different groups of subjects played the coordination and discoordination games. As before, subjects were told that they will be randomly paired with another player, and that five pairs of players will be selected at random and will be paid according to the outcome of the game.

3. Results

We first describe the results of the non-competitive games, and then turn to the analysis of the competitive games, which is the focus of this paper.

Non-Competitive Games

Tables 1a and 1b present the percentage of subjects who chose each of the four items in each of the coordination and the discoordination games, respectively.

1a - **Coordination Games (N=50)**

Set	1	2	3	4	p
1	**86**	0	10	4	0.001
2	6	**54**	12	28	0.001
3	6	6	14	**74**	0.001
4	8	**76**	16	0	0.001
5	6	**88**	6	0	0.001
6	2	6	**88**	4	0.001

1b - **Discoordination Games (N=49)**

Set	1	2	3	4	p
1	**39**	14	18	29	NS
2	28	**20**	32	20	NS
3	17	27	23	**33**	NS
4	6	**20**	55	18	0.001
5	17	**40**	29	15	0.05
6	16	29	**26**	29	NS

Table 1. Percentage of subjects who chose each of the four items in each of the six sets, in the coordination games (1a), and the discoordination games (1b). The focal item in each set appears in bold face. The significance level associated with the Chi-square test of each set is denoted by p.

The data in table 1a show, as expected, that in all six coordination games, the majority of subjects (overall mean=78%) chose the focal item. The hypothesis that each item is selected with equal probability (i.e., .25) is rejected by a Chi-square test as indicated by the p-values in the right hand column. The pattern of

responses in the discoordination games , displayed in table 1b, is quite different. Here, there is no tendency to select the focal item, and the distribution of choices in most games, does not depart significantly from random selection.

To investigate individuals' responses, we counted for each player the number of games (from 0 to 6) in which the player selected the focal item. Figure 2 presents, for both the coordination and discoordination games, the distributions of these values, along with the binomial distribution (with p=.25), that is expected under the hypothesis that all items are selected with equal probability.

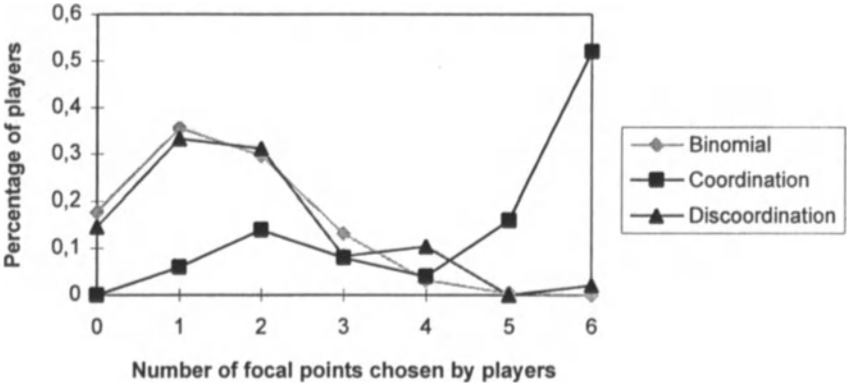

Figure 2. Distribution of patterns of focal point choices for the coordination and discoordination games compared to the predicted binomial distribution.

Figure 2 shows that in the coordination game the distribution of choices was markedly different than expected by chance. In particular, more than 50% of the players selected the focal item in all six games. In the discoordination game, in contrast, the distribution of choices of the focal item is very similar to that expected under random selection. These data indicate that in the presence of a single distinctive item, subjects were able to achieve a reasonable level of coordination (about 65%). In the absence of an effective method of achieving discoordination, players chose more or less in random.

Competitive Games

Tables 2a and 2b present the results of the competitive games for the seekers and the hiders respectively. The data show that, contrary to the game theoretical analysis, the players did not choose the four items in each game with equal probability. A Chi-square test indicates that with only one exception (set 3 for the hider) all the distributions of choices depart significantly from random selection.

We next discuss, in turn, the effects of endpoints, focal items, and the players' role.

2a - **Seeker (N=62)**

Set	1	2	3	4	p
1	**29**	24	42	5	0.001
2	8	**40**	40	11	0.001
3	7	25	34	**34**	0. 01
4	13	**31**	45	11	0.001
5	16	**55**	21	8	0.001
6	20	21	**55**	14	0.001

2b - **Hider (N=53)**

Set	1	2	3	4	p
1	**23**	23	43	11	0.01
2	15	**26**	51	**8**	0.001
3	21	26	34	**19**	NS
4	9	**36**	40	15	0. 01
5	15	**40**	34	11	0.01
6	11	23	**38**	28	0.05

Table 2. Percentage of subjects who chose each of the four items in each of the six sets, in the Competitive games (2a : Seeker, 2b : Hider). The focal item in each set appears in bold face. The significance level associated with the Chi-square test of each set is denoted by p.

Perhaps the most striking feature of these data is players' tendency to avoid the endpoints. Overall, the two endpoints were selected by the hiders and the seekers on 31% and 28% of the games, respectively, significantly less than the 50% expected under random choice ($p<0.001$). The tendency to avoid the endpoints is even stronger in sets 2,4,5,6 in which the endpoint is not a focal item.

To investigate individuals' choices, we counted for each player the number of games (from 0 to 6) in which the endpoint was selected. Figure 3 presents, for the hiders and seekers separately, the distributions of these values, along with a binomial distribution ($p=0.5$) expected under random choice. It is evident from the figure that both hiders and seekers were reluctant to select the endpoints: Avoiding the endpoints in all six games was the single most common pattern, selected by 28% of the hiders and 37% of the seekers. The probabilities of obtaining such results by chance are vanishingly small.

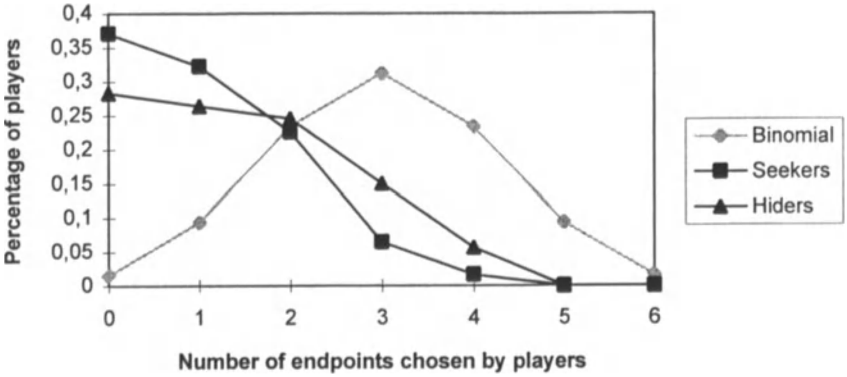

Figure 3. Distribution of patterns of endpoint choices for seekers and hiders compared to the predicted binomial distribution

We have initially hypothesized that hiders will tend to avoid the focal item, in order not to be "predictable". Thus, we expected the hiders to choose the focal item less frequently than the seekers and less frequently than expected by chance (i.e., 25%). The results of the present study support the former hypothesis but not the latter. Overall, the focal items were selected by the seekers 41% of the time and by the hiders only 30% of the time ($p<0.5$), but both groups selected the items significantly more than expected by chance.

Two factors might have contributed to this result. First, the observed tendency to avoid the endpoints is bound to increase the percentage of choice of other alternatives, including the focal items. Second, the popularity of.the focal items was produced primarily by the sets that included a positive focal item. In the verbal sets (2,4 and 6), where the focal items were not the endpoints, the positive focal item (love) was selected 47% of the times (across hiders and seekers) whereas the neutral (B) and the negative (rude) focal items were selected 33% of the times ($p<.05$). The results for the pictorial sets were similar but here the comparison was problematic because the neutral and negative focal points (sets 1 and 3) were endpoints whereas the positive (set 5) was not.

To test whether the endpoint effect depends on the presence of a focal item, we conducted a follow-up study using identical procedure, except that none of the six sets included a focal item. As before three of the sets were verbal and three pictorial. In three of the sets all four items were identical, except for spatial position. In the remaining three sets all four items were distinct, but without a focal item (e.g. south, east, north, west). There were 103 subjects who were divided about equally between the two roles.

In this study too, players tended to avoid the endpoints, although this tendency was less pronounced than before. Seekers chose the endpoints 36% of the time and hiders chose it 43% of the time. Both values are significantly smaller than the value of 50% expected by chance ($p<.05$). To summarize, although the picture is not entirely clear, it appears that seekers tend to avoid the endpoints more than hiders, while hiders tend to avoid the focal point more than seekers.

4. Discussion

The main finding of the present study is that both hiders and seekers tended to avoid the endpoints, thereby departing from the classical game theoretical solution. This tendency was not observed in the coordination game, where players tended to choose the focal items, or in the discoordination game, where players chose more or less at random. The tendency to avoid the endpoints has also been observed in other contexts. When people are faced with the choice among 3,4 or 5 identical items, people tend to avoid the endpoints and select the middle item. This bias has been observed in picking products from a supermarket shelf, selecting a bathroom stall, or picking an arbitrary symbol (Christenfeld, 1995).

In the context of a competitive game, the reluctance to select the endpoints has obvious strategic implications. In the present study it induces a high correlation between the choices of the hider and the seeker, which gives the seeker a considerable advantage despite the symmetry of the game. Note that if the distribution of the seeker's choices is (p_1,p_2,p_3,p_4) and the distribution of the hider's choices is (q_1,q_2,q_3,q_4), then the probability of a match is $m=p_1q_1+p_2q_2+p_3q_3+p_4q_4$, and the expected payoff of the seeker is \$10m. If either player chooses items with equal probabilities than $m=.25$. In contrast, the observed values of m for the six games in the main study were: .31, .33, .26, .32, .32, .31, with an average of $m=.31$. To interpret these values note that if the two players employ the mixed strategy (.4,.2,.2,.2), where one item is chosen twice as often than any other, then m is .28; if both players employ the strategy (.4,.4,.1,.1) then $m=.34$. Thus, the observed m-values reflect substantial departure from the equal-probability strategies.

A possible explanation for the seeker's advantage involves the psychological asymmetry between the hider and the seeker. Although the games are simultaneous, that is, players make their choices without knowledge of the other player's choice, it is natural to regard the seeker as "responding" to the hider's choice. Because it seems easier to contemplate the first than the second move in a game, the seeker may have a better insight into the behavior of the hider than vice versa. Thus, the seeker's tendency to avoid the endpoints could reflect her valid belief that the hider is not likely to select these items.

However, the results of a study we conducted earlier do not support this hypothesis. In that study subjects played a competitive game (using the A B A A set) in which the hider places a mine instead of a treasure. In this game the

seeker's goal is to avoid the item chosen by the hider, whereas the hider's role is to match the choice of the seeker. If the seeker has a psychological advantage, she should be able to avoid the mine more often than expected by chance. The data, however, yielded a positive correlation (m=.28) between the choices of the two players, which favored the hider over the seeker. This result suggests that the advantage of the seeker in the treasure game and of the hider in the mine game reflects a common response tendency rather than a psychological advantage of one role over the other.

In summary, our players were attentive to the payoff function, and employed different strategies in the three types of games. As expected, they selected the focal item in the coordination games, and chose more or less at random in the discoordination games. In the competitive games, however, the players employed a naive strategy (avoiding the endpoints), that is not guided by valid strategic reasoning. In particular, the hiders in this experiment either did not expect that the seekers too, will tend to avoid the endpoints, or else did not appreciate the strategic consequences of this expectation (Shafir and Tversky, 1992).

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Induction vs. Deterrence in the Chain Store Game: How Many Potential Entrants are Needed to Deter Entry?

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Abstract. We report the results of two experiments designed to test competitively the induction against the deterrence theory in Selten's (1978) chain store game with complete and perfect information. Our major purpose is to determine the number of entrants (m) needed for rendering the deterrence argument effective. Our results show that if we increase m from 10 to 15, support for the deterrence theory increases significantly. But even with m=15, the aggressive behavior of the incumbent does not deter most entrants. We then compare these results with previous studies of the chain store game allowing for incomplete information, several choices by the same entrant, and multiple iterations of the game.

Keywords. game theory, experimental economics, chain store paradox, induction, deterrence, sequential equilibrium, reputation

1 Introduction

There is no agreement in the economic literature whether preemptive actions taken by an incumbent firm in order to maintain its monopoly in the face of threatened entry can be maintained as a viable threat. There are those who argue that predatory pricing is an irrational strategy for attempting to maintain a monopoly position and that it is, therefore, unlikely to be adopted in practice (McGee, 1958, 1980; Carlton & Perloff, 1990), and there are others who maintain the opposite position. As noted by Trockel (1986), the crucial point in this debate is the credibility of the threat. It is, therefore, important to determine analytically whether effective threatening of potential entrants by a monopolist is rational.

The first and most influential analysis of this problem is Selten's formal model known as the Chain Store game (1978). The model is presented as a noncooperative (m+1)-person game with a single monopolist (player M) and m potential entrants (players 1, 2, ... , m). The game is played under complete information over a sequence of m consecutive periods (stage games) 1, ... , m. At the beginning of period k, player k (k=1, 2, .. , m) must decide between entering the kth market (IN) or staying out (OUT). Once she has made her decision, all other players are fully informed of her

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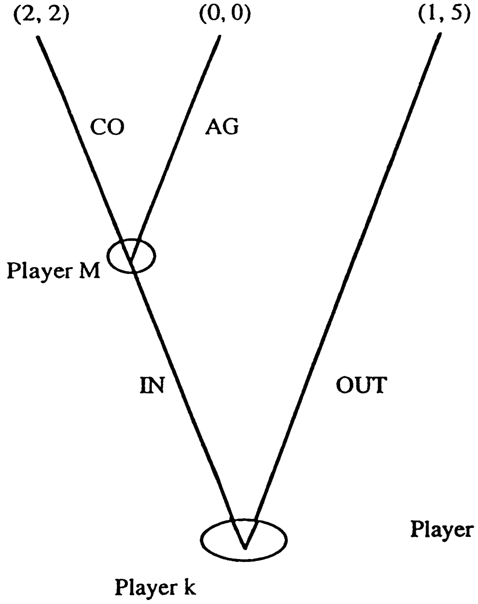
choice. No further decisions are called for in period k, if player k's decision is OUT. If her decision is IN, then player M has to choose between two alternative courses of action called "cooperative" (CO) and "aggressive" (AG). (Here, "cooperative" is better described as peaceful reaction or acquiescence, rather than proper cooperation, and "aggressive" is described as an economic fight). Once player M has made his choice, all other players are immediately informed of his decision, too. Then, for k=1, 2, ..., m-1, the (k+1)th period starts and is played according to the same rules. The game ends after m periods.

It is assumed that all m+1 players are profit maximizers. For the monopolist this means maximization of his profit across all m markets, and for each of the entrants maximization of her profit in the stage game. Players cannot commit themselves to threats or promises, and side payments are not allowed. The payoffs for the stage game are common knowledge; they are displayed in Fig. 1 in the order (M, k). Although the specific choice of numbers in Fig. 1 is consistent with what are believed to be the relative sizes of monopoly and duopoly profits, in fact for each player only the ordinal ranking of the outcomes, independent of the opponent's outcomes, is important.

Figure 1. The Chain Store game in extensive and normal form.



Player k

Player M	IN	OUT
CO	2, 2	5, 1
AG	0, 0	5, 1

The resulting game is, therefore, an m-fold repetition of a two-person game in extensive form with complete and perfect information. Selten proposed two theories for what he calls an "adequate behavior in the game" (1978, p. 130). The first, called

the induction theory, invokes the standard backward induction argument to show that each of the entrants 1, 2, .. , m should always choose IN and the monopolist should always respond with CO. This solution is supported as perfect equilibrium (Selten, 1975) and sequential equilibrium (Kreps & Wilson, 1982a) as well as by repeated elimination of dominated strategies. In particular, the induction theory excludes reputation building by player M and keeps the m markets independent. Player M cannot convince any potential entrant that he is a predator, because they know that he is not. Critical to this theory is the assumption of common knowledge of rationality (Aumann, 1976).

The alternative theory proposed by Selten, called the deterrence theory, reasons that when the stage game is repeated in time, player M has a monetary incentive to respond aggressively to entry by early entrants. He may establish this reputation for toughness by aggressively responding to entry in early rounds, thereby deterring later players k from entering. Given the payoffs for the stage game in Fig. 1 and assuming m=20, player M needs only deter 50 percent of the potential entrants to realize a higher payoff than the always CO choice proposed by the induction theory (assuming cooperative behavior in the last three periods). Specifically, Selten suggests that player M should use his intuition for how many of the last periods of the game he wishes to adhere to the induction argument. Supposing that m=20, and he wishes to accept this argument for periods 18-20 but not 1-17. Then in periods 1-17 he should respond AG to IN, and in periods 18-20 he should respond CO. Anticipating this kind of behavior, players 1, 2, .. , m should behave accordingly. If up to period k-1 not very many of the players 1, 2, ... , k-1 chose IN and player M's response was always AG, then player k should play OUT unless she believes that period k is sufficiently near the end of the game to make it probable that player M will accept the induction argument for periods k, k+1, .. , m. Thus, the deterrence theory gives rise to equilibrium behavior, only it is not sustained as subgame perfect.

Selten concedes that "the deterrence theory does not yield precise rules of behavior, since some details are left to the intuition of the players" (1978, p. 132). This imprecision, argues Selten, should not detract from its plausibility and practicality; rather, he finds the deterrence theory considerably more convincing. "My experience suggests" he writes, "that mathematically trained persons recognize the logical validity of the induction argument, but they refuse to accept it as a guide to practical behavior" (1978, p. 133). The failure of backward induction in two-person ultimatum bargaining games with a relatively short horizon (see Roth, in press, for an extensive review) is in strong support with Selten's conjecture.

There have been several attempts to account for reputation building and predatory pricing by modifying some aspects of the chain store game (Davis, 1985; Kreps & Wilson, 1982b; Milgrom & Roberts, 1982; Trockel, 1986). The starting point for the former three papers is the insight that relaxation of the complete information assumption destroys the subgame structure, thereby leading to a breakdown of the backward induction argument. Kreps and Wilson (1982b) and Milgrom and Roberts (1982) altered the game by assuming a small amount of doubt on the side of the entrants about whether the perceived model of the game is correct. They showed that there exist sequential equilibria for these slightly altered versions of the game prescribing predation in the early stages for player M and, consequently, staying out of the market for most entrants. Trockel (1986) provided yet another

resolution of the paradox, which renders possible reputation building and deterrence, by maintaining the assumption of complete but not perfect information. For experimental evidence supporting predatory pricing see, e.g., Camerer and Weigelt (1988) and Jung, Kagel, and Levin (1994).

Our study departs from these previous experiments by having a closer match between theory and experimental design. In particular, we do not introduce various types of monopolists, as do Camerer and Weigelt and Jung et al., nor do we allow entrants to respond more than once, as do Jung et al. in their sessions without experimenter-induced strong monopolist. Rather, our major focus in this paper is on establishing the necessary conditions for entry deterrence in chain store like games with complete and perfect information. Selten stated that "it will be useful to assume that there are m potential competitors, where m may be any positive integer. Nevertheless, it is convenient to focus attention on m=20, since the game changes its character if m becomes too small" (1978, p.128). We contend that this is not a matter of convenience but of substance. While willing to accept Selten's claim for the deterrence argument when m is relatively high (say, m>20), our major purpose in this study is to investigate the gradual weakening of the force of this argument as m gets smaller. In essence, we ask: how many potential entrants are needed for the deterrence argument to become effective? This question is of some importance if, as Selten claims, the deterrence theory has practical applicability.

When tested experimentally, the induction theory gives rise to the following testable hypotheses:

Hypothesis I1. Player k will always choose IN (k=1, 2, ..., m).

Hypothesis I2. When faced with entry, player M will always choose CO.

Hypothesis I3. The behavior of all m+1 players will be invariant to entrant order.

The hypotheses derived from the deterrence theory are not as specific, because the m potential entrants and the monopolist are not assumed to share the same intuition. Nevertheless, Selten's deterrence theory seems to imply:

Hypothesis D1. When faced with entry (IN), player M will respond AG in early periods, will be less likely to respond AG against middle period entrants, and will respond CO in the last periods of the game.

Put differently, Hypothesis D1 states that the conditional probability of playing aggressively, given entry, p(AG|IN), will decline monotonically in k, starting at one and ending in zero.

With regard to the behavior of the potential entrants, the deterrence theory seems to imply that some early players k are likely to enter in order to test player M's resolve, middle period players are less likely to enter if previous players have met with AG responses, and later period players k are likely to enter as the threat of AG response becomes less credible. Thus,

Hypothesis D2. The probability of entry, p(IN), will first decrease in k and then increase, forming a J-shaped function.

The third and final hypothesis implied by the deterrence theory is:

Hypothesis D3. The behavior of all m+1 players will depend on the entrant order.

2 Experiment 1

2.1 Subjects

Fourty subjects participated in Experiment 1, twenty in each group. The subjects were recruited through advertisements in the school newspaper promising between $5 to $30 contingent on performance. The subjects consisted of male and female undergraduate and graduate students who responded to these advertisements.

2.2 Method

The subjects were run in two separate groups. The twenty subjects in each group were seated separately at one of twenty partitioned desks in a large laboratory (Room A). Each subject was handed a written set of instructions (see Sundali, 1995). These instructions were read aloud, while each subject followed them on his/her written copy.

The instructions explained the structure of the chain store game (referring to player M as "incumbent" and to the twenty competitors as "entrants"). They informed the subjects that they would participate in the game twenty times, ten times in the role of incumbent and ten times in the role of entrant.

The experiment consisted of two phases. In phase I, ten subjects were assigned the role of incumbent and escorted into another room (Room B), and the ten remaining subjects were assigned the role of entrant. On each period, the entrants were assigned a different order to play a given incumbent. The assignment to entrant order was changed from period to period in such a way that each entrant faced a different incumbent once in each of the ten possible order positions. On each period, entrants were informed of the order of play for that period.

At the beginning of the first period (there were ten periods in each phase), an incumbent was selected randomly from the ten subjects in Room B and escorted by one of the experimenters back to Room A. The entrant assigned the first position held up a card indicating her decision for the trial (IN or OUT). This decision was announced by the experimenter and also projected onto a large screen visible to all the subjects. If the first entrant chose OUT, the game proceeded to the next entrant. If she chose IN, then the incumbent responded by raising a card stating his decision to compete (AG) or not (CO). This decision, too, was announced publicly and projected unto the screen. Once the stage game (trial) was completed, the decisions

and payoffs for the trial were recorded on the overhead projector. This procedure was then repeated between the same incumbent and each of the remaining nine entrants.

Once the incumbent faced all the ten entrants, he was escorted out of Room A and moved into a third room (Room C). A second incumbent was then chosen from the remaining nine players in Room B and escorted back to Room A. The order of entry was changed, and the same procedure was repeated. Once each of the ten incumbents faced all the ten entrants in Room A, for a total of 100 stage games, phase I was concluded. Phase II then commenced after switching the roles of entrants and incumbents.

A separate experimenter was placed in each of the three experimental rooms to discourage communication between the subjects.

At the end of phase II, the subjects were administered a written questionnaire. The accumulated payoffs, originally stated in terms of a fictitious currency called "francs", were then converted into US currency (1 franc=$0.50). The subjects were paid individually and dismissed.

The experiment lasted about two hours. The mean payoff per subject was $35.63 ($20.58 for incumbents and $15.05 for entrants).

2.3 Results

To examine the entrants' behavior we conducted a four-way analysis of variance (ANOVA) with the entry decision as the dependent variable and the entrant order, round, group, and phase as the independent variables. Of the four main effects, only the one due to group was significant ($F=8.4$, $p<.01$). Subjects in group 2 entered more frequently (on 91 of 100 trials) than subjects in group 1 (81). Because none of the interaction effects was significant (at $p<.05$), the entrants' decisions were combined over the two groups. Table 1 shows the frequency of entry summed across groups and phases by entrant order and round.

Altogether the entrants chose entry (IN) on 86 percent of the trials. In contrast to Hypothesis D3, there is no evidence of an effect for entrant order. In particular, there is no evidence for the J-shaped function predicted by Hypothesis D2. There is stronger support for Hypothesis I1, although the percentage of entry falls short of the predicted 100 percent.

We turn next to the analysis of the incumbents' behavior. A similar ANOVA to the one described above was conducted on the incumbent's decisions. In contrast to the results of entrant decisions, the main effects due to each of the four independent variables were significant. Incumbents were more likely to compete against early than later entrants ($F=2.8$, $p<.01$), in the second than first group ($F=7.4$, $p<.01$), and in phase II than phase I ($F=8.4$, $p<.01$). In addition, incumbents were more likely to compete in some rounds than others, although we could not discern any systematic pattern in these results. Consistent with the results for the entrant decisions, none of the interaction effects was significant.

Table 2 presents the conditional probability of competition by the incumbent, given entry. Computed across phases and groups, the probabilities are shown by entrant order and round. Whereas Hypothesis I2 predicts that, when faced with entry, player M will always respond with CO, Table 2 shows that, in fact, incumbents reacted with CO only 79 percent of the time. There is strong support, however, for

Hypothesis D3 that incumbent's behavior is not invariant to entrant order. Corresponding to hypothesis D1, p(AG|IN) decreases monotonically in k, although it does not start at one. The (linear) correlation between p(AG|IN) and entrant order is -0.88 (p<.01).

Table 1. Frequency of Entry* by Entrant's Order and Round Summed across Groups and Phases

	Entrant's Order										
Round	1	2	3	4	5	6	7	8	9	10	Total
1	4	3	4	4	3	3	3	3	3	4	34
2	4	3	3	3	4	4	4	4	4	3	36
3	3	4	4	4	3	4	4	3	4	3	36
4	4	4	4	4	4	1	3	3	4	2	33
5	4	4	3	2	3	3	4	3	3	4	33
6	4	4	2	3	4	3	4	3	2	4	33
7	4	4	4	3	4	3	3	4	4	4	37
8	4	3	1	4	4	4	4	3	4	4	35
9	4	3	4	3	2	3	3	4	4	3	33
10	4	3	3	4	3	4	3	4	2	4	34
Total	39	35	32	34	34	32	35	34	34	35	344

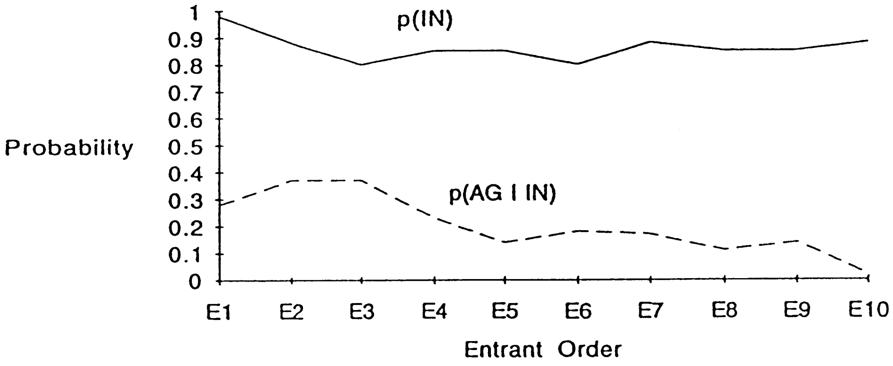
* The maximum number of entries in each cell is 4.

Table 2. Probability of Incumbent's Competition (AG), Given Entry (IN) by Entrant's Order and Round

	Entrant's Order										
Round	1	2	3	4	5	6	7	8	9	10	Total
1	2/4	2/3	2/4	0/4	0/3	0/3	1/3	0/3	0/3	0/4	0.21
2	1/4	2/3	3/3	0/3	1/4	0/4	1/4	0/4	1/4	0/3	0.25
3	1/3	1/4	1/4	1/4	1/3	1/4	1/4	0/3	1/4	0/3	0.22
4	1/4	2/4	2/4	1/4	1/4	0/1	0/3	0/3	0/4	0/2	0.21
5	2/4	2/4	2/3	1/2	0/3	1/3	1/4	1/3	1/3	1/4	0.36
6	1/4	1/4	0/2	1/3	1/4	0/3	1/4	0/3	0/2	0/4	0.15
7	0/4	0/4	0/4	1/3	0/4	1/3	0/3	1/4	2/4	0/4	0.14
8	1/4	1/3	0/1	1/4	1/4	1/4	0/4	0/3	0/4	0/4	0.14
9	2/4	1/3	2/4	1/3	0/2	1/3	1/3	1/4	0/4	0/3	0.27
10	0/4	1/3	0/3	1/4	0/3	1/4	0/3	1/4	0/2	0/4	0.12
P(AG|IN):	0.28	0.37	0.37	0.24	0.15	0.19	0.17	0.12	0.15	0.03	0.21

A more sensitive analysis of the effectiveness of the incumbent's attempt to deter entry considers the entrant's decision conditional on the outcomes of previous trials. The probability of entry by player k was, therefore, computed, given 1) the total number of AG decisions in trials 1 to k-1 in the same round (TC), 2) the number of consecutive AG responses prior to trial k (CC), 3) the number of consecutive AG responses plus the number of OUT decisions in that consecutive run prior to trial k (CC+), 4) the number of trials, prior to trial k, without a CO response (NSP), and 5) the number of consecutive CO responses prior to trial k (TNC). Clearly, these conditional probabilities are highly correlated with each other.

Figure 2. Probability of entry, p(IN), and probability of competition, p(AG|IN), in Experiment 1.



The probability of entry, given TC, assumed the values 0.98, 0.91, 0.88, 0.85, 0.62, 0.78, and 0.56, given no previous AG responses, TC=0, 1, 2, 3, 4, and 5+, respectively. In agreement with the deterrence theory, the probability of entry decreased as the number of the incumbent's previous AG responses increased. Similar patterns were observed with the other four variables: CC, CC+, NSP, and TNC (Sundali, 1995). All of these analyses provide limited support for the deterrence theory. However, although the probability of entry decreased as the incumbent behaved more aggressively, the decline was gradual with entrants more likely to enter than stay out.

In all the analyses reported above, results were summed across subjects. Analyses of individual data show that of the 40 entrants (2 groups by 2 phases), 12 entered against every incumbent they faced. Only 6 of the 40 entrants chose IN in no more than 4 of the 10 encounters. Incumbents were more variable, with 18 of 40 incumbents never competing, 12 competing 1-3 times, and 10 competing more than 3 out of 10 times. In general, competition did not pay off. On the average, incumbents who competed against 0, 1, 2, 3, 4, or 5+ entrants earned \$22.50, \$20.25, \$19.00, \$19.00, \$20.00, and \$16.25, respectively. The correlation between the number of AG

responses (out of 10) and the incumbent's payoff was negative and significant ($r = -0.58$, $p<.01$).

The post-experimental questionnaire (see Sundali, 1995) was designed to ascertain the subjects' motivation and beliefs in their separate roles as incumbents and entrants. The Analytical Hierarchy Process (Saaty, 1980), which presents to the subject all the pairwise choices of a fixed number of statements about motives or beliefs, was used for this purpose. The major findings were 1) incumbents overwhelmingly preferred expected payoff maximization over other possible motives; 2) the belief that the incumbent's decision affects the behavior of subsequent entrants was assigned a higher weight than any other belief (e.g., that stage games are mutually independent); and 3) entrants assigned a higher weight to the belief that their choice was affected by the incumbent's behavior on previous trials than to any other belief (see Sundali, 1995, for details).

2.4 Discussion

When the number of potential entrants in the chain store game is reduced from 20 to 10, support for the deterrence theory is considerably weakened. The incumbent's behavior follows the pattern predicted by the deterrence theory: experienced incumbents (phase II) compete more frequently than inexperienced incumbents (phase I), entrants' decisions are influenced by previous decisions of the incumbent, and the questionnaire results suggest that the subjects are aware of the deterrence argument. However, the relatively high percentage of entry (86%) combined with the low percentage of competition (21%) suggest that when $m=10$, the incumbent's threat to compete is not perceived as very credible and is largely ignored.

Several reasons may explain the ambiguous results of Experiment 1. First and most importantly, as stated above, it is possible that the number of potential entrants was too small for building a credible reputation for entry deterrence. Given the payoffs for the stage game (Fig. 1), and assuming cooperation on the last three trials (as Selten does), the incumbent had to deter four entrants to earn an amount exceeding the one earned by always cooperating. Second, anonymity of the incumbents was not controlled. In Experiment 1 incumbents were visible to all the ten entrants in the room. Because the decision to compete might have been viewed as antisocial, some incumbents might have felt uncomfortable with this decision. Finally, there might have been another methodological shortcoming with subjects being placed in the role of both entrants and incumbents for an equal number of trials. Knowing that they were going to play the game from both positions, the subjects might have developed an implicit norm to accept the (equal) market sharing payoff as appropriate. Experiment 2 was designed to eliminate these potential methodological flaws.

3 Experiment 2

3.1 Subjects

Twenty-five subjects participated in Experiment 2. The same procedure for recruiting subjects was used as in Experiment 1.

3.2 Method

Experiment 2 implemented the same design as Experiment 1 with the following three changes:

First, the subjects participated in a single group, which included 10 incumbents and 15 entrants (m=15). Each incumbent faced the same 15 entrants sequentially, as in Experiment 1, for a total of 150 stage games. The experiment was conducted for a single phase with no switching of incumbent and entrant roles.

Second, incumbents were no longer visible to the entrants. Rather, each incumbent was transferred from Room B to a new room (Room D), which was separated from Room A by a glass partition. Entrants were seated at five foot high partitioned desks in Room A with their backs to the incumbent. They could neither see, hear, or in any other way identify the incumbent. In addition, the incumbent's payoff was never revealed to the other nine incumbents in Rooms B and C. He participated in 15 stage games secure in the knowledge that no other player would be informed of his decisions or payoff. (Of course, the incumbent's identity was known to the experimenters; our method did not implement the double blind design used by Hoffman, McCabe, Shachat, & Smith, 1994).

Thirdly, the payoffs for the stage game were substantially increased from (1,5) to (2,13) and from (2,2) to (6,6). Consequently, incumbents could earn in each stage game $0, $3, or $6.5, and entrants $0, $1.5, or $3.

The experiment lasted about 90 minutes. The mean payoff for incumbents was $44.55 and for entrants $19.70.

3.3 Results

Column 2 of Table 3 shows the entry frequencies by entrant order. Altogether, players k entered the market on 115 of 150 trials (76.7%). This percentage is significantly smaller than the one observed in Experiment 1 ($z=1.83$, $p<.05$). Table 3 shows that, consistent with Hypothesis D3, the entrant's behavior depended on the entrant order. We do not observe the J-shaped function predicted by Hypothesis D2. Rather, the frequency of entry increases monotonically, with only a few deviations, in the entrant order ($r=0.84$, $p<.001$).

Incumbents competed on 34 of the 115 trials (29.6%) in which entry occurred. This percentage is significantly higher than the 20.6 percent observed in Experiment 1 ($z=2.62$, $p<.01$). The right-hand column of Table 3 shows that, consistent with Hypothesis D1, $p(AG|IN)$ decreases monotonically in the entrant order; the correlation between $p(AG|IN)$ and the trial number is -0.80 ($p<.003$).

Incumbents competed aggressively against early entrants and then accommodated entry that occurred later in the round.

Table 3. Probability of Entry, p(IN), and Probability of Competition Given Entry, p(AG | IN), by Entrant Order

Entrant Order	p(IN)	Compete Frequency	p(AG | IN)
1	0.40	4	1.00
2	0.50	3	0.60
3	0.60	5	0.83
4	0.80	4	0.50
5	0.70	3	0.43
6	0.70	1	0.14
7	0.80	3	0.38
8	0.80	0	0.00
9	0.90	2	0.22
10	0.70	1	0.14
11	1.00	4	0.40
12	0.90	2	0.22
13	0.80	1	0.13
14	0.90	1	0.11
15	1.00	0	0.00

Figure 3. Probability of entry, p(IN), and conditional probability of competition, p(AG|IN), in Experiment 2.

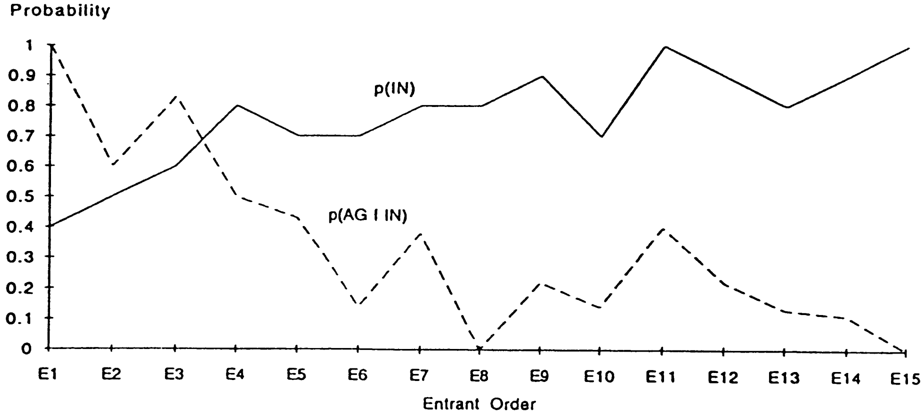


Figure 3 displays the probability of entry as a function of trial number for the entrant, and the conditional probability of competition, given entry, for the incumbent. Comparison of Figs. 2 and 3 shows that increasing m from 10 to 15 (and changing the experimental procedure) enhances the support for the deterrence theory.

Similarly to Experiment 1, we computed the probability of entry conditional on the decisions of the incumbents and entrants on previous trials of the same round. Against new incumbents, four of the ten entrants in the first entrant order position chose to enter. The probability of entry, given that the incumbent had competed (TC) against 0, 1, 2, 3, or 4+ prior entrants was 0.78, 0.67, 0.83, 0.81, and 0.78, respectively. In contrast to Experiment 1, the conditional probability of entry did not decrease as the number of previous compete decisions increased. Additional analyses show that 83 out of 91 entrants (91%) chose to enter if they had seen the incumbent choosing CO at least once. In contrast, only 32 of 59 entrants (54%) chose to enter, if they faced an incumbent who had never chosen CO. Thus, while a majority of players k still entered regardless of the incumbent's behavior, aggressive incumbents were decisively more effective in deterring entry.

Analyses of individual data show that 2, 2, 5, 2, 3, and 1 entrants chose to enter 10, 9, 8, 7, 6, and 5 times (out of 10), respectively. Similarly to Experiment 1, the incumbents constituted a more heterogenous group. Only 2 of the ten incumbents never competed, 3 competed against 50 percent or more of the competitors who chose to enter, and 5 more competed against less than 50 percent of the entrants.

The questionnaire data (see Sundali, 1995) provided stronger support for the deterrence theory than in Experiment 1. Maximization of payoff was still the most preferred motivational explanation for the incumbent's behavior. Establishing reputation for toughness was the second most preferred statement. The belief that the incumbent's decisions would affect all subsequent entrants was assigned five times more weight than any other strategic explanation for the incumbent's behavior. Similarly, the most preferred explanation for the entrant's decision was the dependency of entry decisions on prior decisions by the incumbent on the same round.

4 General Discussion

In explicating the notion of reputation, Wilson writes:

> "In common usage, reputation is a characteristic or attribute ascribed to one person (firm, industry, etc.) by another (e.g., 'A has a reputation for courtesy'). Operationally, this is usually represented as a prediction about likely future behavior (e.g., 'A is likely to be courteous'). It is, however, primarily an empirical statement (e.g., 'A has been observed in the past to be courteous'). Its predictive power depends on the supposition that past behavior is indicative of future behavior" (1985, p. 27-28).

In the context of our experimental study of the chain store game, this supposition or belief depends, among other things, on the information the potential entrant has about

the incumbent. When the entrant is not fully certain about the type of incumbent he faces, the experimental results of Camerer and Weigelt (1988) and Jung et al. (1994) indicate that reputation for toughness leading to entry deterrence is gained rather quickly. Under conditions of complete and perfect information, as in the present study, reputation is built much more slowly. When m=10, as in Experiment 1, some incumbents followed the pattern of behavior prescribed by the deterrence theory, but their threat was not perceived as credible and was, therefore, largely ignored. When the number of potential entrants was increased from 10 to 15, providing the incumbent with a few more opportunities to build his reputation for toughness by competing aggressively against early entrants, support for the deterrence theory increased significantly. Even then, potential entrants were not all deterred; most of the subjects maintaining neither the early nor the late positions in the entrant order still preferred entering over staying out. Our results suggest that Selten's intuition in choosing m=20 was right; the deterrence theory would not gain the support he attributes to it so convincingly with a smaller number of potential entrants. Additional studies are needed to assess the kind of support it will gain when the number of entrants is set equal to or larger than 20.

The relative failure of the incumbent to deter most entrants warrants further discussion. Even if the induction argument can account for the behavior of both incumbent and entrants in the last 3-5 positions in the entrant order, as suggested by Selten, it cannot account for the behavior of earlier entrants; there is ample evidence from different interactive decision making experiments that subjects do not apply the backward induction argument to maximize their individual gain. We suspect that the relative high level of entry reflects the fact that the deterrence argument has different force when viewed from the perspectives of the two types of players. Knowing that he is about to participate in an m-stage game, the deterrence argument has much appeal to the incumbent. He knows that by deterring enough entrants, he can gain more than if he chooses always not to compete. But the same argument does not seem as convincing to the potential entrant, who is only concerned with the outcome of a single-stage game. Clearly, she perceives that if she gets nothing so will the incumbent. She may ask herself why would the incumbent choose to get nothing. The answer that he will do so on that particular stage game because he believes that he can gain more later may not seem so transparent or compelling to her. This is so, because her focus is on the single-stage game she participates in, not on the m-stage game.

If our reasoning is correct, support for the deterrence theory should increase if the original chain store game is modified such that 1) the same entrant moves more than once, or 2) the same set of players participate in the game many times. Both of these modifications are likely to induce the entrant to broaden her perspective by providing some consideration to the m-stage game as a whole rather than restricting her focus to the single stage game. Indirect support for this conjecture comes from comparing Experiment 1 (with m=10) to one of the two experimental conditions of Jung et al. (1994) in which experimenter-induced strong monopolists were not included. This condition incorporated both of the modifications described above. First, in each play of the game the same seven subjects were divided into two sets, one including three monopolists and the other four entrants. In a given play, one of the monopolists faced eight (m=8) entry threats, two from each of the four entrants.

Second, subjects participated in a total of 45 to 90 plays. As shown by Sundali (1995), who provides a detailed comparison of the two studies, the qualitative aspects of the results in the two studies are very similar, despite major changes in the design. However, Jung et al. report considerably stronger support for the deterrence theory and only after the subjects have accumulated considerable experience with the game.

As a final methodological point, it is important to note that because of the interdependency within each group, we actually have only two independent observations in Experiment 1 and a single observation in Experiment 2, corresponding to the respective number of groups. Consequently, the ANOVA results reported above should be interpreted as descriptive statistics, and the conclusions drawn above should be evaluated as tentative. Budget limitations have prevented us (as well as previous investigators conducting experiments with large groups) from collecting more data. Additional experiments, which include a larger number of groups and systematically vary the number of entrants, are needed to clarify the effects of reputations building and entry deterrence in the Chain Store game.

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Cooperation in Intergroup and Single-Group Prisoner's Dilemma Games[1]

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Abstract. We review two experiments in which the Intergroup Prisoner's Dilemma (IPD) team-game was compared with a single-group Prisoner's Dilemma (PD). The first experiment (Bornstein & Ben-Yossef, 1994) compared the games when played once. We found that although the IPD and PD games are strategically equivalent, subjects were more likely to cooperate in the intergroup than in the single-group game. The second experiment (Bornstein, Winter, & Goren, in press) compared the IPD and PD games played repeatedly. We found again that subjects were initially more likely to cooperate in the IPD game than in the PD game. However, cooperation rates decreased as the game progressed and, as a result, the differences between the two games disappeared. This pattern suggests that subjects learn the structure of the game and adapt their behavior accordingly. Computer simulations based on the learning model of Roth & Erev (1995) support this interpretation.

Keywords: Intergroup Prisoner's Dilemma, Cooperation.

1 Introduction

Our concern is with intergroup conflicts and competitions where the payoffs associated with the outcome of the conflict are public goods that are non-excludible with respect to the members of a participating group. Elections, military conflicts, and sports competitions are some of the examples we have in mind. Following the pioneering work of Palfrey and Rosenthal (1983), we model such intergroup conflicts as team games. A team game involves two teams of

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players. Each player chooses how much to contribute towards his or her team effort. Contribution is assumed to be costly (e.g., in terms of effort, time, money, or risk-taking). Payoff to a player is a non-decreasing function of the total contribution made by members of his or her own team, and a non-increasing function of the total contribution made by members of the opposing team. Net of cost, all players on the same team receive the same payoff.

This paper focuses on one such team game, called the Intergroup Prisoner's Dilemma (IPD) game (Bornstein, 1992). The IPD is a simplified team game (i.e., a participation game) where each player makes a binary choice between contributing and not contributing to the public good, and is defined such that a Prisoner's Dilemma game is created both between and within the teams.

The IPD game is operationalized in our experiments as a six-person game in a normal form. The set of players is partitioned into two groups of three players each. Each player in each group receives an endowment at the beginning of the game and has to decide whether to contribute his or her endowment. After decisions are made, payoffs are allocated to each player based on the difference between the number of contributors in his or her group and the number of contributors in the opposing group. Formally, the strategy set of player i is {0,1}, where 1 stands for contribution (C) and 0 stands for no contribution (D). Given a strategy combination ($\sigma 1,\ldots,\sigma 6$) the payoff for i is determined according to the following expression:

$$h_i(\sigma) = \alpha\left(\sum_{j \in A(i)} \sigma_j - \sum_{j \in N \setminus A(i)} \sigma_j\right) + 3\alpha + e(1 - \sigma_i),$$

where N is the set of players and A(i) is the group to which i belongs, $\alpha > 0$ is the players' incremental payoff for each additional contribution by an ingroup member, and $e > 0$ is the initial endowment the individual gets (the cost of contribution). We impose $\alpha < e < 3\alpha$.

Given the first inequality, playing 0 (not contributing) is the dominating strategy for each player. This is because, independently of the strategies of the remaining players, by contributing a player increases his or her reward by α but pays e, hence reducing the payoff by $e - \alpha > 0$. The second inequality ensures that the dominant joint strategy for each team is to have all of its members contribute. This is because, regardless of the actions taken by the outgroup, each player's contribution increases the teams's payoff by $3\alpha - e > 0$. Hence, by our conditions on α and e, the intragroup game defined on each team by fixing a joint strategy for the other team is a three-person Prisoner's Dilemma game.[2] Specifically, there

[2]The constant 3α is added to the payoff function to guarantee positive payoffs to the subjects.

are four intragroup PD games, Γ_0, Γ_1, Γ_2, Γ_3, corresponding to 0, 1, 2 and 3 outgroup contributors, respectively, in the IPD game.

The payoff for player i in game k is given by

$$h_i^k(\sigma)=\alpha\left(\sum_{j\in N}\sigma_j - k\right)+3\alpha+e(1-\sigma_i) \qquad (k=0,1,2,3).$$

Note that for each of the five games (i. e., the four three-person games and the one overall six-person game) the unique Nash equilibrium uses dominating strategies, namely non-contribution. Also note that, although neither game has strong equilibria, the unique Nash equilibrium satisfies the coalition proofness criteria of Berenheim, Peleg, and Whinston (1987). This means that there always exists some group of players that can improve upon the Nash equilibrium outcome by coordinating their strategies, but no such coordination can be self-enforcing.

2 One-Shot Games

Are people less or more likely to cooperate in a PD game when the game is in the context of intergroup conflict? To answer this question, Bornstein & Ben-Yossef (1994) compared the IPD game with an identical PD game. Since the four intragroup PD games embedded in the IPD game (Γ_0, Γ_1, Γ_2, Γ_3) are structurally identical, any one of them could have been chosen as a control condition. Nevertheless, to prevent possible effects of the absolute level of rewards, we included two PD treatments: a High payoff condition (Γ_0, corresponding to the IPD game played by Team A when the number of contributors in Team B is 0), and a Low payoff condition (Γ_3, corresponding to the PD game played by Team A when the number of contributors in Team B is 3).

Also, since the IPD game necessarily involves the co-presence of two distinct groups, we included two (three-person) groups in each session of the PD control conditions as well. However, rather than competing against each other, each group played a separate (independent) PD game. This excludes the possibility that the categorization of subjects into groups is responsible for any potential effects, and ensures that the only difference between the IPD and PD conditions is that in the former condition the two groups are negatively interdependent whereas in the latter they are structurally independent.

2.1 Experimental Procedure

The subjects were 90 male undergraduate students at the Hebrew University of Jerusalem. Subjects were recruited through campus advertisements promising monetary reward for participation in a group decision-making task. Subjects

participated in the experiment in sets of six. Five sets participated in the IPD experimental condition and five in each of the two PD control conditions.[3]

On their arrival at the laboratory, subjects were seated in a single room with arrangements to ensure their privacy. Subject were randomly assigned to two three-person groups and were given verbal and written instructions concerning the rules and payoffs of the game. The instructions were phrased in terms of the individual's payoffs as a function of his own decision and the decisions made by the other players in his set (in the IPD condition) or in his team (in the PD conditions) and were summarized in a table which was available to the participants through out the experiment. Subjects were not instructed to maximize their earnings, and no reference to cooperation or defection was made.[4] Subjects were given a quiz to test their understanding, and explanations were repeated until the experimenter was convinced that all subjects understood the payoff matrix.

Subjects made 10 consecutive decisions between contributing and not contributing. Each decision was recorded on a separate page in a decision booklet. The pages were numbered from 1 to 10 and subjects were told that at the end of the experiment one page would be chosen randomly and their payoffs would be determined by the decisions made by the 6 members of the set (in the IPD condition) or the 3 members of the team (in the PD conditions) on that particular page. Subjects had no opportunity to communicate, and no feedback was provided between one decision and the next. Subjects were told in advance that to ensure the confidentiality of their decisions they would receive their payment in sealed envelopes and leave the laboratory one at a time with no opportunity to meet the other participants. Once all the decision booklets were collected, one of the 10 pages was chosen randomly and the participants were paid based on the decisions made on that page. Subjects were then debriefed on the rationale and purpose of the study and dismissed individually.

2.2 Results

The average number of contributions out of the 10 decisions was 5.47 in the IPD experimental condition, and 2.37 and 3.07 in the High and the Low PD control conditions, respectively. Planned contrasts were performed to test the differences among these means. The first contrast between the IPD experimental condition

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[3] The payoff parameters for the IPD and PD games were e = IS 5 (approximately \$2.00) and α = IS 3.

[4] Subjects were told that each had to decide between keeping the endowment and "investing" it. A bonus was given to each member of a team depending on the difference between the number of ingroup and outgroup investors (in the IPD game), or the number of ingroup investors (in the PD game). In addition to the bonus, each player kept his endowment if he did not invest it.

and the two PD control conditions indicates that the number of contributions in the IPD game was significantly higher than in the PD games $F(1,87) = 12.95$, $p < .001$. The second contrast between the High and Low PD conditions was not significant.

In the IPD game 13 of the 30 subjects (43.3%) used pure strategies (5 subjects defected in all decisions, while 8 subjects cooperated in all 10), and the rest of the subjects used various mixed strategies. In the PD game, 34 of the 60 subjects (56.7%) used pure strategies (29 of them defected in all decisions whereas 5 cooperated). The others used different mixed strategies. Since for all theoretical and practical purposes the games as operationalized in the present experiment were one-shot games, no systematic trend of contribution decisions over time was expected and, indeed, none was found. Subtracting the number of investments in the last five pages from the number of investments in the first five pages, we obtained difference scores of .10, .03, and -.07 in the IPD, PD High, and PD Low conditions, respectively. None of these scores is significantly different from 0. The distribution of subjects according to the nubmer of times (out of 10) each one decided to contribute appears in Figure 2.1.

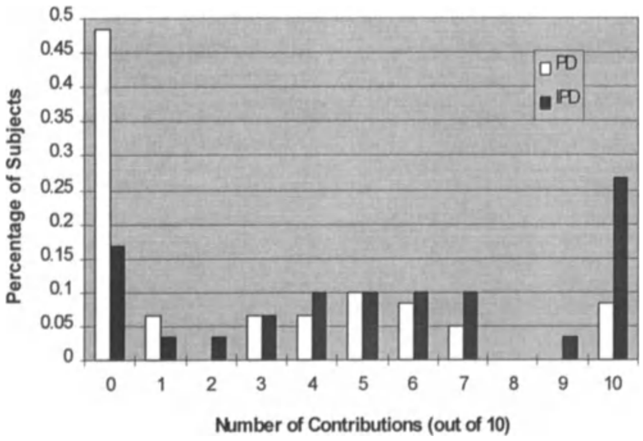

Figure 2.1. Distribution of Contribution Decisions in the IPD and PD.

3 Repeated Games

The experiment by Bornstein and Ben-Yossef (1994) compared the IPD and PD games played once and found that people are more likely to cooperate in the PD game when the game is embedded in the context of an intergroup conflict than when it is played as a single-group game. The objective of the second experiment

by Bornstein, Winter, and Goren (in press) was to examine whether this difference in behavior between the IPD and the PD games persists when the games are played repeatedly.

Repeated games are different from one-shot games in two important ways: First, in an iterated game behavior can be dependent on the earlier choices of other players, whereas in a one-shot game this is not possible. As a result, behavior which may be regarded as irrational in a one-shot game may be rational when the game is repeated. According to the "Folk Theorem" every feasible and individually rational payoff vector is supported by some Nash equilibrium in an infinitely repeated game. In a finitely repeated game with incomplete information about the length of the game (as was the case in our experiment), the set of Nash equilibria is typically also much larger than that in the one-shot game (e. g., Neyman, 1995). In the context of the IPD and PD paradigms this means that the outcome of no one contributing is no longer the only Nash equilibrium of the games.

Second, an iterated game provides players with an opportunity to learn the structure of the game and adapt their behavior accordingly -- an opportunity that they do not have in a one-shot game. The dynamic context of a repeated game thus provides a different, and perhaps more realistic, justification for the theory of equilibrium that does not assume strategic rationality (e.g., Smith, 1984; Harley, 1981; Selten, 1991). Instead, it assumes that equilibrium is a result of learning (Boyd & Richerson, 1985). Learning does not require any insight into the situation. Rather, simple trial-and-error adaptation to success and failure can steer players in the direction of the game-theoretic equilibrium.

3.1 Experimental Procedure

The subjects were 120 male undergraduate students at the Hebrew University of Jerusalem. Subjects participated in the experiment in sets of six with 10 such sets participating in the IPD experimental condition and 10 sets in the PD control condition. As in the first experiment, subjects were randomly divided into two three-person teams. They were given verbal and written instructions concerning the rules of the game and the payoffs were summarized in a table.

In the IPD condition, the two groups competed against each other in 40 rounds (iterations) of the IPD game (with $\alpha = 1$ point and $e = 2$ points). In the PD condition each three-person group independently played 40 rounds of the PD game (Γ_1, corresponding to the case where there is one outgroup contributor in the IPD game with $\alpha = 1$ and $e = 2$). The number of rounds to be played was not made known to the subjects. Each subject had an electric switch that controlled a green or a red light bulb (according to group membership) on an electric board at the front of the room. Each subject could identify his own light bulb (and, thus, his group membership) but could not associate any of the other participants with a particular light bulb (or group).

At the beginning of each round, all the switches were off. Subjects had 15 seconds to decide between contributing and not contributing. If a subject decided to contribute, he was to turn on his electric switch. The room was arranged so that subjects could not see what the others were doing. The decision was not indicated on the board until the 15 seconds were up, at which time the lights were turned on automatically according to the decisions made by each subject (e. g., if there were two contributors in the red group and one in the green group, two red lights and one green one came on). This procedure ensured that each round (constituent game) was played simultaneously. Following the completion of a round the decisions were recorded by the experimenter (subjects were also given 15 seconds to record the outcome of each round, so they could double-check their earnings at the end of the experiment). The lights on the board were then turned off and the sequence was repeated.

Following the last round, the points were added up by the experimenter and cashed in at the rate of IS 1 for 5 points (1 Israeli Shekel was equal to $0.40 at the time the experiment took place, and the average subject earned IS 34.8, or about $14). Subjects were then debriefed on the rationale and purpose of the study, and were paid and dismissed individually.

3.2 Results

Figure 3.1 presents the contribution rate averaged for each five-round block in the IPD and PD games. The mean number of contributions per round in the first five-round block in the IPD game was 2.44 (40.67%) as compared with a mean of 1.78 (29.67%) contributors per round in the PD game. This difference was analyzed using a T-test procedure and was found to be statistically significant both when the analysis was performed at the level of the individual subject, $t(118) = 2.37$, $p < .02$, and when the six-person group was used as the unit for the analysis, $t(18) = 2.07$, $p < .053$. The difference in contribution rates in the second five-round block was significant at the individual level, $t(118) = 2.37$, $p < .02$, but only marginally significant when the analysis was done at the group level, $t(18) = 1.43$, $p < .085$, one-tailed. The same was true for the third five-round block. The mean number of contributors was significantly higher in the IPD game when the analysis was done at the individual level, $t(118) = 1.83$, $p < .035$, one tailed; but only marginally significant when the analysis was performed at the group level, $t(18) = 1.39$, $p < .09$, one-tailed. From the fourth block on, the contribution rates in the two games were not significantly different. Over all 40 rounds, the mean number of contribution was 1.47 out of 6 (24.50%) in the IPD game and 1.17 out of 6 (19.97%) in the PD game. This difference in contribution rates was not significant.

We also calculated the linear trend for each group using the number of the round as the measure of experience. A t-test performed on these trends revealed a significant decrease in contribution rates over time in the IPD game [$t(9) = 3.07$, p

< .02], and a smaller but still significant decrease in contribution rates over time in the PD game [$t(9) = 1.87$, $p < .05$; one-tailed].

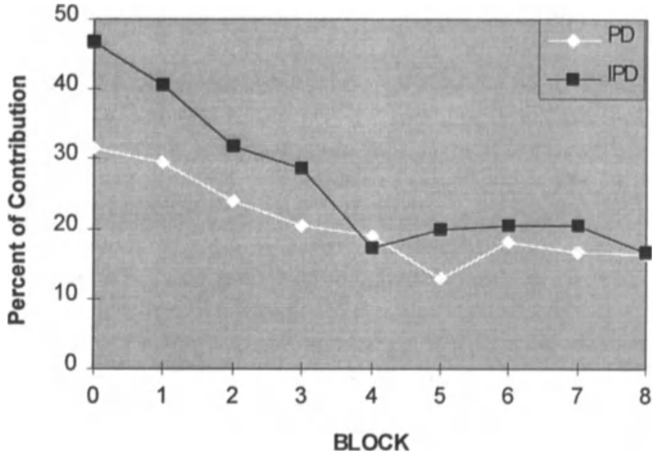

Figure 3.1. Contrinution Rates in the IPD and PD - Empirical Data.

It is clear from the results in Figure 3.1 and the above analyses that as subjects gain more experience they become more likely to choose the equilibrium strategy of the constituent game. In other words, subjects in both games seem to be learning. To further examine this possibility, we applied the learning model recently suggested by Roth and Erev (1995). The basic principle underlying the Roth and Erev model is that choices which have led to good outcomes in the past are more likely to be repeated in the future. This principle, known as the "Law of Effect" (Thorndike, 1898), has been confirmed in a wide variety of environments and seems to be a robust property of both human and animal learning. Roth and Erev (1995) demonstrated that their simple model does a surprisingly good job of reproducing the major features of choice behavior observed in experimental games.

The basic version of the model assumes that Player i, when deciding how to act in the IPD or PD game, considers the two pure strategies: contribution (C) and non-contribution (D). At time t=1 (before any experience with the task has been acquired) Player i has some initial propensity to play each of these two strategies. His propensity to select a particular strategy, say strategy C, at time t=1 is denoted by $q_{iC}(1)$. If Player i plays strategy C at time t and receives a payoff of x, then the propensity to play C is updated by setting $q_{iC}(t+1) = q_{iC}(t) + x$, while for the other strategy, D, $q_{iD}(t+1) = q_{iD}(t)$. The probability $p_{iC}(t)$ that Player i will play strategy C at time t is $p_{iC}(t) = q_{iC}(t)/q_{iC}(t)+q_{iD}(t)$, the ratio of the propensity to play C

divided by the sum of the propensities to play either of the two strategies. Thus, the probability of playing strategy C increases the more successful that strategy has been on previous rounds.

To test this learning model we conducted computer simulations. In performing these simulations we assumed that at t = 1 all players in the same game condition have the same initial propensity to cooperate. In setting these initial propensities we considered two factors: the ratio $q_{iC}(1)/q_{iD}(1)$ of the propensities to play strategy C and D, which determines the probability that C will be played at time t = 1; and the sum of the initial propensities over the two strategies C and D, $S(1) = q_{iC}(1)+q_{iD}(1)$. This second factor can be thought of as the *strength* of the initial propensities. When the value of S(1) is large, the initial propensities are strong and learning is relatively slow. When the value of S(1) is small, the initial propensities are weak and adaptation occurs more quickly. Following Roth and Erev (1995), we set S(1) = 5 and calculated the ratio $q_{iC}(1)/q_{iD}(1)$ for the IPD and the PD games from the observed choices in the first five rounds in each game. Thus, the initial propensities for the computer simulations were set at 2.11 and 2.89 for C and D respectively in the IPD game, and at 1.49 and 3.51 for C and D respectively in the PD game.

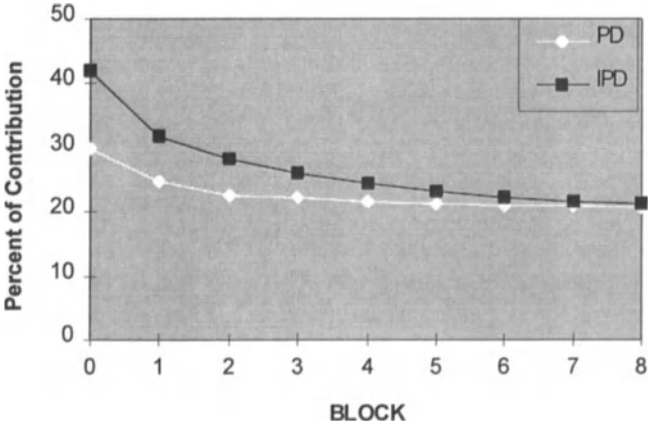

Figure 3.2. Contribution Rates in the IPD and PD - Simulated Results.

Figure 3.2 presents the mean contribution rate in 100 simulated groups, each playing 40 rounds of the IPD or the PD game. As can be seen in the figure, the simulated data reproduce the actual data quite nicely. The simulated subjects, like the real ones, contributed less over time in both games, and most of the decrease in contribution rates occurred in the first 20 rounds of the game. (For a more

detailed description of the computer simulations, see Bornstein, Winter, and Goren, in press.)

4 Discussion

The first experiment demonstrates that subjects are more likely to cooperate in the one-shot IPD than in the one-shot PD game even though both games possess a unique Nash equilibrium with defection as the dominating strategy. Unlike Nash equilibria, the epistemic conditions for playing a dominant strategy require only that players be rational, and therefore the notion of dominant strategies is perhaps the least controversial solution concept in game theory (Aumann and Brandenburger, 1994).

We do not intend to offer a complete explanation for these results. However, our experimental design does enable us to attribute this difference in behavior to the conflict of interests between the groups that exists in IPD game but not in the PD game. It seems that intergroup conflict increases individual willingness to sacrifice self-interest for group causes, as argued by many sociologists and social psychologists (Campbell, 1965, 1972; Coser, 1956; Stein, 1976; Sherif, 1966). This increased in "altruism" in intergroup conflict is paradoxical if one considers Pareto efficiency. In the IPD game universal cooperation is Pareto-deficient, whereas in the single-group PD game it is Pareto-optimal. In his paper on "ethnocentric and other altruistic motives", Campbell (1965), following a similar observation, remarks: "... We have tended to see the altruistic as moral, as the imposed achievement of civilization. Under a broader framework we must now, in some cases, be willing to see altruistic social motives as irrational and immoral, or at least amoral" (p. 307).

The second experiment shows once again that subjects are initially more likely to cooperate in the IPD as compared with the PD game. However, the difference in contribution rates between the two games decreased as the games progressed. This pattern of results supports the hypothesis that subjects learn the structure of the game and adapt their behavior accordingly, and is consistent with the simple learning model of Roth and Erev (1995). The model predicted relatively quick learning at the beginning of the game (particularly in the IPD game), and quite slow learning during later stages. These predictions, as reflected by the simulated results, were nicely confirmed by the experimental data.

The finding that in both the PD and the IPD games the propensity for cooperation decreases in time suggests that the effect of learning is more decisive than the effect of within-group reciprocity. This is because the latter process should have increased the level of cooperation over time. Previous experiments on the repeated Prisoner's Dilemma game (Radlow, 1965; Rapoport & Chammah, 1965, Guttman, 1986) have shown that subjects first learn the payoff structure of

the one-shot (constituent) game, treating the behavior of the other player(s) as invariant. It takes subjects much longer to realize that the behavior of the others may be contingent on their own behavior. The result is a U-shaped trend -- a decrease in contribution rates at the beginning of the game and then a rise or recovery as a higher level of sophistication is acquired.

The present experiment involved a relatively small number of repetitions (namely, 40), focusing primarily on the first, short-term learning phase. Nevertheless, it is possible that, had the game lasted longer, some form of reciprocation would have evolved. We plan future research to examine the long-term interaction effects in the IPD and PD games. We expect to find the typical decline in cooperation at first, followed eventually by a recovery in cooperation rates. Our interest is in identifing possible differences in the onset and extent of the recovery between the IPD and PD games.

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On Styles of Relating to Bargaining Partners

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Abstract. Group researchers like Bales (1970) argue that a lot about outcomes of group discussions can be inferred from observations on the preferred styles of the partners in group communications. There exists convincing evidence supporting this point of view. This evidence, however, is supported primarily from the observation of discussion groups. In our study we try to apply methodological and theoretical concepts to bargaining experiments. Our data is based on face-to-face bargaining. The experimental settings provide an environment in which the socio-emotional dimensions of bargaining acts may become an important factor. Whereas group researchers prefer a fundamental measurement of communication styles either by questionnaires or by rating videotapes, we introduce a classification tool that can also be used on the basis of protocol analysis. From detailed observations we derive four stereotypes that correspond to typical bargaining styles. Choosing two completely different conflict situations we demonstrate the emergence of different styles and their effects.

Keywords. Bargaining behaviour, experimental bargaining, social field, communication styles.

1 Introduction

Styles of relating to partners are often thought to essentially influence face-to-face bargaining processes and results. In Selten/Schuster 1970 it is therefore recommended to enhance the influence of bargaining theories which consider forces emerging from the *social field* that "is automatically set up" ... "if you bring together several people in one room" (p.100). In social psychology Bales, and his colleagues established a broad research tradition in measuring the social field either by direct observation or by rating forms.

Ostmann has applied Balesian methods to explain bargaining results (Ostmann 1990b, 1992a, 1992b, 1994a, 1994b). It has been shown that *extreme communication styles* generally provoke exclusion from the group respectively provoke an in-group guarantee. In the corresponding experiments, the game

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structure gives rise to five well distinguished roles with respect to bargaining power. The game used is a rather complicated six-person simple game introduced by Kravitz 1987. The game provides five different roles with respect to power. Two players are symmetric and their role is a weak one (w). There is one more weak player, but in her/his weakness (s)he depends on one of the two strong players. Let us call the roles of the respective players "dependent" (d) and "protector" (p). The other influential player has a "strong" (s) role. One intermediate type called "bourgeois" (b) completes the setting. The four profiles of the five minimal winning coalitions are sp, sbw, pbd, pwwd. In some of the 26 sessions carried out the conflict induction failed. Dataset A consists of the 17 experiments resulting in successful conflict induction. For more details see the articles mentioned above.

Leopold-Wildburger has reported on superadditive three-person sidepayment games (Leopold-Wildburger 1992a, 1992b, 1993). Dataset B refers to two subsets of 54 experiments all coming from 9 different games. All these games have a zero threatpoint and quotas 50, 30, and 10. The value for the grand coalition varies from 80 to 120 in steps of 5. Let us call the player with the highest quota "strong" (power s), the player with the middle quota "medium" (power m), and the low-quota player "weak" (power w). In each subset twelve independent groups, each including nine subjects, bargained over all nine games once. In the first subset (the first 54 sessions) the subjects received substantial monetary rewards, in the second subset the conflict structure was induced without monetary rewards. In Leopold-Wildburger 1992a it is shown that there exists a linear interdependence between the payoffs realized and the value of the grand coalition. Furthermore it has been found that the payoffs are positively correlated with the own first essential offers and negatively correlated with those of the two partners. In Leopold-Wildburger 1992b a convex linear combination of the Shapley-value and the nucleolus (here equivalent with the kernel) is shown to be a good predictor of the payoffs. Nevertheless, there are substantial deviations that need to be explained.

So far, analysis has used the concept of *identical subjects*, but it can be argued that aspirations and payoffs may also depend on the characteristics of the individuals that handle the conflict. For those characteristics we propose to discuss that it is not essential that they are enduring traits (as e.g. a personality trait) but that they are recordable during the bargaining session. We simply try to identify the individual bargaining styles, and ask how these styles can affect certain processes and results. We start off with theoretical considerations on communicative styles and with *methods to identify stereotypes* that may influence bargaining dynamics. Variables considered are mainly data referring to (process-aggregated) behaviour indicators, aspiration levels, and results, and, on the other hand, data referring to the parameters of the game. In bargaining situations like those mentioned above it might happen, for instance, that one subject with a moderate manner of forming aspirations will find him/herself in a powerful role and may thus be induced to a tough bargaining style. Another subject with a rather expectant manner in a weak position may signal withdrawal out of perceiving the

situation as desperate. As a first orientation, let us formulate the following *hypotheses*:

Hypothesis 1: For certain roles subjects prefer analogue styles: Weakness enhances passivity, strength enhances dominance.
Hypothesis 2: The more dangerous the conflict, the more variable the performed styles.
Hypothesis 3: Certain patterns of aspiration adaptation correspond to certain styles.
Hypothesis 4: High initial aspirations and resistance favour success.
Hypothesis 5: A friendly non-aggressive style favours success.

Before narrowing these hypotheses and testing them for the above datasets (datasets are documented in the discussion paper Ostmann/Leopold-Wildburger 1995) we introduce the basics for measuring and interpreting style differentiation in the context of multilateral bargaining situations.

2 Four Different Styles of Relating to Partners

Based on studies about social cognition and social perception we conceptualize agents in conflict situations as preparing their actions through building up not only a situational view but, especially, partner images. A first task in conflict situations can be supposed to be the identification of potential opponents and potential supporters. While the observational basis for those judgements is very weak in the beginning of the process, people tend to judge their partners at a rather early stage, i.e., after having met for the first time. A further step to come to a more operational judgement consists in identifying what type of opposition, or what type of support can be expected. Obviously, a weak opposition should be answered in a different way than an aggressive opposition. Furthermore, a weak sympathetic support gives rise to other considerations and actions than a strong reliable one. Based on such a model, Ostmann 1992a identified extreme styles called "fighter" and "initiator" that obviously determine coalition formation in face-to-face bargaining experiments. The styles are operationalized by means of an extreme group-related hit frequency with respect to two of the Orlik clusters "withdrawal", "fight", "sympathy" and "accomplishment" (Orlik 1987), namely the clusters withdrawal and fight. It follows from the definition used in the above-mentioned paper that these extreme types are rare. Thus, further research should try to find out about the characteristics of prevailing styles used in bargaining communication and how they interact with power, aspirations, and results. In the following we propose four types:

style 1: insistent
style 2: leadership
style 3: cooperation
style 4: low profile

In contrast to the above model of perception, the classification scheme of the four prevailing styles introduced here makes a first distinction between two styles aiming at leadership of the group, and, on the other hand, two styles that induce the subjects rather to abstain from leading the group. The proposed styles can be ordered monotonically with respect to the dominance dimension of Bales' SYMLOG. The current discussion of *leadership styles* and *participation styles* of collaborators in management science and in social psychology mainly refers to improving the effectiveness in group effort. There exist two important differences between our approach and these research programs. First we consider leadership and participation within the same framework, and second we deal explicitly with conflict situations.

On the basis of videotapes (dataset A) and protocols (dataset B) we try to fix the above stereotypes within the SYMLOG space. The list of items of the questionnaires and the evaluation programmes are documented in the discussion paper by Ostmann/Leopold-Wildburger 1995.

Tab.2.1. UPF-coordinates of stereotypes

style	dominance (U)	friendliness (P)	task orientation (F)	mean arc within style
1 insistent	0.8026	-0.5224	0.2880	0.936
2 leadership	0.6259	0.4343	0.6478	1.116
3 cooperation	0.3017	0.8026	0.5146	1.081
4 low profile	-0.8891	0.4214	-0.1788	0.992

The spherical model of the *SYMLOG space* uses the representation of the three dimensions dominance, friendliness and task orientation on the unit sphere. The entries for the three dimensions in table 1 give the mean position of the judgement. Every judgement is associated with a certain spread. In order to quantify the spread we calculated the mean arc between the characterising items and the mean judgement (mean arc within style). Notice that the values of mean arc within style for our stereotypes are within the interquartile range of judgements on individuals (Ostmann 1995).

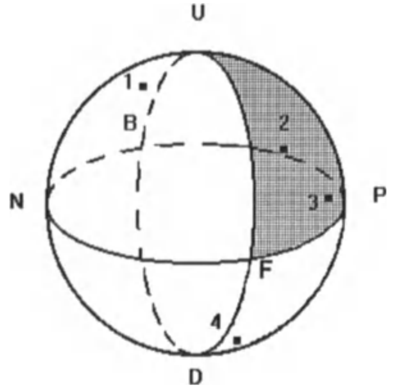
The style representing judgements can also be transformed into a set of corresponding vectors of Orlik cluster-counts.

Tab.2.2 Orlik cluster-counts for stereotypes

styles	withdrawal	fight	sympathy	accomplishment
1	1	19	3	12
2	1	7	11	19
3	3	1	18	23
4	16	0	16	9

The computed values for the mean judgements of the styles are represented graphically in the following figure 2.1.

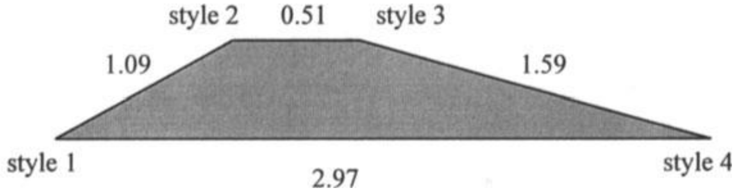
Fig.2.1. Mean location of styles on the sphere



The shaded area in the figure corresponds to the convex hull of the extreme directions U, F, and P. In this positive area most "normal" judgements are (al)located. Style 1 and style 4 deviate from these tendencies, each in a different ways. Type 1 exhibits negative values for friendliness due to his/her insisting way of bargaining. Type 4 reveals negative values for dominance (and task orientation). Evidently, the majority of the extreme negative directions on the sphere are not represented here in our framework. This is also due to rare occurrence.

Figure 2.2 represents the distances between the mean judgements of the four styles. From the values for the mean arcs within style it becomes clear that styles overlap.

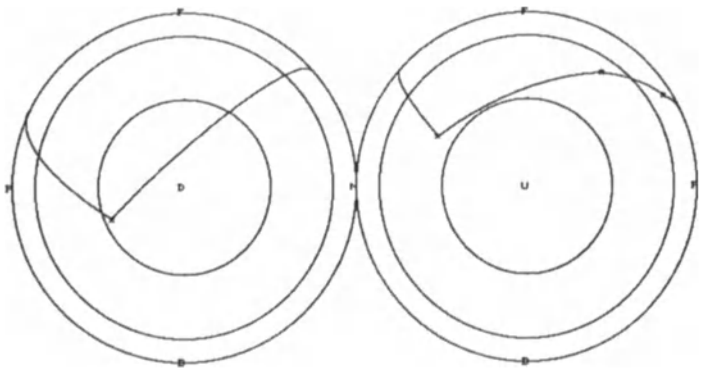
Fig.2.2. Mean arc between styles (2π=360°)



For our purpose it is helpful to consider the shortest connecting lines corresponding to those distances. In the following figure, we give a graphical representation on a pair of polar maps. On the right side the upper (U) hemisphere is located, on the left side the lower (D) hemisphere is located (traditionally, D is

used for -U, N for -P, and B for -F). The circles correspond to latitudes of 0, 30 and 60 degrees.

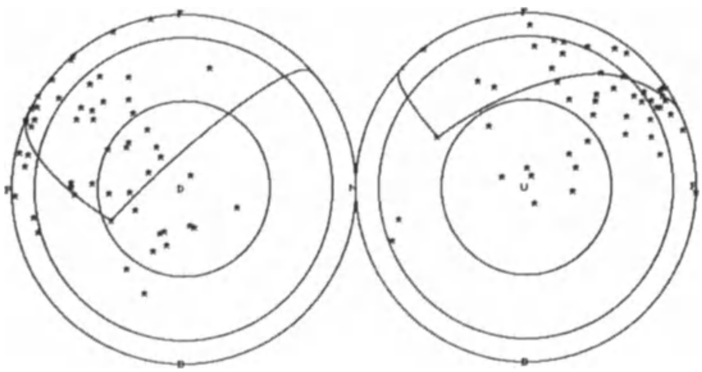
Fig.2.3. Connecting arcs for styles



Assuming we have judgement data in the form of UPF-coordinates, we can try to assign styles to the judgements. Judgements that are not too far (say about half of the mean arc within style) from the connecting arcs can be identified by assigning the style of the *nearest neighbour*.

The following Figure 2.4 gives the locations of the judgements of all 102 subjects of dataset A. In 18 cases a style cannot be assigned using the above method.

Fig.2.4. Connecting arcs and judgements in dataset A



It has been discussed whether the styles of relating to others can be interpreted as enduring traits that can be associated to personality factors. For example, one of the "big five" that can be introduced as 'introvert-extrovert dimension' might be connected with the corresponding dimension of behaviour used here (cf Brandstätter 1992).

3 Power and Style

Before we start to examine how power and styles are interrelated, we use data from dataset A in order to test the reliability of the introduced styles.

As noted in the introduction, the six-person game of dataset A provides five different roles with respect to power. From questionnaires in the 17 experiments mentioned in Ostmann 1994a, 1994b we know the mean judgement of the partners on the behaviour during bargaining for each role:

Tab.3.1. UPF-coordinates of roles in the Kravitz game (dataset A)

role	dominance U(-D)	friendliness P(-N)	task orientation F(-B)	mean arc (within subjects)
s strong	0.28	0.64	0.71	1.24
p protector	0.40	0.67	0.62	1.19
b bourgeois	0.46	0.57	0.68	1.10
w weak	-0.58	0.69	0.39	1.11
d dependent	-0.41	0.81	0.41	1.12

A first test on the *adequateness of the categorial scale* of styles is to compare the mean locations of power roles according to the continuous UPF-scale with those calculated by weighting style locations with the corresponding counts.

Tab.3.2. Identified cases in dataset A according to questionnaires and according to style counting

	basic measurement			counts	weighting styles			error
role	U(-D)	P(-N)	F(-B)	style 1-4	U (-D)	P (-N)	F (-B)	arclength
s strong	0.40	0.43	0.81	2,5,2,3	0.41	0.62	0.67	0.23
p protector	0.48	0.60	0.64	3,2,6,3	0.36	0.70	0.61	0.16
b bourgeois	0.62	0.46	0.63	4,6,1,3	0.63	0.35	0.69	0.11
w weak	-0.66	0.68	0.33	0,2,7,19	-0.68	0.73	0.07	0.26
d dependent	-0.44	0.72	0.23	1,2,5,8	-0.40	0.87	0.31	0.09

not classified: 18 out of 102 cases

An identification error of less than 15 degrees (arclength ≤ 0.26) is regarded as low even for the basic measurement (this is the same order of size as the one observed for the test-retest reliability). For this reason we regard the style classification as justified.

After these preparations we can now consider the *variation of styles by power* for dataset A. Compared with the identification error < 0.26 in dataset A all roles are clearly distinct from the population mean (UPF-mean = (-0.18, 0.87, 0.46); arc distances = (0.83, 0.75, 0.95, 0.54, 0.38)).

A direct chi-square test for the style counts cannot be used to detect a significant variation of styles for the five powers because of the extremely low numbers in some of the cells. Grouping data in two groups ({s,p,b} versus {w,d}) leads to the detection of a significant difference (Chi-square test statistic for 12 D.F. = 32.5, p=0.001).

Let us now apply the style classification to the data from the three-person sidepayment games of dataset B. The distribution of styles for dataset B is concentrated on the "centre-styles" 2 and 3.

Tab.3.3. UPF-coordinates according to style counting for dataset B

role	counts style 1-4	weighting styles U(-D)	P(-N)	F(-B)
s strong	8,16,32,5	0.46	0.65	0.60
m middle	5,23,28,4	0.47	0.64	0.61
w weak	7,16,29,13	0.30	0.75	0.59

not classified: 138 out of 324 cases (32 out of 162 cases in subset 1)

Notice that under monetary conflict induction both games yield a similar proportion of non-identifiable cases. In contrast to dataset A, style patterns in dataset B do not vary with roles (UPF-mean = (0.42, 0.68, 0.60); arc distances = (0.05, 0.07, 0.14) - remember the identification error < 0.26). A direct chi-square test for the style counts should not be used to detect a significant variation of styles for the three grades of power because of the highly concentrated distribution (low numbers in the cells corresponding to style 1 and style 4). In analogy to dataset A, let us pool the data in the two groups {s,m} and contrast them to those in {w}. This leads us to the detection of a significant difference (Chi-square test statistic for 3 D.F. = 6.7, p=0.08). The corresponding small difference between the two groups {s,m} versus {w} in the continuous UPF-location slightly below the identification error calls for more detailed investigation with respect to the extend of the possible effect.

Nevertheless *for both setups it is confirmed* that in weak roles subjects take the opportunity in developing style 4 more often than subjects in other roles. A powerless role may cause withdrawal and a feeling of ending up a loser even

before the bargaining has finished. Subjects who are provided with resources of power have less reason to show low profile.

Obviously, not all conflicts give rise to the same style differentiation with respect to power. For instance, the situation behind dataset A produces high frequencies in style 1 and 4. A possible explanation is coalition formation in a six person simple game which usually results in the exclusion of some players. Such an exclusion represents another qualitatively higher *danger perceived* by the subjects. In such a really difficult situation communication gets stiffer between the opponents. On the other hand, in situations that make available attractive solutions in the grand coalition, more cooperative styles can be expected and are actually realized as is to be seen from dataset B.

4 Aspirations and Style

In dataset A three *different aspiration levels* are recorded. Two of them are filled in a questionnaire by the subjects before they begin their bargaining.

- The variable "*start*" measures what the subject plans to demand for him/herself at the beginning of the bargaining.
- The variable "*min*" measures a level below which the subject plans never to accept a proposal.
- The variable "*last*" measures either the accepted share at the end or, in case that there is no accepted share, the last demand or last accepted share for the corresponding subject (in fact there was only one case observed without both elements of informations; in this case we set last=min). This third aspiration level was recorded after bargaining ended.

In this context we are interested in the following questions:

- Question 1: Do planned aspirations influence performed styles, or do they have a common root?
- Question 2: Do performed styles correspond to the last revealed aspiration?

Before we can check some data on the corresponding hypotheses, we have to find a *method to compare aspiration levels* across roles or even across games. The usual technique consists of a comparison with a payoff-standard (for example the kernel). But for the Kravitz game the dependent player gets nothing according to classical concepts like the *kernel*. This is why we chose a demand standard (here the Banzhaf-value) of a standard of comparison. As it is not very well known that the *Banzhaf-value* provides a demand standard, it has been shown that the Banzhaf-value equilibrates demands with those claims that can be formed by merging pairs of players (Lehrer 1988, Ostmann 1990a).

Tab.4.1. Mean aspiration levels by styles for dataset A

style	start-Banzhaf	min-Banzhaf	last-Banzhaf
1	0.027	-0.119	-0.005
2	0.056	-0.105	-0.034
3	0.062	-0.094	0.022
4	0.098	-0.043	0.028

Group total is set to 1.

First let us consider the situation *before bargaining* took place. A Kruskal-Wallis analysis for the variables start-Banzhaf and min-Banzhaf by four styles yields test statistics of 2.7 and 4.1. These are far from significant differences in mean ranks. However, comparing style 4 with the group {1,2,3} yields a significant difference for min-Banzhaf (test statistic = 3.7, p=0.055; mean rank for style 4 = 48, mean rank for group {1,2,3} = 38). This means that *style 4* usually occurs in combination with hidden high demand profiles. For some subjects with high demand profiles it may be an obvious reaction to exhibit low profile. Either partners accept or do not accept these high demands. In case they do accept, there is reason for struggling for more, in case they do not accept, the subject may signal withdrawal motivated by perceiving the situation as desperate.

Considering the data referring to the *situation at the end of the bargaining process,* results (with respect to the variable last-Banzhaf) highly support the former view. The corresponding Kruskal-Wallis analysis on the set of all four styles yields significant differences for the variable last-Banzhaf in mean ranks (test statistic = 7.8, p=0.05). The aspirations at the end of the process exhibit a clear ordering according to styles: lowest aspirations come from style 2, next is style 1. Style 4 goes along with the highest aspirations.

Let us now compare the above results from dataset A with those from *dataset B.* Dataset B includes data for the variable "start" (in the same meaning as defined above, of course) but not for "min " and "last".

Tab.4.2. Mean aspiration levels by styles for dataset B

style	start-Banzhaf
1	-3.85
2	-2.23
3	-5.54
4	-7.98

Group total payoff varies between 120 and 80.

Again, for the four styles we are far from identifying significant differences by the Kruskal-Wallis analysis. *In contrast* to dataset A, "leaders" (style 1 and 2) demand more than "collaborators" (test statistic for these two groups = 4.9, p=0.03). Moreover, *style 2* claims significantly more than the grouping {1,3,4}

(test statistic = 4.2, p=0.041; mean rank for style 2 = 106, mean rank for group {1,3,4} = 88).

5 Resistance and Style

It has sometimes been argued that different reduction patterns or, equivalently, patterns of resistance characterize groups of subjects that differ in "style" ("hard" vs. "soft" opponents) and in "success" ("above" vs. "below" standard). The following tables, both for our datasets, suggest that reduction or resistance is a variable not only dependent on style. In both cases, the distributions of the variable for different styles are very similar (as far as one can imagine for the small numbers). Indeed no significant differences for the set of all four styles can be detected by Kruskal-Wallis techniques (test statistic = 1.3 and 1.1 for dataset A and 5.2 for dataset B). In dataset B, for style 1 subjects showed a significantly lower resistance than the group of other styles {2,3,4} (test statistic = 3.7, p=0.05). Similarly, for style 3 subjects showed a significantly higher resistance than the group of other styles {1,2,4} (test statistic = 3.1, p=0.07). In contrast to this, in dataset A no variation by style can be detected.

Tab.5.1. Resistance (= payoff-start) by styles for dataset A

	all cases		only members	
style	mean	mean rank	mean	mean rank
1	-0.18	45	-0.11	19
2	-0.22	39	-0.05	23
3	-0.08	47	-0.03	25
4	-0.19	41	-0.08	21

Tab.5.2. Resistance by styles for dataset B (all cases)

style	mean	mean rank
1	-6.9	72
2	-2.3	92
3	-0.9	101
4	-2.5	89

Contrary to the theoretical argument that "a tough bargaining style more often leads to success than a weak one", our data reveal that the insisting style triggers off a process that frequently leads to lower rate of success.

6 Style and Success

A first idea how styles may affect payoffs for the different power levels can be formed from the two next tables. For *dataset A* it can be asked whether it pays off for the strong players to strive for leadership. In both datasets the "friendly collaborators" of *style 3* are relatively successful.

Tab.6.1. Payoff by power for dataset A (number of cases)

			power		
style	s	p	b	w	d
1	0.15 (2)	0.32 (3)	0.24 (4)	- (0)	0.00 (1)
2	0.29 (5)	0.23 (2)	0.15 (6)	0.07 (2)	0.10 (2)
3	0.53 (2)	0.43 (6)	0.00 (1)	0.04 (7)	0.12 (5)
4	0.50 (3)	0.22 (3)	0.00 (3)	0.01 (19)	0.03 (8)

Tab.6.2. Payoff by power for dataset B

		power	
style	s	m	w
1	40.25	30.80	15.57
2	51.75	37.09	19.31
3	46.50	33.64	20.31
4	41.20	32.73	19.15

Kruskal-Wallis techniques do not reveal any differences according to power in any case of *both datasets*. This result is mainly due to the small case numbers. For this reason it is important to use a standard of comparison between power types. Comparison with the Banzhaf-value, a demand standard, yields stronger results which we will discuss now. In Tables 6.3 and 6.4 you can find the *mean success* measured by the variable "payoff-Banzhaf".

Tab.6.3. Mean success for dataset B

style	all cases
1	-10.75
2	-4.52
3	-6.46
4	-10.52

Group total payoff varies between 120 and 80.

In *dataset B* styles 1 and 4 can be grouped as of *minor success* (Kruskal-Wallis test statistic = 7.6, with p=0.01).

Tab.6.4: Mean success (= payoff - Banzhaf) for dataset A

style	all cases	only members
1	-0.155	-0.011
2	-0.163	-0.013
3	-0.015	0.013
4	-0.095	0.002

Group total is set to 1.

For *dataset A*, a corresponding grouping of style of minor success (here style 1 and 2) also shows significance (only members; test statistic = 3.1, p=0.08).

The question now arises why we observe a *different group of minor success* in the two different types of conflict-situations. This difference for style 2 and 4 may be due to the different distribution of power and to the different relevance of membership. Indeed, in dataset A about half of the subjects were excluded, and by the corresponding variance of the variable "success" we cannot expect to separate styles considering all cases (test statistic = 5.1 for all styles and 2.3 for the above grouping). When members are considered, the Kruskal-Wallis analysis yields a significant result, even without grouping (test statistic = 10.1, p=0.02).

7 Discussion

It was shown in the *first part* of the paper that it is possible to identify four styles of relating to bargaining partners which affect processes and results. Styles were defined by corresponding item lists. For the validation of the classification scheme, the styles of behaving insistent, of striving for "positive" leadership, of demonstrating cooperation and of showing low profile, a specific procedure was applied with positive results.

In the *second part* style effects in two bargaining experiments with very different conflict structures were compared. Whereas some of the results seem to be independent of the exact conflict type and show clear similarities, others can be interpreted as proving high sensitivity with respect to conflict type.

Although a powerless role in diverse conflicts may be characterized by very different "resources", subjects weak in power show a clearly distinguished distribution of styles. For both datasets weakness leads to an increase in cases of low profile.

The results concerning the relation between aspirations and styles show a sharp contrast between the two different conflict types. Whereas in the six-person simple game the low profile style usually comes along with high (hidden) demands, in the three-person sidepayment game the high demands occur especially within the grouping of "leaders" represented by style 1 and 2. This contrast calls for further research on how aspirations are formed and revised in different multilateral bargaining situations. A first hint as to what effects to expect might come from our data on resistance.

In both settings success is similarly related to styles. Especially the style of the "friendly collaborator" is favourable to the degree of sucess.

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What Makes Markets Predict Well? Evidence from the Iowa Electronic Markets*

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Abstract. We use the data from the Iowa Electronic Markets to study factors associated with the ability of markets to predict future events. These are large-scale, real-money experimental markets with contract payoffs determined by political election outcomes. They provide data about individual trader characteristics and market micro-behavior which is not available from larger exchanges. In this study we find that market characteristics motivated by financial theory and previous experimental research account for most of the variance in predictive accuracy across sixteen markets. Three variables are particularly important: 1) the number of contract types traded, 2) pre-election market volumes and 3) differences in election eve (weighted) market bid and ask queues.

1 Introduction

Do markets correctly impound information about future events? While a large body of evidence shows that financial markets respond in predictable ways to information releases,[1] whether this response is optimal is open to debate. In large part, this debate is difficult to resolve using data from naturally occurring markets because we can neither observe private information nor determine whether all information is completely impounded in prices. Further, the fundamental values of many assets are never revealed. These factors make it impossible to determine the informational efficiency of a market since we have no way of knowing whether prices truly reflect underlying fundamental values.

Controlled laboratory markets provide a complementary way to study information aggregation and efficiency in markets. In these markets, information and market

* We thank Reinhard Selten for his continued support of the Iowa Electronic Markets and its predecessor, the Iowa Political Stock Market. With Professor Selten's help and encouragement, we were able to bring this technology to Germany so that we could conduct a Unified Germany Election Market on the outcome of the December 1990 German Federal election. We also wish to thank Daniel Friedman, Forrest Nelson and participants in the 1995 Economic Science Association meetings for many helpful comments and suggestion.

[1] For example, see Fama, 1991, for a review of the event study literature. As he puts it, "event studies can give a clear picture of the speed of adjustment of prices to information....The results indicate that on average stock prices adjust quickly..."

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structure can be controlled and manipulated. Further, one can easily measure information aggregation and pricing efficiency.[2] However, laboratory market evidence is often criticized because the markets are very short-term, consists of a handful of traders[3] and, while cash incentives are used for motivation, traders do not self-select into these markets nor invest their own cash, as they would in most naturally occurring financial markets.

Here, we use data from the U.S. political markets conducted on the Iowa Electronic Markets (IEM) to identify factors which determine when markets accurately predict future events.[4] The IEM overcomes some criticisms of traditional laboratory research while retaining some of its advantages. In contrast typical laboratory markets, the IEM consists of longer term markets ranging from ten days to eleven months long. Large numbers of traders self-select into these markets and put their own money at risk in trading.[5] In contrast to typical futures markets, there are no simultaneous spot markets in the underlying fundamentals (vote shares). Thus, there are no arbitrage conditions that drive prices in the futures and spot markets together.[6] This results in a range of prediction errors (which we use as our dependent variable). In addition, the contract design insures risk neutral pricing with very few assumptions about traders or the outside hedging opportunities available to them. There are also no explicit transaction costs in this market, removing any resulting distortions. Finally, the IEM provides a great deal of detailed information on the markets and traders. This allows us to consider variables such as the number of active traders and their experience levels, all information about the queues (instead of simply considering bid/ask spreads), etc.

Because of these advantages, many studies, beginning with those of Forsythe, Nelson, Neumann and Wright, 1991a,b, 1992, use IEM technology to study market

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[2] Laboratory evidence shows that markets can be quite efficient in aggregating information. (Forsythe and Lundhom, 1990 and Plott and Sunder, 1982.) However, sometimes even very simple laboratory markets can exhibit anomalies such as price bubbles and information mirages. (Camerer and Weigelt, 1991; Rietz, 1995; Smith, Suchanek and Williams, 1988.) The evidence also suggests that the structure and distribution of information affect the ability of the markets to aggregate information. (Lundholm, 1991.)

[3] Most laboratory experiments consist of fewer than fifteen traders and are conducted over a period of three hours or less.

[4] In this paper, we have excluded all non-U.S. political markets. This allows us to disregard differences in cultures and political systems. These will be examined in a subsequent study.

[5] On the eve of the November 1992 U.S. election, 1102 traders were registered to trade with a total investment of approximately $83,000. On the eve of the November 1994 election, 3,150 individuals had trading rights in the political markets with a total investment of approximately $42,000.

[6] In this sense, our markets correspond more closely to markets such as corn yield futures or pari-mutual betting than traditional markets for futures on an underlying, market-traded asset.

and trading behavior.[7] Each of these studies has focused on describing and analyzing behavior in a single market. Ours is the first to use the data from a number of markets to study factors affecting predictive accuracy. To do this, we use the average absolute prediction error for each market and study differences in accuracies across markets. While the IEM predicts political outcomes well on average, some markets perform better than others.[8]

Here, we ask what election and market characteristics affect how well IEM prices predict actual U.S. election outcomes across sixteen markets and elections. Our goal is a descriptive one: to use existing data to develop a parsimonious model which explains variations in accuracies across markets. We find that characteristics motivated by financial theory and previous experimental research account for a great deal of the variance in predictive accuracy across these markets. In particular, three variables can explain most of the variation in predictive accuracy across markets (with an adjusted R^2 of 93%). These variables are: 1) the number of contract types traded in a market (corresponding roughly to the number of major candidates), 2) the pre-election market volumes, and 3) differences in election eve (weighted) market bid and ask queues. In addition, two more variables appear to have additional independent significance, but add little to our ability to explain the variance across markets (boosting the adjusted R^2 by 2%). These variables are: 4) pre-election (weighted) market spreads and 5) the average experience level of active traders in the market shortly before the election. We also observe that many other variables were highly correlated with predictive accuracy, but were also highly correlated with one or more of these five variables and do not have significant independent effects.

2 Description of the IEM and Political Vote-Share Markets

The IEM is a real-money futures market operated over the Internet as a research and teaching tool by the University of Iowa College of Business Administration.[9] Participants invest their own funds, buy and sell listed contracts, and bear the risk of losing money as well as earning profits. The method of issuing contracts and making final payoffs on these contracts ensures that the IEM does not realize financial profits, nor suffer losses, and that all monies invested are redistributed to traders based on their holdings. No commissions or transaction fees are charged.

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[7] These include Bohm and Sonnegärd, 1995, Davis, Forsythe, and Holt, 1994, Forsythe, Frank, Krishnamurthy and Ross, 1995, Jacobsen, Potters, Schram, van Winden and Wit, 1995, Kuon, 1991, Lombardo, 1993, Ortner, Stepan and Zechner, 1994, Stepan and Ortner, 1995. The only political market study which does not use the IEM technology is reported by Beckman and Werding, 1994a,b who manually conducted a call market at Universität Passau.

[8] Average absolute prediction errors in the markets studied in this paper range from 0.06% (the 1992 Bush-Clinton market) to 8.59% (the 1992 Michigan Democratic Primary market).

[9] The interested reader can examine current markets by connecting to them over the World-Wide-Web. Our homepage address is: http://www.biz.uiowa.edu/iem.

The IEM operates under the regulatory purview of the Commodity Futures Trading Commission (CFTC). The CFTC has issued a "no-action" letter to the IEM, stating that as long as the IEM conforms to certain guidelines,[10] the CFTC will take no action against it. However, we do not file reports that would be required by regulation so, technically, the IEM is not regulated by, nor are its operators registered with, the CFTC.

Contracts in the IEM are based on the outcomes of political and economic events. In this study, we focus on the "vote-share" political markets because prices in these markets can be used to explicitly measure predictive accuracy. A vote-share market consists of two or more contracts each identified with a particular candidate or party.[11] The liquidating value of a contract in a vote-share market is determined by the percentage of the popular vote actually received by each candidate times $1. If, for example, a candidate receives 54 percent of the vote, the owners of contracts in that candidate receive a 54-cent liquidation payment for each contract held.[12]

The political markets are open to any individual worldwide. The total amount invested by an individual cannot exceed $500, although accumulated earnings can easily push a trader's balance beyond the $500 limit. To establish an account, the trader submits an application form and initial investment. The total amount invested by the trader is placed in his or her cash account with the IEM. Participants then use their funds to buy and sell contracts. Thus, traders have the opportunity to profit from their trades and bear the risk of losing money.

Contracts are placed into circulation when traders purchase bundles of contracts, termed "unit portfolios," for $1 each from the IEM.[13] Each unit portfolio consists of one contract of each candidate in the market. The contracts in a market are structured so that the total liquidation value for each unit portfolio is $1. Once purchasing a unit portfolio from the market, traders can unbundle the contracts immediately and trade them individually. The IEM also stands ready to repurchase unit portfolios for $1 each from any trader at any time.

Each contract is traded in a continuous electronic double auction with queues. Traders can issue bids to buy or asks to sell, or trade at the best outstanding bid or ask. Bids and asks are limit orders with defined prices, quantities and expiration

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[10] These are that 1) the IEM be conducted as a not-for-profit enterprise, 2) it engages in no paid advertising, 3) traders are limited to investing no more than $500 in their accounts, and 4) it does not charge transaction-based commissions.

[11] In some of these markets there is also a contract which is identified as Rest-of-Field. Its liquidation value is determined by the aggregate vote-share of all other candidates or parties not explicitly included in the market.

[12] There are two minor exceptions to this rule. First, in the markets run for the 1990 Iowa and Illinois senate races, the payoff was the fraction of the vote received times $2 for each candidate. Second, we are including the results from two markets conducted to predict the composition of the 1994 US Senate and House. These contracts paid off $1 times the fraction of the seats taken by each party.

[13] For the 1990 Iowa and Illinois Senate markets, unit portfolios cost $2 since such portfolios would ultimately return $2.

dates. Traders are not required to maintain an inventory to cover all of their outstanding asks, nor must they maintain cash to cover all their outstanding bids. However, if a bid or ask reaches the top of its queue, the trader must be able to trade at least one unit or the offer is ruled infeasible and canceled.

There can be many bid and ask prices in the system at any time. They are maintained in bid and ask queues ordered first by price and then by time entered (on a first-in-first-out basis). These queues function as continuous electronic limit order books. When an offer is entered, it remains in the queue until either (1) it is withdrawn by the trader, (2) it expires, (3) it reaches the top of the queue and is found to be infeasible, or (4) it reaches the top of the queue and is accepted by another trader. Purchases on margin and uncovered short sales are not permitted.[14]

In addition to these trading actions, this computerized market provides information about the trader's account and market activity. Trader-specific account information available to each trader includes the number of contracts held in each candidate, the cash account balance, a list of outstanding offers and a list of transactions. Market information available to all traders includes: the current high-bid, low-ask and last transaction price as well as a summary of all previous trading activity by day. Information about the depth of the bid and ask queues is not revealed. The market software records all market activity (logins, logouts, bids, asks, trades, withdrawals, expirations, etc.) in an audit trail.

For this paper, we analyze data from election vote-share markets run to predict outcomes in sixteen major US elections and primaries.[15] The particular markets and elections are detailed in Table 1. As discussed above, traded contracts correspond to candidates in these markets. When the election outcome is announced, the payoff for a candidate's contract equals the fraction of the vote received by that candidate times the price of a unit portfolio. The nature of the market results in no aggregate risk and insures that prices should reflect expected values regardless of risk preferences. Intuitively, this results because all agents can hold the well diversified, risk-free "market" portfolio consisting purely of unit portfolios. All other contracts can be priced from this portfolio and the risk/return tradeoff inherent in it. The return to holding unit portfolios is the same as the risk free rate which is zero in these markets. Thus, the reward for taking on risk is zero and the expected return for each risky asset must also be zero. This can only be true if all contracts are priced at their expected values.[16] In these markets, expected value pricing means prices should equal anticipated vote shares for each candidate in the ensuing election.

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[14] Traders can synthetically construct the payoffs to a fully covered short position.

[15] This includes all vote-share markets run on US elections to date except for the 1988 presidential election. The original market data necessary to generate the variables of interest here is currently unavailable because of changes in the computer systems used at the University of Iowa.

[16] See Rietz, 1995, for a detailed discussion of this point. The result can be derived from general equilibrium theory, the capital asset pricing model or arbitrage pricing theory.

Table 1. Election and Market Descriptions

Market No.	Election	Contracts	Candidates	Opening Date	Election Date	Predicted Share	Actual Share	Error	Average Absolute Error
1	92 Illinois Primary	I.TS	Tsongas	3/1/92	3/10/92	42.1%	26.2%	15.9%	5.3%
		I.HA	Harkin			0.2%	1.7%	-1.5%	
		I.CL	Clinton			43.0%	51.4%	-8.4%	
		I.KE	Kerry			0.2%	0.6%	-0.4%	
		I.BR	Brown			10.3%	15.3%	-5.0%	
		I.RF	Rest of Field			4.1%	4.8%	-0.7%	
2	92 Michigan Primary	M.TS	Tsongas	3/1/92	3/10/92	40.1%	17.0%	23.1%	8.6%
		M.HA	Harkin			1.0%	0.8%	0.2%	
		M.CL	Clinton			44.3%	51.8%	-7.5%	
		M.KE	Kerry			0.1%	0.2%	-0.1%	
		M.BR	Brown			8.2%	26.4%	-18.2%	
		M.RF	Rest of Field			6.2%	3.8%	2.4%	
3	92 Presidential (Perot vs Democrat and Republican)	D&R	Democrat and Republican Vote Share	5/19/92	11/3/92	80.7%	81.0%	-0.3%	0.3%
		PERO	Perot Vote Share			19.3%	19.0%	0.3%	
4	92 Presidential (Bush vs Clinton)	D.CL	Clinton Vote Share	1/10/92	11/3/92	53.5%	53.5%	0.1%	0.1%
		R.BU	Bush Vote Share			46.5%	46.5%	-0.1%	
5	94 AZ Senate	KYL	Kyl	9/29/94	11/8/94	54.1%	53.7%	0.4%	1.6%
		COP	Coppersmith			41.5%	39.5%	2.0%	
		AZROF	Rest of Field			4.4%	6.8%	-2.4%	
6	94 NJ Senate	HAY	Haytaian	8/29/94	11/8/94	45.9%	47.0%	-1.1%	2.3%
		LAU	Lautenberg			53.7%	50.3%	3.4%	
		NJROF	Rest of Field			0.4%	2.7%	-2.3%	

Table 1. Election and Market Descriptions (Continued)

Market No.	Election	Contracts	Candidates	Opening Date	Election Date	Predicted Share	Actual Share	Error	Average Absolute Error
7	94 NY governor	CUO	Cuomo	8/29/94	11/8/94	47.4%	45.1%	2.3%	4.1%
		PAT	Pakaki			40.8%	49.0%	-8.2%	
		ROS	Rosenbaum			2.5%	0.0%	2.5%	
		NYROF	Rest of Field			9.3%	5.9%	3.4%	
8	94 PA senate	WOFF	Wofford	9/15/94*	11/8/94	46.9%	46.9%	0.0%	1.1%
		SANT	Santorum			51.0%	49.4%	1.6%	
		PAROF	Rest of Field			2.0%	3.7%	-1.7%	
9	94 TX Governor	G.BUSH	Bush	4/13/94	11/8/94	55.1%	53.5%	1.6%	1.1%
		G.RICH	Richards			44.4%	45.9%	-1.5%	
		G.ROF	Rest of Field			0.5%	0.6%	-0.1%	
10	94 TX Senate	S.FISH	Fisher	4/13/94	11/8/94	36.8%	38.3%	-1.5%	3.3%
		S.HUTCH	Hutchinson			57.4%	60.8%	-3.4%	
		S. ROF	Rest of Field			5.8%	0.9%	4.9%	
11	94 US House Seats	HS.DEM	Democratic Fraction of House Seats	6/10/94	11/8/94	51.3%	46.7%	4.6%	3.1%
		HS.REP	Republican Fraction of House Seats			48.5%	53.1%	-4.6%	
		HS.OTH	Other Fraction of House Seats			0.2%	0.2%	0.0%	

Table 1. Election and Market Descriptions (Continued)

Market No.	Election	Contracts	Candidates	Opening Date	Election Date	Predicted Share	Actual Share	Error	Average Absolute Error
12	94 US Senate Seats	SS.DEM	Democratic Fraction of Senate Seats	6/10/94	11/8/94	45.5%	48.0%	-2.5%	3.3%
		SS.REP	Republican Fraction of Senate Seats			49.7%	52.0%	-2.3%	
		SS.OTH	Other Fraction of Senate Seats			4.9%	0.0%	4.9%	
13	94 UT House	V.CO	Cook	4/15/94	11/8/94	17.7%	18.3%	-0.6%	3.6%
		V.GW	Green-Waldholtz			39.3%	45.8%	-6.5%	
		V.SH	Shepard			41.2%	35.9%	5.3%	
		V. ROF	Rest of Field			1.9%	0.0%	1.9%	
14	94 VA Senate	NOR	North	1/31/94	11/8/94	41.0%	42.9%	-1.9%	2.0%
		ROB	Robb			42.5%	45.6%	-3.1%	
		ROF	Rest of Field			1.8%	0.1%	1.7%	
		COL	Coleman			13.4%	11.4%	2.0%	
		WIL	Wilder			1.3%	0.0%	1.3%	
15	90 IA Senate	Hark	Harkin	7/11/90	11/6/90	52.7%	53.9%	-1.2%	1.2%
		Tauk	Tauke			47.3%	46.1%	1.2%	
16	90 IL Senate	Mart	Martin	7/11/90	11/6/90	41.2%	36.0%	5.2%	5.2%
		Simn	Simon			58.8%	64.0%	-5.2%	

3 Model Development

Our goal is to find market characteristics associated with a market's performance in predicting election outcomes. Because of the small number of observations, we focus on a few variables we believe will prove important because of theory and existing experimental evidence. Here, we describe the model development and our reasons for examining particular independent variables. We group our variables according to two broad categories that we call "election properties" and "market properties." We also describe how these variables are related to predictive accuracy and to each other. These variables and their descriptions are listed in Table 2.

3.1 Measuring Predictive Accuracy

Predictive accuracies are easily defined and measured by comparing the price predictions to actual vote shares. Specifically, we compare actual election outcomes to election-eve, normalized closing prices from the IEM. We use the last traded price of each contract at midnight on election eve as the closing price. We normalize prices by dividing each of them by the sum of closing prices for all contracts in the market. This insures that the normalized closing prices sum to the value of a unit portfolio which corresponds to 100% of the relevant vote.[17] The difference between the normalized closing price of a contract and the actual vote share received by the corresponding candidate is that contract's prediction error. Our measure of predictive accuracy is the average absolute prediction error of all contracts in a market. The individual and average absolute errors are given in Table 1 and the average absolute errors are shown in Figure 1. Individual contract errors range from 0% to 23% in absolute value. Average absolute errors range from 0.06% to 8.6%.

3.2 Election Properties: Level of Election

For the market to predict election outcomes accurately, traders must have information about the election and the market must aggregate this information. For traders to have information, it must exist and traders must be sufficiently motivated to obtain it. Our data includes markets run for national, state and primary level elections. The nature and scope of information available to traders varies greatly with the level of the election.[18] Moreover, the fraction of our traders that would be eligible to participate in each election varies with the level.[19] Typically, voter

[17] Prices may not sum to one because of nonsynchronous trading, the bid-ask spread and possible arbitrage violations.

[18] National elections typically generate more widespread interest than statewide elections and, as a result, have more media coverage than local elections.

[19] All traders who can vote in a primary election can also vote in state elections; those who can vote in state elections can also vote in national elections. The reverse is not the case. Each trader in a market who can also participate in the corresponding election has at least one piece of private information: whether and how they will vote in the election.

Table 2. Analysis Variables

Variable	Class	Description
Level of Election	Election Property	Categorical Variable: 1=National, 2=State, 3=Primary
Number of Contract Types	Election Property	Number of contract types traded on the IEM. Corresponds to number of major candidates and (usually) one for rest of field.
Volume	Market Property Market Activity	Total Market Dollar Volume between date of measurement and election. Measured from 1 to 7 days before the election.
Number of Active Traders	Market Property Market Activity	Total Number of Traders submitting limit or market orders between date of measurement and election. Measured from 1 to 7 days before the election.
Average Trader Order Numbers	Market Property Trader Experience	Average Order Number for Traders submitting limit or market orders between date of measurement and election. Measured from 1 to 7 days before the election.
Weighted Ask Queues	Market Property Queue Information	Average (over days and contract types) weighted ask queues between date of measurement and election. Measured at midnight from 1 to 7 days before election.
Weighted Bid Queues	Market Property Queue Information	Average (over days and contract types) weighted bid queues between date of measurement and election. Measured at midnight from 1 to 7 days before election.
Total Weighted Queues	Market Property Queue Information	Total average (over days and contract types) weighted bid and ask queues between date of measurement and election. Measured at midnight from 1 to 7 days before election.

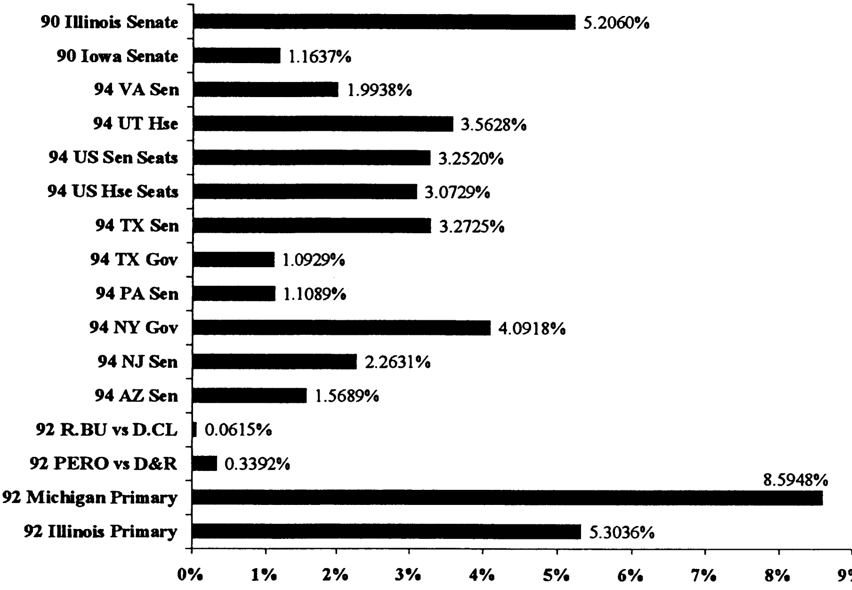

Figure 1: Average Absolute Prediction Errors

turnout also varies with the election level, reflecting the level of interest in the election to some degree.

We create a variable to provide a rough approximation of this by assigning a level of 1 to national (presidential) elections, 2 to state (governor, senate and house) elections and 3 to state primaries. Alone, this variable is significant with national elections resulting in higher predictive accuracy and state primaries in lower. The correlation between this variable and predictive accuracy is 0.7912. However, in our data, this level-of-election variable is also very highly correlated with the number of contract types traded in the market (with a correlation of 0.7883) and volume (with a correlation of -0.4736 with seven-day volume). Including this level-of-election variable with these other variables in a regression would create co-linearity problems. To eliminate the effects of the co-linearity and determine which variables to use, we regressed number of contract types on the level-of-election variable and vice versa. Similarly, we regressed volume on level-of-election and vice versa. We analyzed the resulting residuals by including them in the regressions we ran to explain predictive accuracy. This analysis shows that the number of contract types has significant independent effects while level-of-election does not. Similarly, volume has significant independent effects while level-of-election does not. For this reason, we choose to focus on number of candidates and volume.

3.3 Election Properties: Number of Candidates (Contract Types)

For each election, the IEM allows trade in a separate contract for each candidate deemed "important" by the IEM governors and, often, an additional contract

representing the rest of the field. We include the number of different contract types traded as a potential explanatory variable for two reasons. First, increasing the number of contract types increases the number of variables that the market must predict. In markets with two contract types, the market only predicts one (independent) variable. This variable is the vote share of one of the candidates (since the vote share of the other candidate is simply one minus this vote share). In markets with three contract types, the market must predict two variables: the vote shares for two of the three candidates, etc. Second, the political science literature (e.g., Riker, 1982) suggests that, under the US system of plurality voting, the possibilities for strategic voter behavior and the number of voting equilibria increase with the number of candidates This could make the election outcome more difficult to predict.

For this variable, we use the number of contract types traded, which corresponds roughly to the number of major candidates. This is a significant explanatory variable. More contract types generally decrease predictive accuracy. The correlation coefficient between number of contract types and predictive accuracy is 0.6777. As noted above, this variable is highly correlated with election level in our data. Thus, this may be capturing effects of election level as well as the number of contract types traded. However, as also discussed above, the number of contract types traded has significance above that associated purely with the level-of-election variable.

3.4 Market Properties: Market Activity

3.4.1 Trading Volume

Traditional rational expectations theory predicts no trade if there is asymmetric information (Milgrom and Stokey, 1982). So, volume in the market represents consensus expectations. Under market microstructure theory, more trade means that private information is revealed faster (see O'Hara, 1994). So more volume implies either greater consensus or more information revelation.

As a measure of trading volume, we look at total dollar volume in the market over the last one to seven days before the election. These volumes are highly correlated with each other and with predictive accuracy.[20] Any one of these volume measures would be significant. The total volume over the last five to seven days gives the most explanatory power. The seven-day time span includes the weekend before each election (during which candidates are likely making their last campaign swings). In addition, major polling organizations typically release their final poll results during this period. For these reasons, we use seven-day, total volumes in our regressions.

[20] The correlations of volume with predictive accuracy are -0.3692, -0.3781, -0.3744, -0.4482, -0.5025, -0.4965 and -0.4262 for one through seven-day volumes respectively.

3.4.2 Number of Active Traders

We include the number of active traders in our regressions for two reasons. First, if more traders have more information, then a simple aggregation argument implies that there will be more information reflected in the market price. Second, market micro-structure models show that increasing the number of informed traders increases competition between them and, thus, increases the speed with which this information is incorporated in the market.[21]

We use the total number of traders submitting bid, ask, purchase or sale orders to the market over the last one to seven days before the election. This variable is highly correlated with predictive accuracy, but is also highly correlated with volume. For example, the number of active traders over the last seven days has correlations of -0.2579 and 0.8621 with predictive efficiency and volume over the last seven days, respectively. Including both independent variables would create co-linearity problems. Again, regressing seven-day volume on the number of active traders over this period and using the residuals shows that the number of traders had no independent additional significance. Thus, we choose to use volume in the regressions.

3.5 Market Properties: Participant Experience Level

Evidence from experimental markets shows that the experience level of traders may affect the efficiency of the market. Smith, Suchanek and Williams, 1988, show that markets with more experienced subjects are less prone to obviously inefficient pricing "bubbles." Forsythe and Lundholm, 1990, show that markets with more experienced traders are more likely to aggregate information efficiently. Thus, more experienced traders may increase efficiency levels. Oliven and Rietz, 1995, find that traders in the 1992 market tend to make fewer mistakes as they gain experience. They also show that more experienced traders are more likely to send limit orders to the market and limit orders are less likely than market orders to obviously violate arbitrage and individual rationality constraint. Thus, in this sense, more experienced traders in the IEM are better. Finally, more experienced traders may be those who have been studying this election longer and, potentially, bringing more information to the market.

The IEM provides a proxy for IEM trading experience. It assigns a trader-specific, sequential order number to all orders submitted by each trader. Thus, the order number reflects the trader's previous experience in submitting orders to the market. As a measure of experience level, we use the average order number for all orders submitted to the market over the last one to seven days before the election. While these one to seven-day measures are highly correlated with each other, they do not appear highly correlated with predictive accuracy. (The largest correlation coefficient in absolute terms is -0.0166 for the four-day experience level.) Nevertheless, when included in a regression with volume, number of contract types traded and spread information, the experience level is marginally significant. We

[21] See Foster and Viswanathan, 1994

include the seven-day experience level in one model and drop it in the other. We find that dropping this variable has little effect on overall explanatory power.

3.6 Market Properties: Queue Information

3.6.1 Weighted Spreads

We include information about spreads because of their importance in market microstructure theory. In market microstructure models, the market maker increases the quoted spread in response to higher level of private information since a higher level of private information results in a larger adverse selection problem. To the extent that a consensus exists and information has been aggregated, spreads will shrink. We weight the spreads by the dollars committed at the bid and ask because our markets differ from those discussed in theory. In theoretical models, dealers stand ready to fill the maximum possible order size at all times (see O'Hara, 1995). In our markets, this is not true since traders specify the quantity they are willing to transact at a given price. To measure the "effective" spread, we aggregate across our traders to form a surrogate for an "aggregate" market maker. In aggregating, we weight the spread by the dollar quantity traders have committed to trade at the best bid and ask.

We compute weighted spread measures for the last one to seven days before the election. For each day and contract type, we take the difference between the closing best ask and closing best bid divided by the total dollar quantity committed at this bid and ask. We then average over days and contract types. These (seven) variables are highly correlated with each other and also with one to seven-day volumes. All reflect the level of market activity to some degree. Spreads reflect market-making, limit-order activity while volumes reflect price-taking, market-order activity. To eliminate effects of co-linearity, we use the residuals from a regression of seven-day volume on the seven-day weighted spread. The residuals from this regression show there is a significant independent effect of the weighted spread. However, the results also show that adding this variable does little to increase the explanatory power of the model overall.

3.6.2 Weighted Queues

Market microstructure theory also leads us to include weighted queue information. In market microstructure models, the market makers adjust the bid and ask to reflect the information revealed by each trade. When there is a great deal of private information, spreads are wide and adjustments are large. As this information is revealed, spreads narrow and adjustments become smaller. In our markets, the sensitivity of the market in response to a trade can be measured by the weighted queue, where larger weighted queues mean less price sensitivity.

The weighted bid queue is given by the average (across both contract types and days) of the sum of all closing bids weighted in the following manner. First, we multiply the bid times the dollar quantity committed at that bid. Then we multiply

by the bid over the best bid. Thus, the weighting ranges from 0 for a bid at a price of \$0 to 1 times the dollar quantity committed for the best bid. This serves as a measure of demand or depth on the bid side.

The weighted ask queue is given by the average (across both contract types and days) of the sum of all closing asks weighted in the following manner. First, we multiply the ask times the dollar quantity committed at that ask. Then we multiply by one minus the ask over the one minus the best ask. Thus, the weighting ranges from 0 for an ask at a price of \$1 to 1 times the dollar quantity committed for the best ask.[22] This serves as a measure of supply or depth on the ask side.

The total weighted queue is the sum of the average (across both contract types and days) of the weighed bid and ask queues. These weighted queue variables are all highly correlated with each other and with volumes, each reflecting the general level of market activity. Alone, each is significantly correlated with predictive efficiency. However, there is no significant independent effect after the effects of volume are removed. Thus, we choose to focus on volumes instead of these weighted queue variables.

3.6.3 Differences in Weighted Queues

We include differences in weighted queues for two reasons. First, in the markets reported in Smith, Suchanek and Williams, 1988, imbalances between bids and asks affect subsequent price movements. Second, technical traders often try to determine "support" and "resistance" levels in stock prices and the strength of these levels. Hardy, 1978 (p. 73), defines "support" as "the price level where a declining security may be expected to be supported by buyers." He defines "resistance" as "the price level where an advancing security may be expected to be dumped by sellers." Technical traders cannot determine *actual* support and resistance levels nor their relative strengths. Instead, they estimate these levels and their relative strengths by charting past price movements. At any point in time on the IEM, the bid queue represents the current *actual* support level for the market *directly*. It shows exactly the prices at which buyers are willing to purchase any given quantity at that point in time. Similarly, the ask queue represents the current *actual* resistance level for the market. Their difference is a measure of relative strength. If the weighted bid queue is much smaller than the ask, then there is low support and much resistance and vice versa. Thus, large difference in weighted queues may imply that prices are moving and the market has not yet reached an equilibrium.

The difference in weighted queues is the absolute difference in the average (across both contract types and days) of the weighted bid queues and the average of the weighted ask queues. Measured in this way, the difference in weighted queues does not measure imbalances between buyers and sellers for individual contracts. Instead, it measures the imbalance overall in the market. Again, these measures are highly correlated with each other. We choose to use the election eve difference because this

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[22] Adjustments were made for the \$2 maximum price in the 1990 Senate markets by using two minus the ask over two minus the best ask to weight the ask queues.

should reflect whether the market contains an imbalance between buyers and sellers at the time we measure predictive accuracy (with election eve prices). The election eve difference is very significant and, surprisingly, not particularly highly correlated with volume. (The correlation between this and the one day volume is only 0.0499.)

4 Regression Results

Analyzing the correlation structure indicates a strong relationship between many of our potential independent variables. To a large degree, they all represent the same things: interest in and information about the election, market activity and the level of "balance" from the bid and ask sides of the market. To construct a parsimonious model, we have systematically selected variables that reflect this information, have the most explanatory power and are not highly correlated with the other independent variables in the regression. To choose which of two correlated variables to place in the regression, we analyzed the errors from regressions of these two variables against each other. This showed whether the orthogonal components of the variables had independent significance. In only one case did they both have independent significance. The orthogonal components of both volume and average weighted spreads were significant. Therefore, we use volume and the component of spreads that was orthogonal to volume (i.e., the residuals from the regression of volume on spreads) as explanatory variables.

The final regression results are given in Table 3. Recognizing that there may be election specific effects, we report both OLS standard errors and standard errors corrected by Huber's, 1967, method for analyzing clustered data.[23] The regressions show that the number of contract types, seven-day dollar volumes and average election-eve differences in weighted queues were all significant in explaining variances in predictive accuracy across markets. Further, these three variables provided a great deal of explanatory power with an adjusted R^2 of 93%. Model II in Table III adds average trader order number (as a measure of experience) and the residuals from a regression of volume on average weighted spreads to the regression. While both variables were significant, they did little to increase explanatory power, increasing the adjusted R^2 only 2%. Figure 2 shows the actual predictive accuracy of each market and the accuracies predicted by Models I and II. Predictions correspond very closely to the actual data.

Given the correlation structure of the independent variables, we interpret the results to mean that there are several broad factors determining market efficiency: complexity, level of interest and market convergence. The Model I regression

[23] Election specific effects may be present since some election outcomes were collected on the same election day. For example, the 1992 Illinois and Michigan primaries were on the same day and in both, Tsongas received unexpectedly low vote totals. This could have been due to a common factor such as speculation regarding the recurrence of his cancer. Similarly, unexpectedly low Democratic support during the 1994 election, possibly effecting errors in each 1994 election market we conducted. In adjusting the standard errors, we assumed each election date constituted a cluster of data. We thank Dan Friedman for pointing out how this may effect our results.

Table 3. OLS Models of Average Absolute Prediction Errors

Variable	Model I Coefficients (OLS Std. Err.) [Huber Std. Err.]*	Model II Coefficients (OLS Std. Err.) [Huber Std. Err.]*
Number of Contract Types	0.693734	0.8412334
	(0.0928902†)	(0.1455499†)
	[0.0894716†]	[0.1047139†]
Total Dollar Volume During	-0.009712	-0.0117971
Seven Days Before Election	(0.0025225†)	(0.0021884†)
	[0.0017686†]	[0.0017826†]
Average Difference in	0.0515215	0.0600437
Weighted Bid/Ask Queues at	(0.0096603†)	(0.0091836†)
Midnight Before Election	[0.0076805†]	[0.0086082†]
Average Submitted Order	--	-0.0002093
Number Over Seven Days		(0.0001291‡)
Before Election		[0.0000649†]
Average Residual Weighted	--	16.66211
Spread Over Seven Days		(5.582632†)
Before Election		[2.741727†]
F:	75.48†	71.14†
Adjusted R^2:	0.9332	0.9564
N:	16	16
Root MSE:	0.92465	0.74718

*Corrected using Huber's, 1967, method clustering on election date.
†Significant at the 99% level of confidence
‡Significant at the 85% level of confidence

coefficients indicate adding one contract increases the prediction error by about 0.7%. Since the number of contract types is correlated highly with election level, this likely shows the differences between, and differences in complexity between, state primaries, state elections, and national elections. Similarly, increasing volume by an average of $10 a day over the last week decreases prediction error by about 0.7%. Volumes are highly correlated with election levels, numbers of active traders, average weighted queues and average weighted spreads. Thus, the volume results also likely indicate a higher general level of interest in the election, which increases accuracy. Finally, the results indicate increasing the election eve difference in weighted queues by $10 increases the prediction error by about 0.5%.

5 Conclusion

Prices from the Iowa Electronic Market can be used to predict election outcomes. Just as pollsters are concerned with the margin of error in their polls, we are

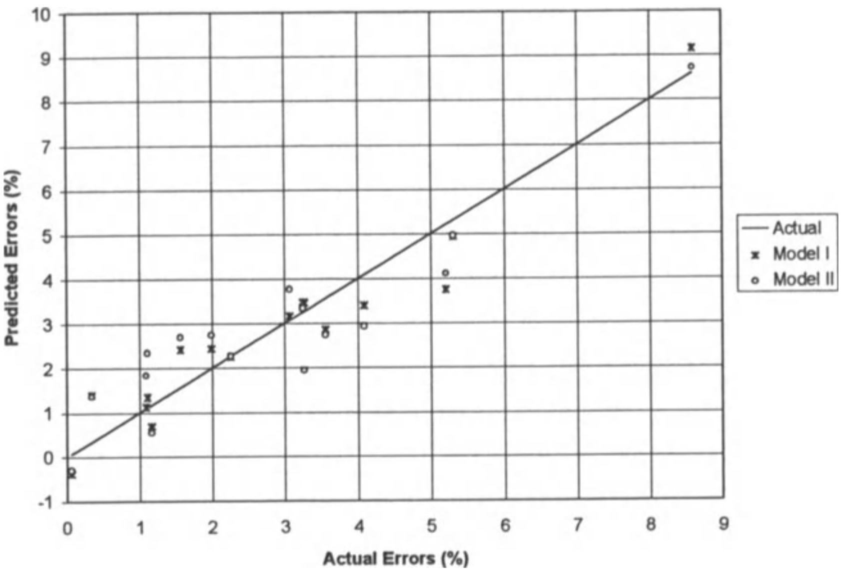

Figure 2: Actual versus Predicted Levels of Average Absolute Market Prediction Errors

concerned with the predictive accuracy of our markets. Here, we relate the accuracy of the markets in predicting election outcomes to various observable election and market specific factors.

In explaining market accuracy, we find many significant factors motivated by financial theory and previous experimental research. In particular, across the markets we examine, variations in accuracy are largely explained by: 1) the number of contract types traded in a market (corresponding roughly to the number of major candidates), 2) the dollar volume of trade over the week before the election and 3) differences in election eve (weighted) market bid and ask queues.

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Sequencing and the Size of the Budget Experimental Evidence*

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Abstract. We present the results of an experiment designed to test the proposition that sequencing budget decisions has systematic influence on the size of the budget. The results suggest that individuals vote strategically, leading to the subgame perfect equilibrium of the underlying voting game. The evidence yields no support for the claim that top-down budgeting provides for more fiscal discipline than bottom-up budgeting.

I. Introduction

In times of fiscal crisis, there is acute interest in the government budget process and often proposals for its reform are made. In the USA, concern about excessive government spending and deficits has led to a series of legislative reforms of the federal budget process, including the Budget Act of 1974, the Gramm-Rudman-Hollings Act of 1985, and the Budget Enforcement Act of 1991. In Europe, procedural reform of member government budget processes to enhance fiscal discipline is mandated by the Treaty of Maastricht, although the treaty does not provide for any specifics.

In the broadest sense, a budget process is a system of rules, both formal and informal, governing the decision-making process that leads to the formulation of a budget, its passage through the legislature, and its execution. The rules divide this process into steps, and determine which steps are taken when. The rules also assign roles and responsibilities to participants and thus distribute strategic influence among them.

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Interest in the budget process derives from the widespread belief that the process itself can shape the decisions made according to its rules. In particular, it is argued that by changing the budget process, the results of the process can be changed in a predictable way. As Irene Rubin puts it, budget processes can be designed to emphasize and facilitate any of the following goals: growth or reduction of expenditures, balance or deficit spending, trade-offs between programs and goals, competition, and financial and managerial control.[2] Game theory is a natural tool to investigate the consequences of such a system of rules for the fiscal performance of governments, and we show that Reinhard Selten's (1965) concept of subgame perfect equilibrium provides a solution for the game that arises from a budget process.

One particular aspect of the budget process that has drawn much attention is the sequence of budget decisions. There are two opposing models, bottom-up budgeting and top-down budgeting. Bottom-up budgeting starts by collecting the budget requests of the spending agencies. Votes are taken on each agency request. The total budget results from adding up the individual agency appropriations. By contrast, top-down budgeting starts with a decision on the total budget. Once the total has been determined, it is divided among the spending agencies. Note that top-down should not be confused with centralization, i.e., giving the head of state or the finance minister superior strategic power over the budget process. That these two concepts are very different becomes clear when one compares the budget processes among members of the European Union.[3] For example, the French process is very centralized and top-down. In contrast, the British process, which is also very centralized, is a bottom-up process.

Conventional wisdom holds that top-down budgeting is more appropriate than bottom-up for limiting the growth of spending and deficits. For example, a comparative analysis of budgeting by the OECD (1987) reports that several countries switched to top-down procedures in the early 1980s, on the grounds that this meant progress towards fiscal discipline.[4] In commenting on this study, Alan Schick (1986, p. 125) argues that "... If they are not blocked in advance, powerful spending claims can impel the government to spend ... more than it wants." Schick goes on to explain that a process starting with the determination of the overall budget total is appropriate to achieve greater fiscal discipline. In the USA,

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[2]See Irene S. Rubin, The Politics of Public Budgeting: Getting and Spending, Borrowing and Balancing. New York: Chatham House, 1993, p. 63.

[3]The classic study in this field is Aaron Wildavsky, Budgeting, Oxford, Transaction Publishers, 1975. For more recent treatments, see Jürgen von Hagen, "Budgeting Procedures and Fiscal Performance in the EC," European Commission (DG II) Economic Papers, 1992, and Jürgen von Hagen and Ian Harden, "National Budget Processes and Fiscal Performance," European Economy, Reports and Studies 3, 1994, 311-418

[4]See OECD, The Control and Management of Government Expenditure. Paris, 1987.

the Budget Act of 1974 introduced elements of top-down budgeting expressly to enhance spending control.[5] We call this the

Superiority Proposition: Top-down budgeting leads to greater fiscal discipline.

Theoretically, however, there is little reason to believe that the sequence of decisions *per se* has unambiguous implications for the size of the budget. This point is explained by Ferejohn and Krehbiel.[6] In their model, rational decision makers recognize the constraints they create for themselves in the first step of a top-down procedure. Such decision makers take their objectives in later votes into account when they decide upon these constraints in the first place. Given strategic voting behavior, one cannot predict the size of the budget solely on the basis of the sequence of decisions: one must also know the exact preferences of the decision makers. In particular, for any sequence of top-down voting, there exist preferences such that the analogous bottom-up voting procedure leads to a smaller total budget.

One has thus a contradiction between the conventional wisdom of budget processes and formal theoretical results. Empirical evidence is needed to shed light on this conflict. Empirical research into this question can proceed in either of two ways. The conventional approach is to compare the fiscal performance of governments operating under bottom-up and under top-down. This has the obvious advantage of using data produced by professional decision makers, but it also has the disadvantages of all comparisons across countries. In addition, real life budget decisions depend on a host of factors besides sequencing, many of which are unobserved, making it hard to identify the role of sequence in the outcome. Another approach is to conduct controlled experiments in laboratory settings, an approach itself pioneered by Reinhard Selten.[7] The experimental design allows one to abstract from other determinants of budgeting decisions, and to focus on the role of sequence in those decisions. The two approaches should be regarded as complementary rather than competing. The purpose of this paper is to provide such empirical evidence from controlled experiments. Such data should be reinforced with data from real world voting. In any event, our goal is not to single out a single budget process as ideal, but rather to gain further insight into the impact of sequence on budget outcomes.[8]

The paper proceeds as follows. Section II sets out a basic voting model, which has top down and bottom up voting as special cases. Section III considers briefly some existing empirical results. Section IV describes the experimental design and procedures. Section V reports preliminary results, relying heavily on the measure

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[5]See Committee on the Budget, United States Senate, Gramm-Rudman-Hollings and the Congressional Budget Process-An Explanation. Washington, US Government Printing Office, 1985.

[6]See John Ferejohn and Keith Krehbiel, "The Budget Process and the Size of the Budget," American Journal of Political Science 31 (1987), 296-320.

[7]Selten ran his first experiment in Frankfurt in 1959.

[8]This also means that we do not consider the problem of a just budget here.

of predictive success introduced by Reinhard Selten (1991), while section VI concludes.

II. A Model of Budgeting[9]

Let R_+^2 denote the positive orthant of the two-dimensional Euclidean space, the space of all possible budgets in our setting. Let the vector $\mathbf{x} = (x_1,x_2) \in R_+^2$ denote a typical budget. The restriction to two dimensions here is made only in the interests of simplicity--none of the results depends crucially on it. One can think of each dimension as a spending category in the overall budget.

There are n voters, represented by the set {1,2,...,n}. We assume that the number of voters is odd.[10] Each voter i has Euclidean preferences, centered at his or her ideal point **x*(i)**. Letting d(.,.) be the Euclidean distance function, voter i's utility function can be written

$$(1)\ u_i(\mathbf{x}) = -d(\mathbf{x},\mathbf{x}^*(\mathbf{i})).$$

(1) says that the further away the actual budget is from voter i's ideal budget, the less voter i likes the outcome.[11] We assume that no two voters have the same ideal point. One story behind this assumption is that the voters are the various spending ministers in the cabinet, and each spending minister has his or her own spending priorities reflected in the ideal point. A budget **x** is Pareto optimal if there is no other budget y with the property that every voter prefers y to **x**. Given our assumptions, the set of Pareto optima consists of the convex hull of the set of ideal points of the voters. Thus, the Pareto set is itself a convex set in R_+^2.

Budgets are chosen by majority vote of the n voters. To avoid cycling, the budget process divides the choice of the budget into two one-dimensional decisions. Our experiments compare two budget processes differing in the sequence of decisions, namely a bottom-up and a top-down process.

The bottom-up game begins with the forced move $x_1 = 0$. This is the proposal on the floor. There follows a random recognition move, where one of the n voters is recognized to make an alternative proposal, either a new value of x_1, or a motion to end debate. A majority vote decides between the proposal on the floor and the alternative proposal. If the alternative proposal is a new value of x_1 and it wins, this becomes the proposal on the floor. There then follows another random recognition move, and so on. If the alternative proposal is to end debate, and it

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[9]This model closely follows Ferejohn and Krehbiel (1987).

[10]This assumption guarantees that the median on any 1-dimensional vote is unique. It is worth noting that many real world voting groups are instituted with an odd number of voters to avoid ties and other multiplicities in the voting. Again, some bodies with an even number of voters, like the United States Senate, add an extra vote (the Vice President) in the case of ties--thus becoming an odd-numbered body.

[11]This assumption is substantive, and the results are at least somewhat sensitive to it.

wins, then the proposal on the floor is accepted. The value x_1 implied by the current proposal on the floor is adopted and the voting process enters stage 2. Once voting has entered stage 2, questions concerning the value of x_1 can never again be brought up. This is the crucial structural feature which prevents voting cycles.

Stage 2 begins with the forced move $x_2 = 0$ which becomes the proposal on the floor. It then proceeds analogously to stage 1. Stage 2 ends when a majority has voted to end debate and, thereby, has accepted the value x_2 implied by the current proposal on the floor. We denote the outcome of bottom-up voting by the budget vector $\mathbf{x}_{BU}$.

Top-down budgeting follows similar rules to bottom-up. The main difference is that the total budget colume, $x_1 + x_2$, is determined at the first stage and the value of x_1 is determined at the second stage. Thus, the forced move at the beginning of the top-down game is $x_1 + x_2 = 0$. At this point the first random recognition move takes place and alternative proposals for $x_1 + x_2$ are considered. Once the group has accepted a proposal to end debate in stage 1, the value of $x_1 + x_2$ implied by the last proposal on the floor is accepted and is fixed for the rest of the game. Again, this structural feature prevents cycling. Stage 2 begins with the forced move $x_1 = 0$ which becomes the proposal on the floor and is followed by a random recognition move. Stage 2 ends when a majority has voted to end debate, thereby accepting the value of x_1 implied by the last proposal on the floor. We denote the outcome of bottom-up voting by the budget vector $\mathbf{x}_{TD}$.

In general, for a fixed set of voters' ideal points, $\mathbf{x}_{BU}$ and $\mathbf{x}_{TD}$ differ. Consider a simple example with three voters, whose ideal points are

$$\mathbf{x}^*(1) = (8,8), \quad \mathbf{x}^*(2) = (12,8), \quad \mathbf{x}^*(3) = (8,12).$$

Then we have $\mathbf{x}_{BU} = (8,8)$, since the median x_1 is 8, and the median x_2 is also 8. On the other hand, $\mathbf{x}_{TD} = (10,10)$, since the median $x_1 + x_2$ is $x_1 + x_2 = 20$, while the median x_1 given $x_1 + x_2$ is $x_1 = 10$. Graphically, the first vote is determined by the median -45° line through the ideal points, and the second vote is determined by the median 45° line through the ideal points. These two lines meet at (10,10). Notice in this example that bottom-up voting leads to a smaller total budget, 16, than does top-down voting, 20.

For any such example, we can reverse that inequality by a reflection of the ideal points. For the above example, replacing voter 1's ideal point by $\mathbf{x}^*(1) = (12,12)$, a reflection around the line $x_1 + x_2 = 20$, leads to the budgets $\mathbf{x}_{BU} = (12,12)$, $\mathbf{x}_{TD} = (10,10)$. Now bottom-up voting leads to a larger budget than top-down voting. Conversely, if for a given set of ideal points bottom-up voting leads to a smaller budget than top-down voting, we can reverse this inequality by reflecting ideal points around the -45° line $x_1 + x_2 = \mathbf{x}_{TD}$.[12]

[12]One can show that a sufficient condition for $x_{BU} = x_{TD}$ is that there exist a Condorcet alternative x, in which case $x = x_{BU} = x_{TD}$.

We can also interpret the voting equilibria of top-down and bottom-up voting as game equilibria. To see this, let G be the extensive form game that is induced by bottom-up voting over budgets. After each proposal that is recognized, voters vote simultaneously. One subgame perfect equilibrium path involves all voters proposing the median value in the x_1 direction, followed by a majority of voters voting simultaneously for that proposal. Then all voters propose closure, with a majority voting for closure--and similarly for stage 2. The subgame perfect equilibrium outcome is x_{BU}.[13] The argument for top-down voting is similar. Thus, one can summarize the above argument by saying that if voting over budgets is strategic, then the Superiority Proposition need not be true.

III. Existing Empirical Results

There are two kinds of empirical results which could shed light on the Superiority Proposition. One kind would compare the outcomes of real world settings that differ in the use of top-down or bottom-up voting. One such study is due to Crain and Miller (1993). In a statistical study of the fiscal constitutions of the 50 American states, they run regressions with the growth of state government spending on the left hand side, and various explanatory variables on the right hand side. The critical explanatory variable is a dummy variable for whether the state in question uses bottom- up or top-down budgeting. The resulting coefficient suggests than top-down budgeting reduces the rate of growth of state spending by 1.3%. However, that the standard error of this coefficient exceeds 1.3%, and thus the result is not conclusive.[14]

A second source of empirical data is provided by voting experiments.[15] Although we know of no economics experiments involving voting over budgets per se, voting experiments may provide data to shed light on whether human subjects vote strategically or not. For instance, Eckel and Holt (1989) in a series of voting experiments find some evidence in favor of strategic voting. This provides indirect evidence in favor of subgame perfect equilibrium play, and thus against the Superiority proposition. It must be noted, however, that subjects first had to acquire considerable experience in these voting experiments before they exhibited strategic behavior. Inexperienced subjects, by contrast, voted in a rather naive fashion.

We now turn to the present set of experiments, which we hope provide direct evidence concerning the Superiority Proposition.

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[13]This is not the only subgame perfect equilibrium path, but it is the only subgame perfect equilibrium outcome.

[14]See Crain & Miller (1993) for further details.

[15]See Davis and Holt (1993) for an insightful survey of voting experiments over the past 25 years.

IV. Experimental Design and Decision Setting

Two series of trials were conducted, one at the University of Mannheim during June and July 1994, and another at Indiana University during October 1994. Both series were run by hand, each with 5 subjects.[16] In both series, volunteers were recruited from undergraduate economics courses. Before volunteering, subjects were informed that they would participate in a decision-making experiment, would be paid in cash, and could expect the experiment to last about an hour. Subjects entered a classroom, and were seated in such a way that no two subjects were face to face. Subjects privately went through a set of instructions in their native language (English and/or German instructions available upon request). The instructions consisted of two main parts. The first part described the payoff function, the second the voting rules. The instructions were then read aloud to the subjects. Subjects had to complete a short quiz before proceeding into the main part of the experiment. At this point, subjects were given their own payoff functions (both as a formula and as a payoff table), as well as the ideal points of all other subjects. Subjects thus had complete information about the voting game. Subjects had the opportunity to ask an experimentor a question at any time during the experiment, but were not otherwise allowed to speak. At the end of a session, subjects filled out a debriefing questionnaire and then were paid privately in their home currency. No subject participated in more than one trial.

The basic design treatments were the distribution of ideal points and the budget process, whether bottom-up or top-down. The four distributions of ideal points used are shown in Table 1. The distributions work in pairs, (IA, IB) and (IIA, IIB), each of which differs in exactly one ideal point. By reflection, in each pair of distributions, $\mathbf{x}_{BU}$ costs more in one and $\mathbf{x}_{TD}$ costs more in the other. Distribution IA has the feature that at a subgame perfect equilibrium top-down voting leads to a 20% smaller budget than bottom-up voting. Distribution IB has the feature that at a subgame perfect equilibrium bottom-up voting leads to a 20% smaller budget than top-down voting. These distributions have a further important feature, which we call strong median. A strong median is any median which is shared by more than one voter. Consider for instance $\mathbf{x}_{TD}$, which costs 20 for both distributions IA and IB. This median is strong, since 4 voters (numbered 1,2,4, and 5) have the ideal value $x_1 + x_2 = 20$. Again, consider $x_1 = 8$, part of $\mathbf{x}_{BU} = (8,8)$ in distribution IB. Two voters (numbered 2 and 3) have ideal points with $x_i = 8$ in the x_1 direction. Since the subgame perfect equilibrium depends so strongly on median voter considerations, such an equilibrium is most likely to be observed when a median is strong.

[16] The experiment has now been programmed for the Business Behavior Laboratory at the University of Mannheim. Subsequent experiments will be computerized.

Table I: Experimental Design

	Ideal Points			
	Design I		Design II	
Individual	A	B	A	B
1	7, 13	7, 13	6, 13	6, 13
2	8, 12	8, 12	7, 9	7, 9
3	8, 8	12, 12	8, 16	8, 16
4	12, 8	12, 8	11, 12	9, 9
5	13, 7	13, 7	11, 14	11, 14
Subgame Perfect Equilibrium Predicition				
Bottom up	12, 12	8, 8	8, 13	8, 13
Top Down	10, 10	10, 10	10, 13	8, 11

The pair of distributions (IIA, IIB) do not have strong medians. Indeed, the medians for these distributions never equal any individual ideal point. Thus, we would expect more deviation in observed voting from subgame perfect equilibria in these designs. As with the pair (IA, IB), these distributions have the feature that exactly one ideal point differs between them. The predicted budget under top-down voting exceeds the predicted budget under bottom-up voting by 9.5 percent in IIA, while the reverse is true in IIB.

V. Experimental Results

Results are reported for a total of 11 trials, conducted over the four ideal point distributions and the two budget processes, top-down and bottom-up. Aggregate results are portrayed in Table 2. Data from both German and American subject pools are pooled in this portrayal; we have not found any significant difference between the behavior in these two subject pools.

To discuss the results, we rely on a descriptive measure of predictive success developed by Reinhard Selten.[17] A descriptive measure of predictive success tells whether an observation is consistent with a theoretical prediction (a hit) or

[17]See Reinhard Selten, "Properties of a Measure of Predictive Success," Mathematical Social Sciences 21, 1991, 153-200.

inconsistent with it (a miss). The hit rate, r, is the ratio of hits to total observations. The relative area of a theory, a, is the number of points that count as a hit relative to the number of all points which might be expected to be observed in a single trial. Selten axiomatizes the following measure of predictive success:

$$(7)\ m = r - a,$$

where m is the predictive success measure, r is the hit rate, and a is the relative area.

We consider the predictive success of the following three theories here. The first theory, T1, is subgame perfection. This theory predicts a single point outcome. The second theory, T2, is subgame perfection plus small errors. According to Selten's doctrine of trembling hand perfection (Selten, 1975), a theory should be capable of withstanding the effects of small errors in play. Here we define a small error as one which is at most one unit away from the subgame perfect equilibrium in either direction. Thus, T2 counts as a hit any observation in the square with side of length 3, centered at the subgame perfect equilibrium outcome. T2 thus encompasses T1. The third theory, T3, predicts a Pareto optimum.

We can compare these three theories in terms of relative areas, once the set of outcomes which might be expected in a single trial has been established. For our purposes, we take any outcome which lies within the bounds of the set of ideal points as one which may be expected. More precisely, let min x_1* be the minimum value of the ideal points in the x_1 direction; max x_1*, the maximum value of the ideal points in the x_1 direction. Similarly, let min x_2* be the minimum value of the ideal points in the x_2 direction; max x_2*, the maximum value of the ideal points in the x_2 direction. Then the area relative to which we measure any theory's predictions is the rectangle with the following vertices:

$$(\min x_1^*, \min x_2^*),\ (\max x_1^*, \min x_2^*),\ (\min x_1^*, \max x_2^*),\ (\max x_1^*, \max x_2^*)$$

In the design I treatments, there are 49 such points. For this treatment, the relative area of T1 is 1/49, or 2%; of T2, 9/49, or 18%; of T3, 21/49 or 43%. For the design II treatments, the corresponding relative areas are 3%, 24%, and 58% respectively.

Table II reports the results of the 11 trials performed. Consider first T1, subgame perfection. For this theory, there were 4 hits out of 11 trials, for a hit rate of 36%. Since the range of relative areas is from 2% to 3%, the predictive success of this theory ranges from 34% to 36%. Next consider T2, subgame perfection plus small errors. For this theory, there were 9 hits out of 11 trials, for a hit rate of 82%. Since the range of relative areas is from 18% to 24%, the predictive success of this theory ranges between 58% and 64%. Finally, consider T3, Pareto optimum. For this theory, there were 10 hits out of 11 trials, for a hit rate of 91%. Since the range of relative areas is from 43% to 58%, the predictive success of this theory ranges from 38% to 43%. The theory with the best predictive success for this data is T2, subgame perfection plus small errors.

Table II: Empirical Results

Budget Process	Design	Obser-vation	Thoretical Prediction		
			T1	T2	T3
	IA	12, 13	0	1	0
Bottom	IB	8, 8	1	1	1
Up	IIA.1	11, 13	0	0	1
	IIA.2	8, 13	1	1	1
	IIB	8, 14	0	1	1
	IA	10, 10	1	1	1
Top	IB	12, 8	0	0	1
Down	IIA.1	11, 12	0	1	1
	IIA.2	11, 13	1	1	1
	IIA.3	10, 13	0	1	1
	IIB	7, 10	0	1	1
relative area (%)					
Design I			2	18	43
Design II			3	24	58
Predictive Succes (%)			34- 36	58 - 64	38-43

VI. Conclusion

This paper has presented a model of strategic voting over budgets, along with data from a controlled experiment to see the extent to which observed behavior is consistent with strategic behavior. Our main finding is that, in 2-dimensional voting games with 5 players, observed behavior is best predicted by subgame perfection plus small errors.

This result should be of interest to budget reformers. To the extent that budget outcomes are nearly subgame perfect, hence nearly Pareto optimal, the role of reform is purely redistributional. In particular, we find no evidence in favor of the Superiority Proposition that top down budgeting is isystematically conducive to

smaller budgets than bottom up budgeting. It appears that, with strategic voting, neither budget process can dominate the other.

Of course, these results must be looked upon as provisional, since 11 observations are not nearly enough to build a fully satisfactory theory. Still, it is interesting to note that the observations fall so close to subgame perfect equilibrium outcomes. It is often the case in experiments that subgame perfect equilibrium has little if any predictive success, and so one might wonder why the success here. One possibility is that this success is actually due to myopic behavior. That is, the players, when voting on one dimension, simply vote as if that were the only dimension. Myopic behavior in this sense would also lead to the subgame perfect equilibrium outcomes, although for rather different reasons than subgame perfection. We hope to explore the potential difference between myopic behavior and backward inductive behavior more fully in the sequel, by exploiting non-Euclidean preferences.

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Intertemporal Speculation Under Uncertain Future Demand: Experimental Results*

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Abstract

This paper explores a market in which a subset of agents acting independently without direct communication can purchase commodities to carry forward in time in the face of uncertain future demand. The hypothesis that the equilibrating properties of markets will coordinate decentralized decisions to speculate as if all information was public gives no theory about the mechanism through which such information transfer might take place. The results provide general support for the validity of the equilibration suggestion. The mechanisms of information transfer seem to be located in the local nature of the price formation and carry-forward decisions coupled with a tendency for traders to specialize their activities.

1. Introduction

This paper explores the behavior of markets in which a subset of agents can purchase commodities in one period and resell in the subsequent period. At the time of purchase the demand in the second period is unknown, and no limitations are placed on the number of units carried forward either by an individual agent or the agents as a group. The decisions take place in an environment in which speculative decisions are made without knowledge of demand and supply parameters, without knowledge of the number of other speculators and without knowledge of the number of units that might be carried forward by other agents. Received theory and existing experimental work suggests that the equilibrating properties of markets will coordinate decisions as if all information was public. This paper explores the validity of that suggestions and the mechanisms through which information dissemination and coordination might take place.

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Intuition about the behavior of markets under the circumstances described above can be drawn from several studies. In part these results can be viewed as a replication and extension of the experiments reported in Agha and Plott (1983) who suggest that the rational expectations model functions well in carryover markets with uncertain future demand. The relationship with that study will be reviewed in detail below. Such an optimistic message is muted somewhat by the work of Camerer and Weigelt (1991) who have demonstrated that market equilibration can (but not always) reflect mirages that mislead agents about the nature of the private information that exists in the environment. The capacity of markets to form expectations or belief "bubbles" has also been demonstrated by Smith et al (1988). In addition, the work of McKelvey and Palfrey (1992) suggests that rationality of agents is not public information and the markets as well as games can result in "bubble-like" patterns of choices based upon misperception about the rationality of others. A strong presumption exists that time and experience produce elements that place beliefs in closer proximity to the rational expectations model, Forsythe and Lundholm (1990), but the conditions under which such beliefs can develop are unknown. Thus, while there is reason to suspect that the classical economic models are fundamentally reliable, exceptions do exist. Institutions and processes that have a substantial information aggregation component must be examined independently and carefully until the principles that govern the phenomena of information formation are more fully understood.

The Agha and Plott paper demonstrates markets in which the decisions of speculators, whose identity is public, cause markets to operate substantially in accord with the rational expectations model. The nature of the specific results of the paper, the experimental procedures, and the market parameters, suggest two concerns that the conclusions might be sensitive to, if not dependent upon, specific features of the experiments. The first concern is about the equilibrating properties of the market as the number of agents, the number of speculators, and the optimal amount of carry-forward increases. The Agha and Plott parameters involved only two traders with an option to carry forward and the optimum carry-forward was only four units. Will a larger, more complex market with more information to be aggregated from a larger number of sources still find the optimum allocation? If so, then how does the structure of the market act to bring the carry-forward close to the optimum quantity? The traders act in ignorance of each others' decisions. If there are price differences between the two periods, which suggests the existence of an expected profit, then risk-neutral agents would want to carry forward an infinite quantity. If the story of the Agha and Plott experiments is correct, then agents do not seek an infinite carry-forward as individuals, and as a group they decide on exactly the amount that should be carried forward and how this total should be divided up among them, without ever

explicitly communicating. The paper focuses on examining the mechanism through which this coordination might occur.

2. Experimental Environment and Parameters

A total of six markets were created and studied over a total of four experimental sessions, divided into two series. In Series 1, consisting of experiments 1124 and 1208, one market per session was conducted. In Series 2, experiments 0322 and 0323, each session consisted of two markets conducted simultaneously and connected in a manner to be outlined below. For consistency of reference we shall refer to each market as a separate experiment. All experiments were conducted at the California Institute of Technology using twelve subjects drawn from undergraduate and graduate students and staff at the Institute. The markets were organized as a computerized multiple unit double auction. (Plott (1991)) All participants were trained to use this software, and had experience in at least one experiment using the software. When subjects arrived at the experiment, instruction sheets were distributed and read aloud.[1] Any questions asked by a subject were answered aloud to the group as a whole. Additionally, each participant completed a short comprehension test at the conclusion of the instructions. The sessions, including the instructions, test, and market periods, lasted about three hours each.

Each market of each of the two series consisted of a series of ten "years". Each "year" consisted of two trading periods, which will be referred to as the "first period" and the "second period" of a given year. Thus, each experiment lasted a total of twenty periods. The periods themselves were indexed starting with zero and consecutively numbered so the first period of a year was always an even numbered period and the second period of a year was an odd numbered period. Sometimes it is convenient to refer to the period number as opposed to the period of a given year.

Market years were independent but the periods within a given year were related by the possibility that units could be carried forward from the even numbered periods (the first period of a year) to the odd numbered periods (the second period of a year). Figure 2.1 captures the relationships. Some agents, called "traders", had the capacity to buy in the first period of a year and hold the units as inventory and sell them in the second period of the year. Carry-forward from the second period of a year to the first period of the next year was not allowed, and any units held by traders at the end of the second period of a year were lost without compensation. The induced market demand and supply for the first period of each year was the same for all years throughout an experiment. In the second periods, however, the demand

[1] The complete instructions and parameters are available in the working paper version of this paper.

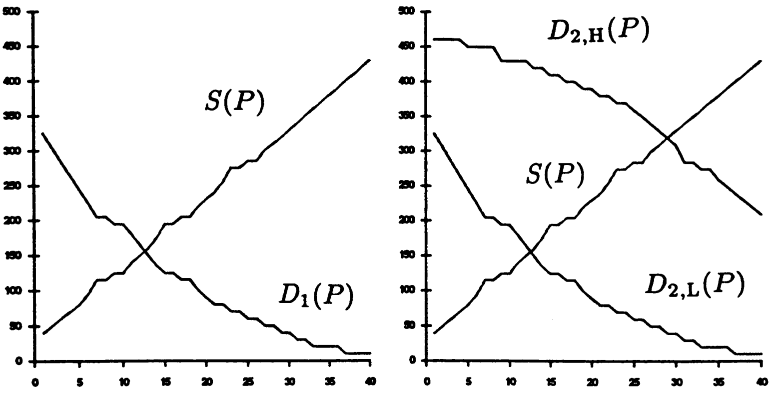

Fig. 2.1. Redemption values and costs: first and second periods of a year

was randomly determined from one of two possible demand functions. Which function was used was determined by the roll of a single six-sided die between the first and second periods of the year, and was publicly announced. It was publicly known that both demand functions were equally likely to occur.

Twelve agents participated in each market. Of these, four agents were specifically designated as "buyers" and were only permitted to submit bids and accept asks during the course of the experiment. Four other agents were designated to be "sellers", and were only permitted to submit asks and accept bids. The remaining four agents in each market, called "traders", were permitted to buy and sell units freely throughout the experiment. The traders had no induced supply or demand and realized profits only by the purchase and subsequent sale of units. In Series 1, traders had no other function in the experiment other than as a trader; in Series 2, however, the distribution of roles was such that no agent served the same function in both markets. Therefore, traders had the option to ignore their speculative role in a market to concentrate on earning profits by their actions in the other market.

3. Models

From an analytical and modeling perspective, only one model, the rational expectations model, seems to have a capacity to fit the complexity of the situation. The basic rational expectations model is extended in the discussion below to factor in potential risk aversion of the traders in a market. In addition, consideration of the decision of traders whether or not to speculate is amenable to a game-theoretic perspective, and suggests that the theory

of contestable markets might apply in this case. Application of the rational expectations model in a situation where there is no public information on the speculative decisions leads us to the problem of coordination, which is treated at the end of this section. An autarky model could be used for background measurement of speculative activity. Its predictions can be deduced by a glance at Figure 2.1.

3.1 Risk-Neutral Rational Expectations Model

Prior research suggests that a rational expectations model might be appropriate in the context of these markets. This implies, assuming that the traders are risk-neutral, that the market price in the first period will be adjusted by the speculative activity of the traders until the expected profit on a unit carried forward is exactly zero. For each period and demand state, we have the following three equilibrium equations:

$$D_1(P_1) + Y = S(P_1) \tag{3.1}$$
$$D_{2,\mathrm{L}}(P_{2,\mathrm{L}}) = S(P_{2,\mathrm{L}}) + Y \tag{3.2}$$
$$D_{2,\mathrm{H}}(P_{2,\mathrm{H}}) = S(P_{2,\mathrm{H}}) + Y \tag{3.3}$$

together with an equation that captures the rational expectations principle:

$$P_1 = \frac{1}{2}P_{2,\mathrm{L}} + \frac{1}{2}P_{2,\mathrm{H}} \tag{3.4}$$

The subscript 1 refers to the first period of a year and the subscript 2 refers to the second period of a year. The subscript L indicates the low-demand condition in the second period of a year and the subscript H indicates the high-demand condition. The variable Y is the aggregate amount of carry-forward. Thus $P_{2,\mathrm{L}}$ is the price in period 2 of a year under the low-demand condition. The first three equations state the supply equals demand equilibrium condition for each of the three possible states of the market within a year. In each of these equations, the quantity Y, the aggregate carry-forward of all traders in the market for the year, is the amount that equates quantity supplied during the period with quantity demanded at the market price. The fourth equation defines the rational expectations condition for risk-neutral traders, stating that the price in the first period is equal to the expected value of the price in the second period.

Given the supply and demand functions induced by the experimental parameters, the predictions of the rational expectations model for each of the markets studied are listed in Table 3.1.

Table 3.1. Predictions of the risk-neutral rational expectations model

Markets	P_1	V_1	Y	$P_{2,L}$	$V_{2,L}$	$P_{2,H}$	$V_{2,H}$
1124, 1208	200	16	8	120	16	280	32
0322-1, 0323-1	180	14	8	100	14	260	32
0322-2, 0323-2	345	15	10	195	15	495	35

3.2 Risk Aversion and the Rational Expectations Model

The preceding discussion makes the assumption that traders are risk neutral, in that the value of one unit in the first period is exactly the expected value of the unit in the second period. Research suggests, however, that agents tend to be risk averse. In the case of this market, risk aversion implies that the value of one unit to a trader in the first period is less than the expected value of the unit to that trader in the second period. That is, risk-averse traders will demand some risk premium to offset the risk of carrying units forward into an uncertain future.

We introduce a risk aversion parameter ρ, which is a measure of the ratio of the certainty equivalent for an agent to the certainty equivalent for a risk-neutral agent. A ρ of unity indicates risk neutrality, a ρ of less than unity is risk aversion, and a ρ of greater than unity would indicate risk-seeking behavior. Then, our rational expectations equation becomes

$$P_1 = \rho(\frac{1}{2}P_{2,L} + \frac{1}{2}P_{2,H}) \tag{3.5}$$

If we assume that traders are risk averse with the same parameter $\rho < 1$, we can derive that

$$Y = S(\frac{\rho}{2}(P_{2,L} + P_{2,H})) - D_1(\frac{\rho}{2}(P_{2,L} + P_{2,H})) \tag{3.6}$$

As ρ decreases, the price in the first period decreases, and the supply decreases and demand increases, thus we conclude that Y decreases. Therefore, under the classical conditions of negative sloped demand and positive sloped supply, risk aversion among traders decreases aggregate carry-forward, according to the rational expectations model.

3.3 Contestable Markets and Monopoly Traders

A decision to speculate in the transport from the first period to the second can be viewed as a decision to enter a very special industry. If only one agent exists in the industry then, in the position of a monopolist, extraordinary profits can be made. An agent that exists in the position of a single seller

has an interest in discouraging entry. If prices are too high, entrants may be attracted. Since there are no barriers to entry other than the inherent risk, the entry preventing price is near the competitive price of no profits. Thus, according to contestability theory, only a single agent might exist as a speculator but the carry-forward may approach the competitive quantities.

3.4 Coordination Failure

The speculative decisions of the traders, both in the current period and in the past years, are not public information. Therefore it is possible for traders to act in a sufficiently uncoordinated fashion that the aggregate carry-forward fails to converge to the competitive equilibrium. If the rational expectations equilibrium is observed, then coordination failure can be rejected. In such a case, the institution should be examined to find the mechanism through which coordination is achieved.

4. Statistical Models

Two special statistical issues should be discussed before the results of the experiments are reviewed. The first is the model that is used to measure convergence and equilibration. The second is the methodology for measuring market efficiency. Becuase of the two period nature of the economy and the fact of uncertainty, both of these issues are addressed with special statistical tools, assumptions, and conventions.

4.1 Convergence Analysis

Statistical analysis of the data will follow the procedures and conventions introduced by Noussair, Plott and Riezman (1995). A problem exists because the theoretical models are static equilibrium models, while the data are clearly generated by a dynamic process that is not understood. Any statistical model must be sufficiently forgiving of the lack of theory of dynamics to permit an analysis of convergence in the data. Thus all conclusions must be evaluated with reference to tentative assumptions upon which the statistical analysis rests. The problem is addressed by the use of a simple dynamic model they call the "Ashenfelter-El Gamal model". The explanation and notation is taken from Noussair, Plott and Riezman.

This model is built on an assumption that for any particular dependent variable, each experiment may start from a different origin but that all markets will experience adjustment, as described by a common functional form.

Furthermore, the model assumes that the variable will converge to a common asymptote. Formally, the model is as follows:

$$z_{it} = B_{11}D_1\frac{1}{t} + ... + B_{1K}D_K\frac{1}{t} + B_2\frac{t-1}{t} + u \tag{4.1}$$

where i is the index of the experiment, D_j are dummy variables that take value 1 if $i = j$ and value 0 otherwise, t is time measured in terms of experimental period number, K is the number of experiments, and u is a random variable distributed normally with 0 mean.

In some cases the model will be slightly generalized to replace the single B_2 term with a term B_{2i} for each experiment i. The model becomes

$$z_{it} = \Sigma_i D_i B_{i1}\frac{1}{t} + \Sigma_i D_i B_{i2}\frac{t-1}{t} + \epsilon_{it} \tag{4.2}$$

Notice that the statistical model has useful properties. It allows for the possibility that variables make take different values at the start of different experiments. The model then captures the specification that the experiments are converging to a common asymptote. During the early periods the asymptote gets no weight because the term $\frac{t-1}{t}$ is small but as t gets large the term goes to 1 while $\frac{1}{t}$ goes to 0. Thus, in equation (4.1) the weight of the end of the experimental session is on the common term B_2 and in (4.2) the individual B_{2i} variables carry a similar interpretation.

Thus the model can be used to test the hypothesis that the data are converging to the predictions of various models by testing whether or not the estimates of B_2 are significantly different from the predictions of the models. In addition, a notion of *weak convergence* can be used to assess the models. Comparison of the B_{1j} terms with the B_2 term reveals the direction of convergence. If the B_2 term is closer to the model's prediction than are the B_{1j} terms, we say that the data are *weakly converging* to the model's predictions. If the B_2 term is not significantly different from a model's prediction, we say that the variable is *strongly converging* to the prediction.

4.2 Market Efficiency

The presence of uncertainty and intertemporal speculation in this market calls for extensions of the standard market efficiency measures to capture the special structure of the market. Several interdependencies exist so there is a possibility that part of the system is working efficiently given what happened elsewhere. The first and second periods are each dependent on the other, since the demand in the first period and the supply in the second period are functions of the actual carry-forward. Also, the actual profits realized depend in large part on the state which occurs in the second period, which information is not known at the time of the speculative decisions.

We consider the general efficiency measure as an aggregation of partial efficiency measures of each of the two periods. These partial measures are restricted to taking into account only information present within that period, and therefore will better mirror the state of the market and the closeness of the market to optimality.

Consumption in the first period derives from two sources: purchases by buyers for redemption, and purchases by traders for carry-forward into the second period. The value of the redeemed units is given, and the value of the units carried forward is the expected value of those units to the trader in the second period, given the actual aggregate carry-forward. Thus, an appropriate partial efficiency measure for the first period is

$$\Psi_1 = \frac{\Pi_{A,1} - C_{A,1} + E[P_2(Y_A)]Y_A}{\Pi_{O,1} - C_{O,1} + E[P_2(Y_O)]Y_O} \tag{4.3}$$

where $\Pi_{A,1}$ is the actual earnings from consumption, $C_{A,1}$ is the actual cost of production, $\Pi_{O,1}$ is the rational expectations earnings from consumption, $C_{O,1}$ is the rational expectations cost of production, Y_A is the actual number of units carried forward, Y_O is the rational expectations prediction for units carried forward, and $E[P_2(Y)]$ is the expected price of a unit in the second period, given carry-forward Y. This measure is always less than or equal to unity, and penalizes for deviations from the optimal carry-forward.

In the second period, the state of the market is known. There is now only one source of consumption, that of the buyers. There is one source of costs, inventory use costs of the sellers, since there is no additional cost inherent in carrying units forward from the previous period. Thus, the second period partial efficiency measure is simply

$$\Psi_2 = \frac{\Pi_{A,2} - C_{A,2}}{\Pi_{O,2} - C_{O,2}} \tag{4.4}$$

The general efficiency measure is obtained by adding the numerators and denominators of the two partial measures, giving an index of the overall closeness of the market to the optimal allocation:

$$\Psi_G = \frac{\Pi_{A,1} - C_{A,1} + \Pi_{A,2} - C_{A,2} + E[P_2(Y_A)]Y_A}{\Pi_{O,1} - C_{O,1} + \Pi_{O,2} - C_{O,2} + E[P_2(Y_O)]Y_O} \tag{4.5}$$

5. Results

A time series of contract prices for experiment 1208 is shown in Figure 5.1. The figure contains impressions of most of the results that are reported below. Prices and volumes appear to approach the magnitudes predicted by the

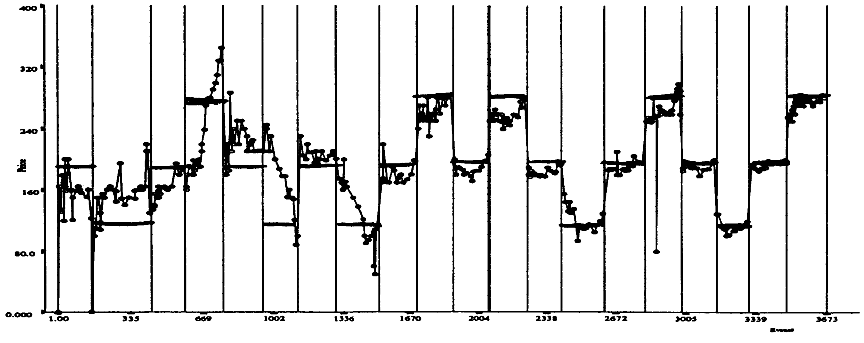

Fig. 5.1. Price time series for experiment 1208

rational expectations model. Result 1 through Result 6 make this impression precise. Since the rational expectations equilibrium makes certain assumptions we have reason to suspect are not accurate, the convergence leads us to ask what features of the market contribute to enforcing the competitive equilibrium. Result 7 through Result 9 indicate that the markets are behaving in ways that are consistent with many other experiments at a market level. Result 10 and Result 11 explore in detail the actions of individual traders and provide clues to the special properties of these markets that generate the results.

Result 1 demonstrates that the aggregate quantity carried over from the first period to the second within a year is converging toward the rational expectations aggregate quantity. Thus, the sum of the quantities acquired by the individual traders is approaching the quantity predicted by the model.

Result 1. Aggregate carry-forward converges weakly toward the rational expectations equilibrium.

Table 5.1. Convergence of aggregate quantities carried forward: all experiments

Market	B_{1i}	B_{2i}	Y^*	$\overline{B_2}$	R^2
1124	10.3 (2.9)[a]	2.6 (1.6)			
1208	5.4 (2.9)	8.5 (1.6)	8	6.9 (1.6)	0.331
0322-1	5.8 (2.9)	9.0 (1.6)			
0323-1	4.4 (2.9)	7.4 (1.6)			
0322-2	3.1 (1.3)	7.1 (0.6)			
0323-2	0.2 (1.3)	5.3 (0.6)	10	6.2 (0.6)	0.584

[a] Throughout the tables, numbers in parentheses are standard errors for parameter estimates.

Support. Table 5.1 presents the estimated coefficients of the convergence model applied to the carry-forward data. Asymptotes are presented for each market, and a common (average) asymptote is reported for markets that have directly comparable carry-forward predictions. The estimates indicate that in five of the six markets the asymptotic value of the carry-forward was closer to the rational expectations prediction than was the initial period carry-forward. For example, in experiment 1208, the estimated quantity carried forward has an estimated origin at 5.4 which is 2.6 below the rational expectations prediction of 8, and the estimated asymptote is 8.5, which is only .5 higher than the predicted value. Convergence is in the direction of the rational expectations equilibrium quantities and therefore weak convergence is observed. ▪

In all but the anomalous 1124 market, the predicted aggregate carry-forward value is approached from below, suggesting this result which will also be useful in the discussion of price behavior.

Result 2. Traders demonstrate risk aversion.

Support. The rational expectations model predicts that risk-averse traders will carry forward fewer units than their risk-neutral counterparts. Reexamining Table 5.1, we note that, for the $Y^* = 8$ markets, the average of the four asymptotic estimates was 6.9 units carried forward, 1.1 below the risk-neutral prediction, and that in the $Y^* = 10$ markets the carryforward approaches 6.2, falling 3.8 units short of the risk-neutral prediction.

Furthermore, the risk-averse rational expectations model implies that the market price in the first period will be less than that predicted by the risk-neutral version. The first period average price in the last year of each of the markets fell below the risk-neutral prediction in all cases. Thus, we conclude that the traders' behavior was consistent with risk aversion. ▪

With quantities adjusting in a systematic way, one would expect the prices to be behaving in a manner that is understandable in terms of the model. However, because of the randomness, the price patterns cannot be understood in terms of a single time series. The market behavior must be conditional on both the carryover decisions of the traders and the randomness inherent in the market. The next result demonstrates that given the speculative decisions and the state, the markets adjust along the lines that are predicted by the model.

Result 3. If carry-forward is treated as exogenous and fixed then within both the first periods and second periods the market prices converge toward the equilibrium implied by the law of supply and demand.

Support. We consider a simple exponential model of convergence toward the temporary equilibrium price:

$$\text{abs}(P_t - P_{\text{eq}}) = ae^{bt+u} \tag{5.1}$$

where P_t is the price at contract number t, P_{eq} is the temporary equilibrium price, and a and b are the parameters to be estimated. If the system is moving toward the temporary equilibrium, the value of b to be less than zero.

Table 5.2. Simple exponential model of convergence toward temporary equilibrium

Period	b	R^2	nobs
1	-0.034 (0.008)	0.141	974
2,L	-0.077 (0.011)	0.256	497
2,H	-0.046 (0.005)	0.305	949

Table 5.2 provides the estimates of the coefficients fitted to the market data. For each state, the estimate of the value of the coefficient b is significantly less than zero. Therefore, we conclude that the absolute value of the deviation of prices from the temporary equilibrium value is decreasing over time, and that the market price is converging within a period towards the temporary equilibrium price. ∎

Results 1 and 3 suggest the summary result that the prices and volumes in these markets are converging in a way that is understandable through the lens of the rational expectations model. Result 4 is the statement for prices and Result 5 is the statement for volumes.

Result 4. The market prices converge toward the rational expectations prediction for price.

Support. We consider the sequence of average prices in the second periods in each market. If this result holds, the difference between these prices and the rational expectations prediction for each period should be decreasing over time. Table 5.3 describes the convergence of the distance from the rational expectations price for all markets.

Table 5.3. Convergence of price to rational expectations predictions

Market	B_{1i}	B_{2i}	eq	$\overline{B_2}$	R^2
1124	-31.3 (34.9)	17.9 (18.2)			
1208	22.1 (34.9)	-14.8 (18.2)			
0322-1	48.8 (34.9)	3.0 (18.2)	0	27.7 (18.2)	0.577
0323-1	25.7 (34.9)	-11.6 (18.2)			
0322-2	-83.5 (34.9)	74.7 (18.2)			
0323-2	-213.6 (34.9)	97.4 (18.2)			

In each of the six markets, the estimated asymptote is closer in absolute value than the estimated origin to the equilibrium prediction of 0, so we may conclude that the market prices are weakly converging toward the rational expectations prediction. In four of the six markets (1124, 1208, 0322-1, 0323-1), the asymptote estimate is not significantly different from zero, implying that these markets strongly converged to the rational expectations prediction. In the remaining two markets, the convergence is less robust; however, taking into consideration the carryforward data, we conclude these markets also were converging to the rational expectations equilibrium. ■

Given that the market prices are tending towards the rational expectations predictions, one would expect that the quantities traded in the market are also tending towards the rational expectations value. The next result demonstrates that this is the case.

Result 5. The net volume in the market corresponds to the rational expectations predictions.

Support. The convergence model was applied to the net volume data for each of the three groupings of parametrically similar markets, and the results for the markets 1124 and 1208 are summarized in Table 5.4.

Table 5.4. Analysis of net market volume, markets 1124 and 1208

Period	Market	B_{1i}	B_{2i}	eq	$\overline{B_2}$	R^2	nobs
1	1124	25.5 (0.94)	19.2 (0.49)				
1	1208	14.9 (0.94)	17.8 (0.49)	16	18.5 (0.49)	0.824	20
2,L	1124	43.1 (5.77)	10.4 (1.43)				
2,L	1208	27.3 (1.37)	14.2 (0.95)	16	12.3 (1.19)	0.906	12
2,H	1124	36.5 (1.67)	37.0 (1.78)				
2,H	1208	23.9 (4.24)	35.7 (1.25)	32	36.4 (1.52)	0.656	8

In 14 of the 18 cases, the market volume weakly converges toward the rational expectations prediction. For example, for the even periods of 1124, the volume is initially near 25.5 units, according to the estimates, and then asymptotes to 19.2 units, which is a movement toward the rational expectations volume of 16 units. In three of the four remaining cases, the divergence away from the rational expectations prediction is not statistically significant. Thus, we conclude that the market volumes observed are consistent with the rational expectations predictions. ■

If the market volumes and prices are converging toward the rational expectations predictions then it would not be surprising to know that the underlying allocations were approaching the most efficient. Result 6 demonstrates this is the case.

Result 6. System efficiencies are high and comparable with efficiency levels of multiple market double auction organizations.

Table 5.5. Market efficiencies for market 1208

year	Y	state	Ψ_1	Ψ_2	Ψ_G
1	6	low	78.0	1.8	33.3
2	1	high	95.9	92.6	92.1
3	12	low	95.0	78.8	84.5
4	11	low	97.2	96.1	96.3
5	7	high	97.3	99.6	99.6
6	9	high	94.5	98.9	98.2
7	6	low	97.3	98.0	97.4
8	7	high	98.9	95.5	96.2
9	8	low	100.0	100.0	100.0
10	9	high	99.1	99.8	99.6

Support. The market efficiencies were computed according to the measures described earlier. Table 5.5 provides a typical sequence for an experiment. The efficiencies over all periods of experiments are generally high. The total efficiency measure exceeds 95% in 33 of the 60 market years, and 90% in 44 of the 60 market years, averaging 90.6% overall. Efficiencies in the first period are greater than 95% in 46 years (including 11 instances of 100% efficiency) and greater than 90% in 51 years yielding an average efficiency of 91.5%. Second period efficiencies exceed 95% 39 times, and 90% 46 times for an overall average efficiency of 91.7%. This consistently strong behavior indicates that the market activities and magnitudes are close to the rational expectations prediction. ■

Having established that the market as a whole tends toward a rational expectations equilibrium, it is instructive to examine the mechanisms by which this type of equilibrium is achieved. The first question is whether or not these markets exhibit any special characteristics not present in other markets in which speculators are not present. Since a double auction institution was used, one might ask if the principal characteristics of double auction market equilibration in general are also found in these markets.

Prior experience indicates a tendency that, when a market is trending upward, asks will tend to be taken more frequently than bids, and when a market is declining, contracts will take place due to bids being accepted. As the next result shows, this property is present in the markets with speculators.

Result 7. The side of the market which is consummating trades corresponds with the direction of prices in the market.

Table 5.6. Correlation of price change direction with bid/ask behavior

Market	taken	down	same	up
1124	ask	43	54	94
	bid	111	54	67
1208	ask	101	54	178
	bid	84	18	38
0322-1	ask	62	59	84
	bid	66	61	33
0322-2	ask	18	19	56
	bid	104	116	116
0323-1	ask	38	46	83
	bid	78	67	71
0323-2	ask	77	96	95
	bid	37	18	18

Support. Table 5.6 summarizes the number of bids and asks being taken in relation to the change in price since the last transaction. The result indicates that transactions will tend to be classified into the upper right (ask taken when market is moving up) and lower left (bid taken when market is moving down) cells.

In four of the six experiments, the number of bids taken is greater than the number of asks taken when the market was headed downward (the left column in the table). For example, in the 1124 experiment, 111 transactions occurred when a bid was taken in this situation, and only 43 transactions represent an ask being taken. The number of asks taken exceeds the number of bids taken when the market was trending upward (the right column in the table) in five of the six experiments. Again taking 1124 as an example, taking asks was responsible for 94 transactions during upward moves as opposed to only 67 bids being taken. Therefore, we conclude that the bid/ask behavior of this market is consistent with double auctions in general. ∎

A second standard feature of the double auction institution is that at the end of a period the time series of transaction prices exhibits a "tail" pointing towards the equilibrium price. In this particular market, one would therefore expect the time series to tail towards the temporary equilibrium established by the actual carry-forward.

Result 8. Toward the close of a period, the price tends in the direction of the temporary equilibrium.

Support. We consider the sequence of the last six transactions to take place in each period. If this tailing phenomenon is present, than we would expect both of the following to hold:

- The net change in price from the beginning to the end of the sequence is not away from the temporary equilibrium as seen from the beginning of the sequence.
- The change in price between any two successive transactions in this sequence is not away from the temporary equilibrium.

Both conditions hold in a substantial majority of the cases. In all experiments, the first condition holds in at least 72.2% of the market periods, and overall holds 86.1% of the time. The second condition holds somewhat less frequently due to the greater noise inherent in examining individual transactions; even still, it holds at least 65.6% of the time in all markets, and overall 70.7% of the transactions agree with the condition. Thus, we conclude that there is some "tailing" effect at the end of a period toward the temporary equilibrium. ■

The next three results describe characteristics of the traders' behavior that help establish and enforce the rational expectations equilibrium. Central to this issue is how the market is able to coordinate, given that the traders make their decisions with no information about the concurrent decisions of their peers. In view of the previous results, indicating that the market is in fact coordinating to some extent, this implies that there is some mechanism which gives traders some notion as to what their best choice is, even though they do not have direct information.

One possible mechanism is an implicit feature of the price discovery aspect of the double auction. A general property of the double auction institution is the property of preference revelation near the end of the period. In particular, sellers tend to reveal their costs at the margin. If sellers reveal their costs and if as a result the prices would be above the expected value of the unit if it is purchased and carried forward, then the market itself, together with the marginal decisions of buyers, is a coordinating mechanism. The next result attempts to characterize the central feature of the mechanism.

Result 9. Asks at the end of the first period of a year tend to reveal the marginal supply cost that exists in the market at the moment.

Support. The analysis first identifies the final asking price that existed at the close of each of the first periods of each year. This final asking price was then compared to the market's marginal supply cost, the lowest marginal cost that existed among sellers given all previous trades during a period. If the asking price was cost revealing then the difference should be zero. Table 5.7 contains for selected levels of difference the percentage of final asking prices that diverged from the marginal supply price by that amount or less. Fifty percent of the final asks were within ten francs (approximately three cents) or less. Seventy percent of the final asking prices were within thirty francs (ten cents) of the supply costs. ■

Table 5.7. Distribution of difference between final asking price and marginal supply cost in the first period of a year

percentile	final asking price minus marginal supply cost
10	0[a]
20	1
30	2
40	5
50	10
60	20
70	25
80	50
90	75
100	165

[a] There were a few instances of the final asking price being less than the marginal supply cost. This is attributable to errors in ask submission on the part of sellers.

A second potential mechanism for coordinating carry-forward decisions would be "specialization", in which traders find a niche or role in the market and stay within that niche or role every year. In the context of this market, specialization amounts to carrying forward the same or similar number of units each market year.

Result 10. Traders specialize, behaving in a similar fashion from year to year.

Support. Table 5.8 summarizes the carry-forward behavior of all traders for the last five years of each experiment. The series of numbers (0,0,1,1,1) indicates that this trader in experiment 1124 carried forward 0 units in year 6, 0 units in year 7, 1 unit in year 8, and so on.

Table 5.8. Sequence of quantities of units carried forward

Market	Trader 1	Trader 2	Trader 3	Trader 4
1124	(0,0,1,1,1)	(0,0,0,0,0)	(0,0,0,0,1)	(2,0,0,0,0)
1208	(0,1,1,1,1)	(2,2,1,0,0)	(3,2,2,4,4)	(4,1,3,3,4)
0322-1	(5,5,4,5,0)	(2,1,2,1,0)	(2,3,0,0,5)	(0,2,3,3,1)
0322-2	(3,3,3,3,3)	(1,2,1,2,0)	(2,0,2,2,3)	(0,0,0,1,1)
0323-1	(6,7,8,9,5)	(0,0,0,0,0)	(0,0,1,0,2)	(0,0,0,0,0)
0323-2	(1,1,2,1,0)	(0,1,0,0,0)	(1,3,3,0,3)	(1,2,2,2,2)

Given that a trader carried forward n units in the previous year, the trader carried forward that same number of units in 48 of the 96 cases, and carried forward on the interval $[n-1, n+1]$ in 79 of the 96 cases. The correlation

coefficient between the number of units carried forward by a given trader in a given year with the number of units that trader carried forward in the previous year is .733. Therefore, the traders exhibited a tendency toward specializing. ■

An interesting feature of the individual carry-forward data is that in market 0323-1, one trader was responsible for the vast majority of the aggregate carry-forward. This suggests this final result.

Result 11. A contestable market principle governs the entry decisions of the traders.

Support. The contestable market principle implies that, for a market in which the entry cost is zero, a monopolist is forced to act as if here were in a competitive situation. In this market, there is no entry cost, and so one would expect this type of behavior from a monopolist.

In fact, in 0323-1, one trader emerged as a monopolist, carrying forward over 95 percent of the total units carried forward over the course of the experiment. His individual carry-forward over the last half of the experiment averages to just under the rational expectations prediction for aggregate carry-forward. However, despite his essentially monopoly, his purchasing and selling behavior was consistent, in terms of prices, with the rational expectations predictions, and so he gained no special advantage due to his monopoly. Thus, we conclude that the contestable market principle is applicable to this market. ■

Result 9 holds the key to understanding the coordination process. Asks tend to reveal marginal cost near the end of the first period. In a double auction market buyers are able to make unit by unit decisions about carry-forward. When asks become too high given second period price expectations, the carry-forward is increased no more. Results 10 and 11 simply show how the speculative activity becomes specialized and disciplined.

6. Conclusions

This paper was motivated by a series of questions that have now been answered. Do the results of Agha and Plott replicate in the sense that speculators who carry forward into an uncertain future act such that the price in the first period is equal to the expected value of the price in the second period given the carry-forward? Not only can this be observed, it happens even when the number of speculators and their intentions are unknown to the agents. The laws of the markets work as theory would naturally suppose given the decisions of the speculators. The decisions of the speculators, in turn, evolve to create the efficiency creating capacity of optimal allocations. All of this is done in the absence of any public information.

What is the mechanism through which all of these coordinated decisions are made? The markets' stability owes to the speculators tending to specialize in the quantity of carry-forward. The optimality is guided by decisions made at the margin. Overall, speculators take advantage of myopic profits, thinking perhaps that if left to grow in the fashion of monopoly, another speculator will take advantage. In this sense, contestability seems to be an important feature of the entry process as agents consider entering a market for speculative purposes. The myopia extends itself to a comparison of the profit potential of each unit. Thus when asking prices of the individual units are too high, the speculators do not purchase them for carry-forward. The asking prices themselves are a creature of the competition among sellers and the preference revealing forces at the end of the period of the double auction market. Sellers compete for access to the buyers and in the process the lowest marginal cost is revealed to the speculators for use in their calculation of whether to carry forward or not. In this manner, at the margin, the decisions are guided toward the optimum.

In some respects the results of these experiments are encouraging because they demonstrate the decentralized power of market forces. In other respects they are discouraging because individuals do not reach the rational expectations equilibrium through global, systems-solving deductions. The decisions seem to be locally based and the information is locally generated. As a consequence, general inefficiencies perpetrated by local conditions becomes a theoretical possibility that is in need of further research.

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Endowment Effect for Risky Assets

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Abstract. Endowment effects have been mainly investigated for choice under certainty. In this paper we present two experimentes which directly test the effect of endowment and framing for choices under risk. In both experiments subjects were endowed with cash and some quantity of state contingent claims. They were offered the chance to buy additional claims which could cover their risky position or which would take them to another risky position. Standard utility theory gives strong predictions about the amount subjects should bid for the additional claims. In addition, the bid should not depend on the way the initial endowment is framed. Our results show that subjects did not follow the prediction of standard utility theory. Their responses were influenced by both framing and initial endowment - although the relation strength of these effects appeared to vary significantly, depending on the incentive scheme used.

Introduction

In recent years there has been growing interest in the 'endowment effect' (Thaler, 1980; Knetsch, 1989; Kahneman, Knetsch and Thaler, 1990) whereby individuals attach significantly different values to a commodity depending on whether or not that commodity is part of their current endowment.

Much of the existing evidence of endowment effects and their role in buying/selling price disparities concerns what is traditionally regarded as choice under certainty (e.g. buying/selling coffee mugs or chocolate bars, or exchanging mugs for chocolate, etc.) and a recent model which offers an account of the endowment effect - Tversky and Kahneman's (1991) 'reference dependent utility'

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model - is explicitly addressed to "the analysis of riskless choice". However, we were interested in the extent to which endowment effects might be observed in the context of risky assets.

There is some literature which compares buying and selling prices for simple risky lotteries using a between-subject design (Knetsch and Sinden, 1984; Casey, 1990) or a within-subject design (Kachelmeier and Shehata, 1992). Some other recent papers compare prices for short selling and prices for covering a short position (Harless, 1989; McClelland and Schulze, 1990; Eisenberger and Weber, 1995). All these studies show that the ratio of selling to buying price is significantly greater than one.

Our study takes a somewhat different approach. We shall focus exclusively on buying prices for risky assets in the form of simple state contingent claims, and examine the extent to which the pattern of those buying prices is consistent with the existence of a significant endowment effect of the kind proposed in the literature cited above.

1. Theory

State contingent claims are a central concept in a wide variety of economic models. Our experiments will therefore focus upon preferences over simple portfolios of state contingent claims. Two questions will be studied. We will investigate whether initial endowments have systematic effects upon subjects' valuations of claims. In addition, we will test whether different ways of framing the same initial endowment may have some further influence on the valuations. We shall derive hypotheses relating to each of these questions in turn.

Endowment Effects

Consider the case where there are only two mutually exclusive states of the world. In our experiments, the state which occurs is determined by the colour of a single ball drawn at random from a bag containing 20 balls which are some mix of green and yellow: hence we shall label the two possible states G and Y. There are also two kinds of state contingent claim: claim i, which involves a strictly positive money payoff only under G, and its complement, claim j, which involves the same strictly positive money payoff only under Y. Besides holding such claims, individuals may also hold cash (which, being equally valuable under both states, may be thought of as equivalent to holding

some equal quantity of both claims).

The implications of conventional analysis in such circumstances can be illustrated with the aid of Figure 1. The vertical axis shows the total value of claims held by an individual in the event that G occurs, and the horizontal axis shows the position under Y.

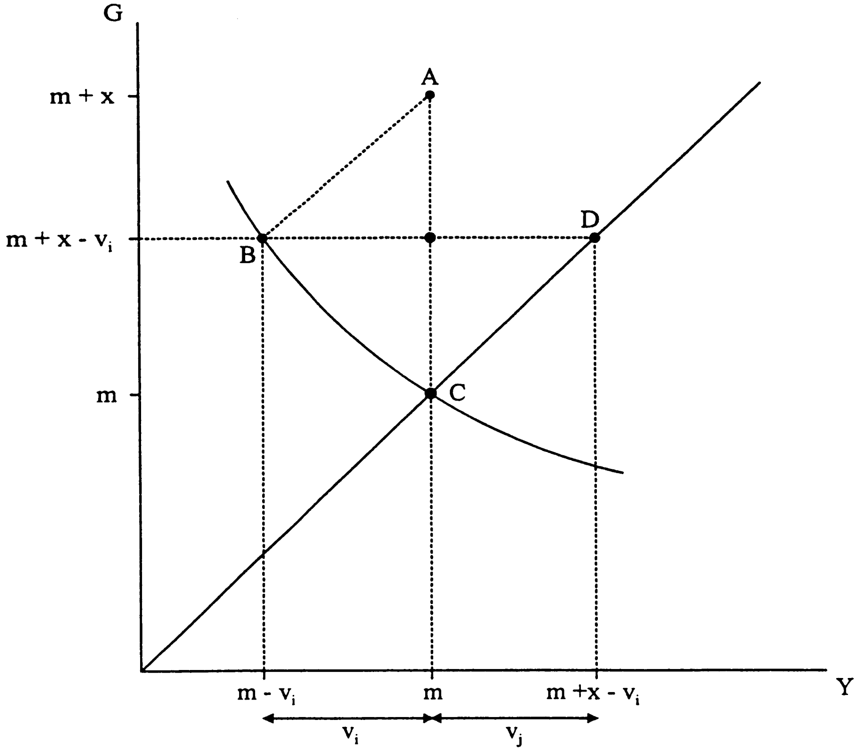

Fig. 1. Indifference Curve

First, consider an individual endowed with m in cash i.e. initially located on the certainty line at point C. Now suppose she is given the opportunity to acquire a claim i, worth x if G occurs and worth zero if Y occurs. If this claim could be acquired for nothing, the individual's portfolio would now be represented by point A. However, if the individual has to purchase the claim for cash, she will be willing to pay anything up to v_i, where a payment of v_i takes her to point B, which is on the same

indifference curve as point C.

Alternatively, suppose that the same individual starts with an initial endowment represented by point B, and is offered a claim j, worth x if Y occurs and zero otherwise. If this claim would be aquired free, it would move the individual's portfolio to point D. However, if required to buy the claim, the maximum price she will be prepared to pay is the amount that takes her back to the indifference curve that passes through B and C: that is, she would be willing to pay up to v_j, where $v_j = x - v_i$.

This provides us with the null hypothesis, namely that the sum of each individual's two bids for such a pair of state contingent claims should equal x, the strictly positive money payoff contingent upon the two complementary states. Notice that although this hypothesis clearly applies to standard expected utility theory, it is derived on the basis of indifference curve analysis, and therefore holds for a broad class of non-expected utility models which respect transitivity and thereby entail reversible indifference loci.

According to conventional analysis, she should be willing to pay anything up to v_j, where $v_i + v_j = x$. This provides us with our null hypothesis.

Now contrast that with the case, illustrated in Figure 2, where an otherwise identical individual is subject to the endowment effect. Starting from point C and paying cash to obtain a claim worth x under G and zero under Y effectively means giving up part of the endowment under Y in order to obtain some increment under G. If the endowment effect entails the idea that the loss of (part of) something an individual currently holds is weighted more heavily than the gain of something on top of the individual's reference endowment, it will be as if the individual's indifference curve is steeper to the north-west of C than was the case in Figure 1. Put another way, the individual will not be willing to pay more than v_i^e, where $v_i^e < v_i$, so that the indifference curve through C passes <u>above</u> B, and through B^e instead.

Alternatively, suppose that this individual is endowed with the portfolio represented by point B, and is offered a claim worth x under Y and zero under G. <u>This</u> transaction involves losing something under G in order to gain something under Y. Weighting losses more heavily than gains now leads to a maximum willingness to pay of v_j^e where $v_j^e < v_j$: thus going southeast from B, the indifference curve passes <u>above</u> C and through C^e instead.

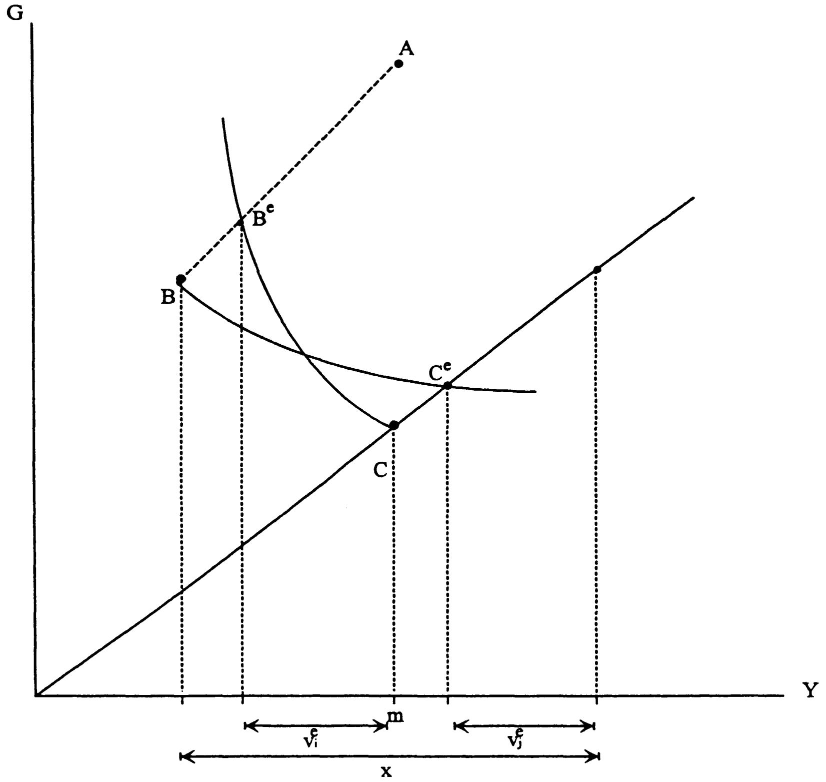

Fig. 2. Indifference Curve depending on Endowment

The overall effect is that the individual appears indifferent between C and B^e, which strictly dominates B, which is indifferent to C^e which strictly dominates C - a set of relations which, viewed conventionally, entail indifference curves that cross. Put another way, our alternative hypothesis is that the amounts offered by the individual for the two claims will sum to less than x: i.e. $v_i^e + v_j^e < x$. Thus we shall attempt to check on a within-subject basis in the context of risky assets the 'nonreversible indifference curve' phenomenon that Knetsch (1989) inferred from aggregate data involving riskless transactions.

Framing Effects

The portfolio at point A in Figures 1 and 2 can be framed in two different ways. One possibility

is to endow a subject with a cash amount m plus a claim worth x if G occurs. We call this the positive frame. Alternatively, the subject could be endowed with a sum of cash (m+x) together with a liability to pay back x if Y occurs. We call this the negative frame. According to standard theory, purely presentational differences of this kind are irrelevant: an individual offered the chance to buy a claim worth x if Y occurs should be willing to pay the same price whichever way the initial portfolio has been framed.

However, there is plentiful evidence from other contexts which suggests that framing may have substantial and systematic effects upon many individuals' responses (see, for example, Tversky and Kahneman, 1986). In the current context, the presentation of the initial endowment as involving a potential loss which can be cancelled out by purchasing the claim would be expected to produce a higher buying price than when the initial endowment is positively framed. Denoting these amounts as v_j^- and v_j^+ respectively, our null hypothesis is that $v_j^- = v_j^+$, while our alternative hypothesis is that $v_j^- > v_j^+$.

2. Experimental Design, Conduct and Results

General Principles

Essentially, the experiments are built around two sets of 10 questions, where each question involves giving subjects a particular endowment (framed in a certain way), then offering them the possibility of purchasing a particular claim and eliciting their maximum willingness to pay for that claim.

Tables 1 and 2 give the parameters of the two sets of questions. The second column shows the initial endowment: either an amount of cash, or else a combination of cash plus claim/liability contingent on either G or Y[1]. Thus, for example, Question 5 in Table 1 involves an endowment of £10.00 cash plus a liability to pay back £6.00 in the event of a yellow ball being drawn. The third column shows the claim being offered - in Question 5 above, £16.00 conditional on a yellow ball

[1] For the experiment conducted at Kiel, payments were in Deutschmarks, converted at the rate of 3DM:£1.

being drawn. The fourth column shows the probability of a green ball being drawn[2], and the fifth column, headed "From:To", indicates whether purchasing the claim offered would take the individual from a risky position in one of the states (G or Y) to the certainty line (C), or to a risky position in the other state; or else from the certainty line to a risky position in one of the states. So in Question 5 above, the initial endowment is located to the north-west of the certainty line (i.e. on the side closer to the G axis), and purchasing the claim would produce a new portfolio to the south-east of the certainty line (i.e. closer to the Y axis).

Notice that each table groups questions into subsets with the same values of Prob(G). Within each subset, questions differ according to whether it is a claim contingent on G, or the complementary claim on Y that is being offered, and whether the initial endowment is positively or negatively framed. In later questions within each subset, the cash element of the initial endowment is represented by a lower case letter rather than by a specific amount. This represents the fact that for each individual this cash element was computed (in a way we shall describe below) on the basis of that individual's responses to earlier questions in the subset.

Table 1 Parameters for Set 1 Questions

Question	Endowment	Offer	Prob(G)	From:To
1	3.00 + 12.00 Y	12.00 G	0.6	Y -> C
4	15.00 - 12.00 G	12.00 G	0.6	Y -> C
7	a	12.00 Y	0.6	C -> Y
2	4.00 + 6.00 G	16.00 Y	0.8	G -> Y
5	10.00 - 6.00 Y	16.00 Y	0.8	G -> Y
8	b - 10.00 G	16.00 G	0.8	Y -> G
10	c + 10.00 Y	16.00 G	0.8	Y -> G
3	7.00	8.00 G	0.7	C -> G
6	d + 8.00 G	8.00 Y	0.7	G -> C
9	e - 8.00 Y	8.00 Y	0.7	G -> C

[2] Since all balls were either green or yellow, so that Prob(Y) = 1 - Prob(G), we simply report Prob(G).

Table 2 Parameters for Set 2 Questions

Question	Endowment	Offer	Prob(G)	From:To
1	2.00 + 8.00 G	8.00 Y	0.7	G -> C
4	10.00 - 8.00 Y	8.00 Y	0.7	G -> C
7	a	8.00 G	0.7	C -> G
2	4.00 + 10.00 Y	16.00 G	0.8	Y -> G
5	14.00 - 10.00 G	16.00 G	0.8	Y -> G
8	b + 6.00 G	16.00 Y	0.8	G -> Y
10	c - 6.00 Y	16.00 Y	0.8	G -> Y
3	7.00	12.00 Y	0.6	C -> Y
6	d + 12.00 Y	12.00 G	0.6	G -> C
9	e - 12.00 G	12.00 G	0.6	G -> C

To illustrate the idea in more detail, consider the first subset of three questions in Table 1. Question 1 endowed each subject with £3.00 in cash plus a claim worth an additional £12.00 if Y occurred, for which the probability was 0.4. In that question, the claim offered for sale paid £12.00 if G occurred. Purchasing such a claim would therefore take an individual from a risky portfolio t a position on the certainty line. Letting v_1 denote an individual's maximum willingness to pay for the claim offered in Question 1, the implication is that the individual is indifferent between the initial risky endowment and £15.00 - v_1 for sure.

In Question 4, subjects were given the same initial endowment as in Question 1, but this time framed as £15.00 in cash plus a liability to pay back £12.00 if G occurred. Otherwise, Question 4 is identical to Question 1, so that the null hypothesis derived from conventional theory (i.e. <u>no</u> framing effects) is that $v_4 = v_1$; the alternative hypothesis is that $v_4 > v_1$.

Question 7 started subjects with a certain cash endowment and offered them the complementary claim to the one offered in Questions 1 and 4, i.e. £12.00 contingent on Y. To allow for the fact that v_1 and v_4 <u>might</u> differ (either because of framing effects, or simply because of random response error), the initial cash endowment in Question 7 was computed for each individual as the mean of the positions on the certainty line implied by responses to Questions 1 and 4: that is, **a** was set equal to £15.00 - $0.5(v_1 + v_4)$. Then v_7 - maximum willingness to pay for the £12.00 claim contingent on Y - was elicited. Conventional theory, which allows neither framing nor endowment effects, entails $v_1 + v_7 = v_4 + v_7 =$ £12.00. If there is a framing effect but no endowment effect, we should observe v_4 +

$v_7 > v_1 + v_7 =$ £12.00. However, if there is an endowment effect of the kind discussed in Section 1 but no framing effect, we should expect $v_4 + v_7 = v_1 + v_7 <$ £12.00. If both framing and endowment effects are at work, we should still expect $v_1 + v_7 <$ £12.00, since the endowments in Questions 1 and 7 are both positively framed. However, we cannot predict whether $v_4 + v_7$ will add up to more or less than £12.00, since the negative frame in Questions 4 will tend to work in the opposite direction to the endowment effect, and we cannot say a priori which of the two effects will be more powerful.

We shall tabulate the full list of hypotheses when we come to discuss the results of each experiment. But first, some general remarks about the reasons for conducting two experiments at different sites, and the similarities and differences between the two.

Experiment 1 was conducted at the University of York, and Experiment 2 at the University of Kiel. In both cases, subjects were randomised between two subsamples, one subsample being presented with the Set 1 questions shown in Table 1, the other subsample answering the Set questions in Table 2. Besides the currency conversion referred to in footnote 1, the key difference between the two experiments lay in the procedures used to elicit willingness to pay and the incentive mechanism employed. In the Kiel experiment, all subjects received a flat rate payment unrelated to their responses to each of the ten questions presented to them. In the York experiment a competitive auction mechanism was used, and subjects participated on the understanding that their payment would depend entirely on the way that their decisions in just one of the questions (chosen at random at the end of the experiment) worked out. Because the Kiel procedure was faster, we were able to include two additional question, i.e. York subjects were presented with only the first eight of the ten questions in each set. We begin with the York experiment.

Experiment 1: Procedure

Letters of invitation were circulated widely among the undergraduate and postgraduate student population, describing the main features of the experiment and inviting would-be participants to sign up for one of eight places in one of ten sessions arranged to start at various advertised times. All lists were quickly filled, but 7 signatories failed to show up, so that 73 people actually took part.

Each session began with two Practice Rounds (common to both subsamples) to allow participants to become familiar with the procedure and, if they wished, to ask points of clarification.

Figure 3 reproduces the slip of paper given to each participant at the beginning of the first

Practice Round. The initial endowment consists of £8.00 in cash plus a further £5.00 contingent on a yellow ball being drawn. The typed numbers at the base of each column of the claim show that in this round there are 9 green balls and 11 yellow balls in the bag. The slip also informs participants of the nature of the claim on offer - in this case, £15.00 if a green ball is drawn.

Round ...P1...   Session ....   Person ....

YOU START WITH ....8.00.............. in cash +

G	Y
0	5.00
9	11

YOU ARE OFFERED

G	Y
15.00	0
9	11

Please write down THE MOST YOU ARE PREPARED TO PAY ....

(mulitiples of 5p)

Fig. 3. Answer Sheet

It was explained to participants that several identical claims were up for sale by auction, and that they were each invited to bid for one. They were asked to imagine that the auction was of the usual (English/ascending) kind, except that all the claims would be sold simultaneously; and that, instead of bidding in person, each participant had to instruct an agent to act on their behalf by writing down, in confidence, the most they were prepared to bid for one of the several claims. In other words, they were asked to imagine that their agent would stay in the bidding as long as the asking price was below the amount written down, but would drop out the moment the price rose above that amount.

With the aid of an example involving four claims being offered for sale to eight bidders, it was made clear that the asking price would rise until four bidders' agents had dropped out, leaving just four in the bidding. At that point, the remaining four would each be deemed to have bought a claim for a price 5p (5 pence) above the level at which the fifth bidder's agent dropped out.

In other words, our intention was in effect to operate a value-revealing (fifth-price) Vickrey auction - see Vickrey (1961) - but to do so under the guise of a form of auction many people are more familiar with. After taking part in two Practice Rounds, participants seemed comfortable with the procedure.

They were then given Record Sheets with space to record the details of the eight auctions that were to be run for real. At the start of Round 1, each participant in a Set 1 session was given a slip, set out like the one in Figure 3, which gave them an initial endowment of £3.00 cash plus a claim worth a further £12.00 if a yellow ball was drawn, but nothing if a green ball was drawn. Each participant was required to write this information in on the Record Sheet, together with the details of the claims being offered for sale, each worth £12.00 if the ball drawn at random turned out to be green, and worth nothing if it turned out to be yellow.

Each participant then had to 'instruct the agent' by writing his/her maximum willingness to pay on the slip and noting this amount on the Record Sheet, before handing the slip back to the experimenter. When all slips had been collected, the experimenter announced the price which was 5p above the fifth-highest bid, and participants wrote that amount into the last column of the Record Sheet. On each occasion, the experimenter reminded participants that those who had bid the purchase price or higher now owned one of the claims on offer, and would have the purchase price deducted; while those who had bid less than the purchase price were left with what they had started with.

The remaining rounds all operated in the same way. After the last round had been completed, a die was rolled to determine which round was to be played out for real. Then a bag was filled with the appropriate number of green and yellow balls, and one ball was picked out at random. Each participant then went in turn to an adjacent office where their slip for that round was consulted, and they were paid accordingly.

Experiment 1: Results

Consider first the framing effect. In each Set there were two pairs of questions - Questions 1 & 4, and 2 & 5 - intended to test for possible framing effects. In both Sets the null hypotheses are that $v_1 - v_4 = 0$ and $v_2 - v_5 = 0$, whereas the alternative hypotheses are that $v_1 - v_4 < 0$ and $v_2 - v_5 < 0$.

Of course, when considering the data we should allow for random error. Thus for Questions 1 and 4 we formulate our null hypothesis as "the number of cases where $v_1 - v_4 > 0$ is not significantly

different from the number of cases where $v_1 - v_4 < 0$", while our alternative hypothesis takes the form that "the number of cases where $v_1 - v_4 < 0$ is greater than the number of cases where $v_1 - v_4 > 0$ to an extent that is unlikely to have occurred by chance". And likewise for $v_2 - v_5$.

Table 3 classifies the 73 individuals' responses accordingly.

Table 3 Framing Effect (York Data)

	Difference	< 0	0	> 0
Set 1	$v_1 - v_4$	29	3	5
	$v_2 - v_5$	17	5	15
Set 2	$v_1 - v_4$	21	4	11
	$v_2 - v_5$	23	5	8

In all four pairs, the number of cases where the difference is less than zero exceeds the number of cases where the difference is greater than zero, which is in the direction consistent with the alternative hypothesis. However, in two of those pairs - Questions 2 and 5 in Set 1, and Questions 1 and 4 in Set 2 - the asymmetry is not sufficient to register as significant at the 5% level. By contrast, in the other two pairs the null hypothesis is decisively rejected in favour of the alternative hypothesis. Thus there appears to be evidence of a framing effect, but its strength is variable: we conjecture that it is strongest when the initial endowment involves a greater-than-50% chance of losing a relatively large sum, and that it becomes weaker as the probability of loss, and/or the magnitude of that loss, diminish. But however that may be, it appears that there is at least some predictable effect due to the way the initial endowment is framed: in all four cases, the average amount subjects were willing to pay was higher for the negatively framed endowment, suggesting that negative framing tends to make subjects more eager to cover the risky position.

Now consider the five pairs which were intended to test for any endowment effect when both sets of endowments were 'positively' framed, i.e. consider $v_3 + v_6$ and $v_1 + v_7$ in Set 1, and $v_3 + v_6$, $v_1 + v_7$ and $v_2 + v_8$ in Set 2. In all these cases, the null hypothesis is that the pair of bids sum to x, where x is the positive payoff contingent on G in one question and Y in the other. To allow for random error, we formulate the null hypothesis as "the number of cases where $v_i + v_j > x$ is not significantly different from the number of cases where $v_i + v_j < x$", while the alternative hypothesis takes the form that "the number of cases where $v_i + v_j < x$ is greater than the number of cases where $v_i + v_j > x$ to

an extent that is unlikely to have occurred by chance". Table 4 reports the results: in four of the five cases there is an asymmetry consistent with the endowment effect which allows us to reject the null hypothesis at the 5% level of significance.

Table 4 Effect for Positive Frame (York Data)

	Sum Total	< x	x	> x
Set 1	$v_1 + v_7$	24	1	12
	$v_3 + v_6$	15	1	21
Set 2	$v_1 + v_7$	24	1	11
	$v_2 + v_8$	30	0	6
	$v_3 + v_6$	26	3	7

The five other pairs of questions involving complementary state contingent claims were all cases where at least one of the endowments was negatively framed. The data from these pairs are reported in Table 5. In four of these five cases, there is no significant asymmetry. This could be interpreted as being consistent with the null hypothesis. But as suggested earlier in this Section, there is an alternative interpretation: namely, that both endowment effects and framing effects may be at work, exerting opposing influences that may to a large extent be cancelling each other out.

Table 5 Endowment Effect with at Least One Negatively framed Endowment (York Data)

	Sum Total	< x	x	> x
Set 1	$v_2 + v_8$	17	1	19
	$v_4 + v_7$	16	2	19
	$v_5 + v_8$	20	1	16
Set 2	$v_4 + v_7$	18	1	17
	$v_5 + v_8$	30	0	6

Overall, then, in cases where it was possible to state an alternative hypothesis in unambiguous fashion, there is evidence of both a framing effect and an endowment effect, although the strength of both effects appears to be somewhat variable. Different conjectures about

the reasons for this variability might be offered. One possibility is that the strength of the effect in each question is related in some way to the parameters of the question. Another possibility is that the variability is no more than one might expect in subsamples of 36 or 37 individuals.

In order to see whether the pattern of variability would be broadly reproduced with another subject pool, or whether the variability would be less apparent in larger sample sizes, we ran a second experiment at the University of Kiel. It is to this experiment that we now turn.

Experiment 2: Procedure

As mentioned earlier, the Kiel experiment involved presenting the same two sets of eight decision problems as in the York experiment, plus two additional problems for each subsample as described in Tables 1 and 2. As with Experiment 1, the Kiel sample was randomised between two subsamples, with 92 people presented with Set 1 questions and 90 people presented with Set 2.

Dispensing with the financial incentive mechanisms used at York allowed us to ask respondents for more information within each question. Specifically, for each problem in turn, respondents were asked not only to state the amount which they regarded as their 'best' estimate of what they would be prepared to pay for the asset being offered, but also to indicate the maximum and minimum they thought the asset would be worth to them.

It was hoped that there would be benefits from using larger subsamples and conducting the experiment on a larger scale. However, it was also necessary to change some aspects of the experimental procedure.

The most significant change involved the computation of individual cash endowments for Questions 6-10, i.e. amounts a to e in Figure 1 and 2. This was handled as follows. In the first part of the session, participants were given a sheet containing Questions 1-5, with all the necessary parameters written in, and were asked to work through them, in each case providing a minimum, maximum and best estimate of what they would be willing to pay for the asset being offered. Besides entering these values on the sheet, which was collected in, they made a record of their best estimates on a separate slip of paper which they kept with them. After all the first sheets had been collected, each participant was asked to calculate the individual amounts a to e on the basis of their

first five answers. Then they were given a second sheet containing Questions 6-10, and transferred the computed amounts from their slips of paper into the appropriate places on the second sheet. When this had been done, they were asked to state their minimum, maximum and best estimates of willingness to pay for each of the assets being offered on that sheet.

Experiment 2: Results

In the course of the procedure described above, a number of participants made arithmetical errors in their calculations, and such responses have been excluded from the results reported in this section. We have also excluded responses on two other grounds: either because the best estimate lay outside the min-max range; or because the stated value was strictly greater than the positive payoff offered by the asset being valued.

Let us first consider framing effects. The addition of Questions 9 and 10 increased the total number of framing effect comparisons to eight - four in each Set. The relevant data are presented in Table 6.

Table 6 Framing Effect (Kiel Data)

	Difference	< 0	0	> 0
Set 1	$v_1 - v_4$	25	12	28
	$v_2 - v_5$	39	8	24
	$v_6 - v_9$	25	10	32
	$v_{10} - v_5$	22	9	34
Set 2	$v_1 - v_4$	30	7	31
	$v_2 - v_5$	28	4	37
	$v_6 - v_9$	19	11	37
	$v_8 - v_{10}$	32	10	30

On this evidence, there appears to be no systematic framing effect of the kind observed to a significant extent in two out of the four comparisons in Experiment 1. In Table 6 the asymmetries, such they are, tend more often to be in the opposite direction to the one entailed by our framing hypothesis. Indeed, the only one of the eight asymmetries which is statistically

significant - the one involving Questions 6 and 9 in Set 2 - is one of the six cases where the asymmetries are in the 'wrong' direction.

We now turn to endowment effects. If, as the data above suggest, there are no significant framing effects, we need not restrict our attention to pairs of questions involving only positively framed endowments. Table 7 therefore reports the breakdowns for <u>all</u> pairs of questions where there are complementary claims on G and Y.

Table 7 Endowment Effect (Kiel Data)

	Sum Total	$<x$	x	$>x$
Set 1	$v_1 + v_7$	52	2	11
	$v_2 + v_{10}$	58	2	5
	$v_3 + v_6$	58	2	7
	$v_2 + v_8$	66	1	4
	$v_3 + v_9$	57	4	6
	$v_4 + v_7$	60	2	4
	$v_5 + v_{10}$	58	2	5
	$v_5 + v_8$	60	1	6
Set 2	$v_1 + v_7$	50	1	16
	$v_2 + v_8$	60	4	7
	$v_3 + v_6$	56	1	14
	$v_2 + v_{10}$	59	2	9
	$v_3 + v_9$	61	1	7
	$v_4 + v_7$	50	4	14
	$v_5 + v_8$	65	1	3
	$v_5 + v_{10}$	65	0	3

The first three rows show cases where both endowments were positively framed; the next four rows report cases where one endowment was negatively framed; and the last row is the case where both endowments were presented in the negative frame. However, in contrast with the York data, these distinctions make little difference: in every Question in both Sets, those subjects stating $v_i + v_j < x$ outnumber those giving $v_i + v_j > x$ to an extent which is always statistically significant at the 1% level.

The robustness of this result can be demonstrated by using the maximum buying prices stated by the subjects. Adding the maximum buying prices for two complementary claims should definitely yield a sum larger than x. However, even these sums of maximum buying prices tended to be significantly smaller than x in all but two cases ($v_1 + v_7$ and $v_4 + v_7$ in Set 2).

3. Discussion

Overall, then, the two experiments both produced behaviour which runs contrary to standard theory - but in rather different ways. In Kiel there were no framing effects, but what appear to be very strong endowment effects; whereas in York there was a good deal more evidence of framing effects, while the endowment effects, although systematic, seemed to be somewhat weaker. So although both experiments provide clear evidence of non-standard behaviour, the obvious and important question to ask is: why was there such a marked difference between the patterns of results emerging from the two experiments? Our initial conjecture is as follows.

Because Experiment 2 involved hypothetical decisions, the various 'endowments' received little attention. The cash endowments were never going to be received, nor were the endowed assets or liabilities ever going to result in additions to or deductions from those notional cash amounts, so we conjecture that there was little motivation for most participants to 'think themselves into' the position of actually having those endowments in addition to the wealth they possessed before the experiment began. Moreover, since they were being asked to provide not one but three responses - minimum and maximum as well as best estimates of willingness to pay - it would not be surprising if the information about endowments had little force, while attention and concentration was drawn instead towards the risky asset on offer.

If this conjecture is correct, and participants do not view the purchase of an asset that pays off under one state in the context of cancelling out a real liability under that state, or balancing against a real asset which pays off only under the other state, it would be natural for them to evaluate the assets in all cases simply as additional unhedged risks to be paid for out of their current actual wealth. If they treat each problem in turn in this way, and give risk averse responses

by valuing each asset independently at something below its expected value, the result will be much as we see in Tables 6 and 7: the framing of the notional endowment will have little systematic effect, and v_i and v_j will typically sum to less than x by the amount of the two (independent) risk premiums.

In short, what appears at first sight to be massive evidence of endowment effects in Table 7 may really reflect the fact that the hypothetical nature of the decisions in Experiment 2 allowed our 'induced endowments' to be largely ignored, so that each asset was being evaluated from the same standpoint, namely, the individual's actual current wealth.

According to this account, the main reason for the contrast between Experiments 1 and 2 is the difference in incentives. In Experiment 1, each endowment was very real, in the sense that it determined the entire payment for anyone who did not buy a unit of the asset offered in the round played out at the end, and was often still an important component for anyone who did buy a unit of the asset. In these circumstances, we suggest that endowments receive much more attention, so that the framing of those endowments may have a real impact, especially when it involves a greater-than-even probability of having to pay back a substantial portion of the initial cash endowment.

If our conjecture is correct, and participants really did pay attention to the endowments in Experiment 1, then the asymmetries observed in four out of the five positively framed pairs cannot be accounted for along the lines of the explanation for Table 7. Instead, it is plausible that genuine endowment effects as well as genuine framing effects exist in the context of risky assets. By way of evidence to support this proposition, we note that in another study of ours (Weber, Loomes, Keppe and Meyer-Delius, 1994), a double auction market - normally such a well-behaved experimental environment - shows clear manifestations of framing effects when initial endowments are negatively framed. We suggest that the implications of such effects for the operation of both experimental and non-experimental asset markets merit further consideration and investigation.

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